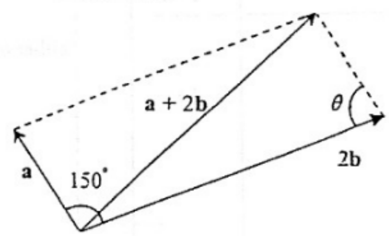
| Solution | Marks | Remarks |
|---|---|---|
| 1. $\frac{d}{dx}[\frac{\sin(2x+1)}{x}] = \frac{x\frac{d}{dx}\sin(2x+1) - \sin(2x+1)\frac{d}{dx}(x)}{x^2}$ | 1M | For quotient / product rule |
| $= \frac{x[2\cos(2x+1)] - \sin(2x+1)}{x^2}$ | 1M | $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$ (accept without $\frac{du}{dx}$) |
| $= \frac{2x\cos(2x+1) - \sin(2x+1)}{x^2}$ | 1A | |
| | (3) | |
| 2. $\cos^2 x - \cos^2 y$ | | Marking criteria:<br>First trig formulas 1A<br>Second trig formula 1M<br>Completed proof 1 |
| $= \frac{1}{2}(1+\cos 2x) - \frac{1}{2}(1+\cos 2y)$ | 1A | |
| $= \frac{1}{2}(\cos 2x - \cos 2y)$ | | |
| $= \frac{1}{2}(-2\sin\frac{2x+2y}{2}\sin\frac{2x-2y}{2})$ | 1M | |
| $= -\sin(x+y)\sin(x-y)$ | 1 | |
| **Alternative Solution (1)** | | |
| $\cos^2 x - \cos^2 y$ | | |
| $= (\cos x + \cos y)(\cos x - \cos y)$ | | |
| $= \left(2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)\left(-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right)$ | 1A | |
| $= -(2\sin\frac{x+y}{2}\cos\frac{x+y}{2})(2\sin\frac{x-y}{2}\cos\frac{x-y}{2})$ | | |
| $= -\sin(x+y)\sin(x-y)$ | 1M + 1 | |
| **Alternative Solution (2)** | | |
| $-\sin(x+y)\sin(x-y)$ | | |
| $= -\frac{1}{2}\{\cos[x+y-(x-y)] - \cos[x+y+(x-y)]\}$ | 1A | |
| $= -\frac{1}{2}(\cos 2y - \cos 2x)$ | | |
| $= -\frac{1}{2}[(2\cos^2 y - 1) - (2\cos^2 x - 1)]$ | 1M | |
| $= \cos^2 x - \cos^2 y$ | 1 | |

| Solution | Marks | Remarks |
|---|---|---|
| **Alternative Solution (3)** | | |
| $-\sin(x+y)\sin(x-y)$ | | |
| $=-(\sin x\cos y+\cos x\sin y)(\sin x\cos y-\cos x\sin y)$ | 1A | |
| $=-\sin^2 x\cos^2 y+\cos^2 x\sin^2 y$ | | |
| $=-(1-\cos^2 x)\cos^2 y+\cos^2 x(1-\cos^2 y)$ | 1M | |
| $=-\cos^2 y+\cos^2 x+\cos^2 x-\cos^2 x\cos^2 y$ | | |
| $=\cos^2 x-\cos^2 y$ | 1 | |
| $\therefore\ \cos^2 x-\cos^2 y\equiv-\sin(x+y)\sin(x-y)$ | | |
| | (3) | |
| 3. $(1-2x+3x^2)^n$ | | |
| $=[1+(-2x+3x^2)]^n$ | 1M | Accept $[(1-2x)+3x^2]^n$ etc. |
| $=1+{}_nC_1(-2x+3x^2)+{}_nC_2(-2x+3x^2)^2+\cdots$ | 1M | For binomial expansion |
| $=1+n(-2x+3x^2)+\frac{n(n-1)}{2}(4x^2+\cdots)+\cdots$ | | |
| $=1-2nx+(2n^2+n)x^2+\cdots$ ------------------------------------------(*) | | (pp−1) if dots were omitted in all cases |
| $-2n=-10$ | 1M | either one |
| $n=5$ | 1A | |
| $k=2n^2+n$ | | Award ONLY when (*) is correct |
| $=2(5)^2+5$ | | |
| $=55$ | 1A | |
| | (5) | |
| 4. $kx^2+x+k>0$ | | |
| $\Delta=1-4k^2$ | | |
| For $kx^2+x+k>0$ for all real values of $x$, | | |
| $k>0$ and $1-4k^2<0$ | 1A+1M | 1A for $k>0$, 1M for $\Delta<0$ |
| $k>0$ and $k^2>\frac{1}{4}$ | | |
| $k>0$ and $(k>\frac{1}{2}$ or $k<-\frac{1}{2})$ | 1M | For solving a quadratic inequality correctly |
| $\therefore\ k>\frac{1}{2}$ | 1A | |
| | (4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 5. $\begin{cases} y = x^2 \\ y = x + k \end{cases}$ | | |
| $x^2 = x + k$ | 1M | |
| $x^2 - x - k = 0$ | | |
| Let $x_1$ and $x_2$ be the $x$-coordinates of $P$ and $Q$. | | |
| $x_1 + x_2 = 1$ | 1M | |
| Let $(a, b)$ be a point on the locus. | | |
| $a = \frac{x_1 + x_2}{2}$ $= \frac{1}{2}$ | 1M | |
| OR | | |
| $x = \frac{1 \pm \sqrt{1+4k}}{2}$ | 1M | |
| $a = \frac{1}{2}(x_1 + x_2)$ | | |
| $= \frac{1}{2}\left(\frac{1+\sqrt{1+4k}}{2} + \frac{1-\sqrt{1+4k}}{2}\right)$ | 1M | |
| $= \frac{1}{2}$ | | |
| Alternative Solution | | |
| $\begin{cases} y = x^2 \\ y = x + k \end{cases}$ | | |
| $y = (y-k)^2$ | 1M | |
| $y^2 - (2k+1)y + k^2 = 0$ | | |
| Let $y_1$ and $y_2$ be the $y$-coordinates of $P$ and $Q$. | | |
| $y_1 + y_2 = 2k + 1$ | 1M | |
| Let $(a, b)$ be a point on the locus. | | |
| $b = \frac{2k+1}{2}$ | 1M | |
| $a = b - k = \frac{1}{2}$ | | |
| $\therefore$ the equation of $L$ is $x = \frac{1}{2}$. | 1A | OR $\begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} + k \end{cases}$ |
| | (4) | |

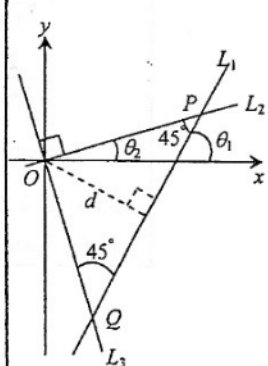
| Solution | Marks | Remarks |
|---|---|---|
| 6. $x\|x\|+5x+6=0$ | | |
| Consider the following cases : (1) $x \geq 0$ | | |
| (2) $x < 0$ | | |
| Case (1) : The equation becomes | | |
| $x(x)+5x+6=0$ | 1A | |
| $x^2+5x+6=0$ | | |
| $x=-3$ or $x=-2$ | | |
| Since $x \geq 0$, the two values are rejected. | 1M | For checking |
| **OR** | | |
| $x^2+5x+6=0$ | | |
| Since $x \geq 0$, so this equation has no solution. | 1M | either one |
| Case (2) : The equation becomes | | |
| $x(-x)+5x+6=0$ | 1A | |
| $x^2-5x-6=0$ | | |
| $x=-1$ or $x=6$ | | |
| Since $x<0$, $x=-1$ | | |
| Combining the two cases, $x=-1$. | 1A | |
| **Alternative solution** | | |
| $x\|x\|+5x+6=0$ | | |
| $x(x)+5x+6=0$ or $x(-x)+5x+6=0$ | 2A | For BOTH correct |
| $x^2+5x+6=0$ $x^2-5x-6=0$ | | |
| $x=-3$ or $x=-2$ $x=-1$ or $x=6$ | | |
| **OR** | | |
| $x\|x\|=-(5x+6)$ | | |
| $x^4=25x^2+60x+36$ | 1A | |
| $x^4-25x^2-60x-36=0$ | | |
| $(x+1)(x^3-x^2-24x-36)=0$ | | |
| $(x+1)(x-6)(x^2+5x+6)=0$ | 1A | For any two correct linear factors |
| $(x+1)(x-6)(x+2)(x+3)=0$ | | |
| $x=-1$ or $x=6$ or $x=-2$ or $x=-3$ | | |
| Put $x=-3$, LHS $=-18 \neq 0$ (rejected) | 1M | For checking |
| Put $x=-2$, LHS $=-8 \neq 0$ (rejected) | | |
| Put $x=-1$, LHS $=0=$ RHS | | |
| Put $x=6$, LHS $=72 \neq 0$ (rejected) | | |
| $\therefore x=-1$ | 1A | |
| | (4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 7. (a) $\mathbf{a}\cdot\mathbf{b}$ | | |
| $=\sqrt{3}\,(2)\cos 150^\circ$ | 1A | |
| $=-3$ | 1A | |
| (b) $\lvert\mathbf{a}+2\mathbf{b}\rvert^2=(\mathbf{a}+2\mathbf{b})\cdot(\mathbf{a}+2\mathbf{b})$ | 1M | |
| $=\mathbf{a}\cdot\mathbf{a}+4\,\mathbf{a}\cdot\mathbf{b}+4\,\mathbf{b}\cdot\mathbf{b}$ | | |
| $=(\sqrt{3})^2+4(-3)+4(2)^2$ | 1A | For $\mathbf{a}\cdot\mathbf{a}=\sqrt{3}^2$ and $\mathbf{b}\cdot\mathbf{b}=2^2$ |
| $=7$ | | |
| $\lvert\mathbf{a}+2\mathbf{b}\rvert=\sqrt{7}$ | 1A | |
| Alternative solution<br><br> | | |
| $\lvert\mathbf{a}+2\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2+\lvert 2\mathbf{b}\rvert^2-2\lvert\mathbf{a}\rvert\lvert 2\mathbf{b}\rvert\cos\theta$ | | |
| $=(\sqrt{3})^2+4^2-2(\sqrt{3})(4)\cos 30^\circ$ | 1M+1A | 1M for cosine formula<br>1A for $\lvert\mathbf{a}\rvert=\sqrt{3}$ and $\lvert 2\mathbf{b}\rvert=4$ |
| $=7$ | | |
| $\lvert\mathbf{a}+2\mathbf{b}\rvert=\sqrt{7}$ | 1A | |
| | | Omitting vector sign / dot sign in most cases, or using the expression $\mathbf{v}^2$ or $\vec{v}^2$, (pp−1) |
| | (5) | |

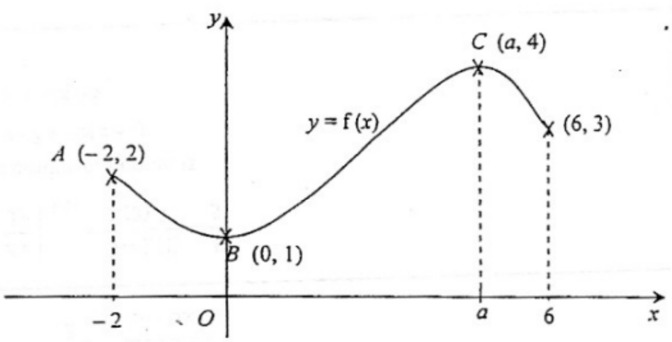
| Solution | Marks | Remarks |
|---|---|---|
| 8. For $n=1$, | | |
| $n^3-n+3=1^3-1+3=3$, which is divisible by 3. | 1 | |
| $\therefore$ the statement is true for $n=1$. | | |
| Assume $k^3-k+3$ is divisible by 3, where $k$ is a positive integer. | 1 | |
| (OR Assume $k^3-k+3=3m$, where $k$ and $m$ are positive integers.) | | |
| $(k+1)^3-(k+1)+3$ | | |
| $=k^3+3k^2+3k+1-(k+1)+3$ | | |
| $=(k^3-k+3)+3k^2+3k$ | 1M | For using the assumption |
| $=3(m+k^2+k)$ | 1 | |
| $\therefore (k+1)^3-(k+1)+3$ is also divisible by 3. | | |
| The statement is also true for $n=k+1$ if it is true for $n=k$. | | |
| By the principle of mathematical induction, the statement is true for all positive integers $n$. | 1 | Not awarded if anyone of the above marks was withheld. |
| Alternative solution | | |
| $n^3-n+3$ | | |
| $=n(n^2-1)+3$ | | |
| $=(n-1)(n)(n+1)+3$ | 2A | |
| $(n-1)(n)(n+1)$ is the product of 3 consecutive integers and hence is divisible by 3. | 2A | |
| As 3 is also divisible by 3, | | |
| $\therefore n^3-n+3$ is divisible by 3 for all positive integers $n$. | 1 | |
| | (5) | |

| Solution | Marks | Remarks |
|---|---|---|
| 9. (a) $\cos\theta-\sqrt{3}\sin\theta=r\cos(\theta+\alpha)$ | | |
| $=r(\cos\theta\cos\alpha-\sin\theta\sin\alpha)$ | 1A | |
| $\begin{cases} r\cos\alpha=1 & \ldots\ldots (1) \\ r\sin\alpha=\sqrt{3} & \ldots\ldots (2) \end{cases}$ | 1M | |
| $r=2$ <br> $\alpha=60^\circ$ | | |
| $\therefore\ \cos\theta-\sqrt{3}\sin\theta=2\cos(\theta+60^\circ)$ | 1A | Accept $2\cos(\theta+\frac{\pi}{3})$ |
| **OR** | | |
| $\cos\theta-\sqrt{3}\sin\theta=2(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta)$ | 1M | For the form of $r(\frac{a}{r}\cos\theta-\frac{b}{r}\sin\theta)$ |
| $=2(\cos 60^\circ\cos\theta-\sin 60^\circ\sin\theta)$ | 1A | |
| $=2\cos(\theta+60^\circ)$ | 1A | Accept $2\cos(\theta+\frac{\pi}{3})$ |
| (b) $\cos 2x-\sqrt{3}\sin 2x=1$ | | |
| $2\cos(2x+60^\circ)=1$ | 1M | using (a) |
| $\cos(2x+60^\circ)=\frac{1}{2}$ | | |
| $2x+60^\circ=360n^\circ\pm 60^\circ$, where $n$ is an integer. | 1M | General solution of $\cos\phi=k$ |
| $x=180n^\circ$ or $x=180n^\circ-60^\circ$ | 1A | Accept radian (pp−1) for mixing degree and radian measures in answer |
| Alternative Solution (1) | | |
| $\cos 2x-\sqrt{3}\sin 2x=1$ | | |
| $2\sin(2x+150^\circ)=1$ | 1A | |
| $\sin(2x+150^\circ)=\frac{1}{2}$ | | |
| $2x+150^\circ=180n^\circ+(-1)^n(30^\circ)$, where $n$ is an integer. | 1M | |
| $x=90n^\circ+(-1)^n(15^\circ)-75^\circ$ | 1A | |
| Alternative Solution (2) | | |
| $\cos 2x-\sqrt{3}\sin 2x=1$ | | |
| $1-2\sin^2 x-\sqrt{3}(2\sin x\cos x)=1$ | 1A | |
| $2\sin x(\sin x+\sqrt{3}\cos x)=0$ | | |
| $\sin x=0$ or $\tan x=-\sqrt{3}$ | | |
| $x=180n^\circ$ or $x=180n^\circ-60^\circ$ | 1M+1A | 1M for either one |
| | (6) | |

| Solution | Marks | Remarks |
|---|---|---|
| 10. $y = \int (3+2\cos 2x)\,dx$ | 1M | (pp-1) for $dx$ omitted<br>1M awarded even if $y$ was omitted |
| $= 3x + \sin 2x + c$ , where $c$ is a constant. | 1M+1A | 1M for $\int \cos u \, d\, u = \sin u$<br>1A awarded even if $c$ was omitted |
| Put $x = \frac{\pi}{4}$, $y = \frac{3\pi}{4}$ : | | |
| $\frac{3\pi}{4} = 3(\frac{\pi}{4}) + \sin 2(\frac{\pi}{4}) + c$ | 1M | |
| $c = -1$ | | |
| $\therefore$ the equation of the curve is $y = 3x + \sin 2x - 1$. | 1A | |
| | (5) | |
| 11. (a) Slope of $L_1 = 2$ | | |
| Let $m$ be the slopes of $L_2$ and $L_3$. | | |
| $\left\lvert \frac{m-2}{1+2m} \right\rvert = \tan 45°$ | 1M+1A | 1M for LHS<br>Accept omitting absolute sign |
| $\frac{m-2}{1+2m} = \pm 1$ | | |
| $m = -3$ or $m = \frac{1}{3}$ | | |
| $\therefore$ the equation of $L_2$ and $L_3$ are $y = \frac{1}{3}x$ and $y = -3x$. | 1A | |
| Alternative solution<br>Slope of $L_1 = 2$<br>Let $L_1$ and $L_2$ make an angle $\theta_1$ and $\theta_2$ with the $x$-axis respectively. | | |
| $\theta_2 = \theta_1 - 45°$ | 1A | |
| $\tan \theta_2 = \tan(\theta_1 - 45°)$ | | |
| $= \frac{\tan \theta_1 - \tan 45°}{1 + \tan \theta_1 \tan 45°}$ | 1M | |
| $= \frac{2-1}{1+2(1)} = \frac{1}{3}$ | | |
| $\therefore$ the equation of $L_2$ is $y = \frac{1}{3}x$.<br>Since $L_3 \perp L_2$, slope of $L_3 = -3$,<br>$\therefore$ the equation of $L_3$ is $y = -3x$. | 1A | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| (b) $\begin{cases} y = 2x-5 \\ y = \frac{1}{3}x \end{cases}$ | | |
| $x=3$, $y=1$ $\quad\therefore$ $L_1$ and $L_2$ intersect at $P(3, 1)$. | 1M | Marking Criteria:<br>1M for finding a point of intersection<br>1M for the correct method of finding the area<br>1A for the answer |
| $\begin{cases} y = 2x-5 \\ y = -3x \end{cases}$ | | |
| $x=1$, $y=-3$ $\quad\therefore$ $L_1$ and $L_3$ intersect at $Q(1,-3)$. | | |
| Area $= \frac{1}{2}\begin{vmatrix} 0 & 0 \\ 1 & -3 \\ 3 & 1 \\ 0 & 0 \end{vmatrix}$ | | |
| $= \frac{1}{2}[1-(-9)]$ | 1M | |
| $= 5$ | 1A | |
| **Alternative solution (1)** | | |
| $L_1$ and $L_2$ intersect at $P(3, 1)$. | 1M | Same as above |
| $OP = \sqrt{3^2+1^2} = \sqrt{10}$ and $OQ = \sqrt{10}$ | | |
| Area $= \frac{1}{2}(OP)(OQ)$ | | |
| $= \frac{1}{2}(\sqrt{10})(\sqrt{10})$ | 1M | |
| $= 5$ | 1A | |
| **Alternative solution (2)** | | |
| $L_1$ and $L_2$ intersect at $P(3, 1)$. | 1M | Same as above |
| $L_1$ and $L_3$ intersect at $Q(1,-3)$. | | |
| $OP = \sqrt{3^2+1^2} = \sqrt{10}$, $PQ = \sqrt{(3-1)^2+(1+3)^2} = \sqrt{20}$ | | |
| Area $= \frac{1}{2}(\sqrt{10})(\sqrt{20})\sin 45°$ | 1M | |
| $= 5$ | 1A | |
| **Alternative solution (3)** | |  |
| The distance from $O$ to $L_1$ is | | |
| $d = \left\vert -\frac{5}{\sqrt{5}} \right\vert = \sqrt{5}$ | 1M | |
| (The angles are $45°$,) the length of (the hypotenuse) $PQ$ is | | |
| $2d = 2\sqrt{5}$ | 1M | |
| The area of $\Delta OPQ$ is | | |
| $\frac{1}{2}(\sqrt{5})(2\sqrt{5}) = 5$ | 1A | |
| | (6) | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| 12. (a) $x^2-xy+y^2=7$ | | |
| $2x-(x\frac{dy}{dx}+y)+2y\frac{dy}{dx}=0$ | 1A+1A | 1A for $\frac{d}{dx}(xy)$, 1A for the other terms |
| $\frac{dy}{dx}=\frac{y-2x}{2y-x}$ | 1A | |
| Alternative solution | | |
| $x^2-xy+y^2=7$ | | |
| $y^2-xy+x^2-7=0$ | | |
| $y=\frac{x\pm\sqrt{x^2-4(x^2-7)}}{2}$ | | |
| $=\frac{x}{2}\pm\frac{\sqrt{28-3x^2}}{2}$ | 1A | |
| $\frac{dy}{dx}=\frac{1}{2}\pm\frac{1}{2}(\frac{-6x}{2\sqrt{28-3x^2}})$ | 1A | |
| $=\frac{1}{2}\pm\frac{3x}{2\sqrt{28-3x^2}}$ | 1A | |
| (b) $\left.\frac{dy}{dx}\right\|_{(1,3)}=\frac{3-2(1)}{2(3)-1}=\frac{1}{5}$ | | |
| Equation of normal is | 1M | |
| $y-3=-5(x-1)$ | | |
| $y=-5x+8$ | 1A | |
| | (5) | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| 13. (a) The curve attains a maximum at $x = a$. | 1A | Withhold 1A if answered |
| The curve attains a minimum at $x = 0$. | 1A | in coordinates form |
| (b) (i) $\int_0^a f'(x)\,dx$ | | |
| $= [f(x)]_0^a$ | | |
| $= f(a) - f(0)$ | 1A | |
| $= f(a) - 1$ (Since $f(0) = 1$) | | |
| Since the area of $R_1 = 3$, $f(a) = 4$. | 1A | |
| (ii) $[f(x)]_0^6$ = Area of $R_1$ − Area of $R_2$ | | |
| $f(6) - f(0) = 3 - 1$ | | |
| $f(6) = 3$ | 1A | |
|  | 1A | Shape |
| | 1M | For points $A$, $B$ and $C$ |
| | (7) | |

| | Solution | Marks | Remarks |
|---|---|---|---|
| 14. (a) (i) | The centre of $J$ is $(0, 0)$ and the radius is $r$. | | |
| | Distance from $(0, 0)$ to $L = r$ | | |
| | $\left\lvert \dfrac{m(0)-0+c}{\sqrt{m^2+1}} \right\rvert = r$ | 1M+1M | 1M for distance formula<br>1M for $d = r$<br>Accept omitting absolute sign |
| | $\dfrac{c^2}{m^2+1} = r^2$ | | |
| | $c^2 = r^2(m^2+1)$ | 1 | |
| | <u>Alternative solution</u> | | |
| | $\begin{cases} x^2+y^2=r^2 \\ y=mx+c \end{cases}$ | | |
| | $x^2+(mx+c)^2=r^2$ | 1M | For elimination of $x$ (or $y$) |
| | $(m^2+1)x^2+2mcx+c^2-r^2=0$ | | |
| | $\Delta=(2mc)^2-4(m^2+1)(c^2-r^2)=0$ | 1M | |
| | $m^2c^2-(m^2c^2-m^2r^2+c^2-r^2)=0$ | | |
| | $c^2=r^2(m^2+1)$ | 1 | |
| (ii) | Since $L$ passes through $(h, k)$, | | |
| | $k = mh + c$ | | |
| | $c = k - mh$ | | |
| | Substitute into $c^2 = r^2(m^2+1)$: | | |
| | $(k-mh)^2 = r^2(m^2+1)$ ..... (*) | 1 | |
| | | (4) | |
| (b) (i) | Equation of $PR$: | | |
| | $y+5=\dfrac{4+5}{7+5}(x+5)$ | 1M (with next lines) | |
| | $3x-4y-5=0$ | | |
| | radius of $J = \left\lvert \dfrac{3(0)-4(0)-5}{\sqrt{3^2+4^2}} \right\rvert$ | | |
| | $= 1$ | 1A | |
| | <u>Alternative solution</u> | | |
| | Equation of $PR$: | | |
| | $y+5=\dfrac{4+5}{7+5}(x+5)$ | 1M (with next lines) | |
| | $y=\dfrac{3}{4}x-\dfrac{5}{4}$ | | |
| | Using (a) (i), | | |
| | $(-\dfrac{5}{4})^2=r^2[(\dfrac{3}{4})^2+1]$ | | |
| | $r^2=1$ | | |
| | $\therefore$ the radius of $J$ is 1. | 1A | |

| Solution | Marks | Remarks |
|---|---|---|
| (ii) Let $m$ be the slope of $PQ$. | | |
| Put $r=1$, $h=7$, $k=4$ into the result of (a)(ii) : | | |
| $(4-7m)^2=m^2+1$ | 1M | Withhold 1M if $(h,k)\neq(7,4)$ |
| $48m^2-56m+15=0$ | | |
| $m=\frac{3}{4}$ (rejected) or $m=\frac{5}{12}$ | | |
| $\therefore$ the slope of $PQ$ is $\frac{5}{12}$. | 1A | |
| **Alternative solution** | | |
| Slope of $PR=\frac{4+5}{7+5}=\frac{3}{4}$ | | |
| Slope of $OP=\frac{4}{7}$ | | |
| Let the slope of $PQ$ be $m$. | | |
| $OP$ bisects $\angle QPR$. | | |
| $\dfrac{\frac{3}{4}-\frac{4}{7}}{1+(\frac{3}{4})(\frac{4}{7})}=\dfrac{\frac{4}{7}-m}{1+(\frac{4}{7})(m)}$ | 1M | |
| $\frac{1}{8}=\frac{4-7m}{7+4m}$ | | |
| $m=\frac{5}{12}$ | 1A | |
| (iii) Put $r=1$, $h=-5$, $k=-5$ into (*) : | | |
| $(-5+5m)^2=m^2+1$ | 1M | Withhold 1M if $(h,k)\neq(-5,-5)$ |
| $24m^2-50m+24=0$ | | |
| $m=\frac{3}{4}$ (rejected) or $m=\frac{4}{3}$ | | |
| $\therefore$ the slope of $QR$ is $\frac{4}{3}$. | | |
| **Alternative solution** | | |
| Slope of $PR=\frac{3}{4}$ | | |
| Slope of $OR=1$ | | |
| Let the slope of $QR$ be $n$. | | |
| $\dfrac{1-\frac{3}{4}}{1+\frac{3}{4}}=\dfrac{n-1}{1+n}$ | 1M | |
| $n=\frac{4}{3}$ | | |

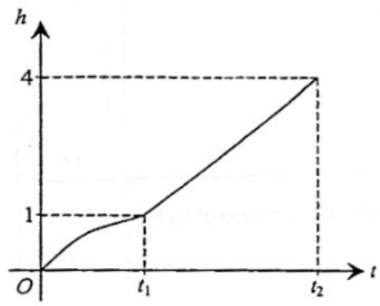
| Solution | Marks | Remarks |
|---|---|---|
| The equation of $PQ$ is<br>$y-4=\frac{5}{12}(x-7)$<br>$5x-12y+13=0$ (1)<br>The equation of $QR$ is<br>$y+5=\frac{4}{3}(x+5)$<br>$4x-3y+5=0$ (2) | 1M | For finding equations of $PQ$ and $QR$ |
| (1)×4−(2)×5 :<br>$-33y+27=0$<br>$y=\frac{9}{11}$<br>Substitute $y=\frac{9}{11}$ into (1), $x=\frac{-7}{11}$. | 1M | For solving (1) and (2) |
| $\therefore$ the coordinates of $Q$ are $(\frac{-7}{11},\frac{9}{11})$. | 1A | |
| | (8) | |

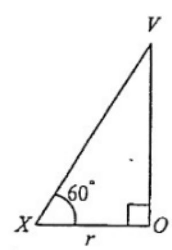
| Solution | Marks | Remarks |
|---|---|---|
| 15. (a) $l = PS + QS + NS$ | | |
| $= 2(\sqrt{1^2 + x^2 + (\sqrt{2})^2}) + 2 - x$ | | |
| $= 2\sqrt{x^2+3} + 2 - x$ | 1A | |
| $\frac{dl}{dx} = 2(\frac{2x}{2\sqrt{x^2+3}}) - 1$ | | |
| $= \frac{2x}{\sqrt{x^2+3}} - 1$ | 1 | |
| | (2) | |
| (b) (i) $\frac{dl}{dx} = 0$ | | |
| $\frac{2x}{\sqrt{x^2+3}} - 1 = 0$ | 1M | |
| $4x^2 = x^2 + 3$ | | |
| $x = -1$ (rejected) or $x = 1$ | 1A | |
| (table below) | 1M | $\frac{d^2 l}{dx^2} = \frac{6}{(x^2+3)^{\frac{3}{2}}}$ ; $\left.\frac{d^2 l}{dx^2}\right\|_{x=1} = \frac{3}{4} > 0$ |
| As $l$ has only one turning point, it attains the least value at $x = 1$. | 1A | |
| (ii) The greatest value of $l$ occurs at one of the end-points. | | |
| At $x = 0$, $l = 2\sqrt{3} + 2$ $(\approx 5.46)$ | 1M | |
| At $x = \frac{3}{2}$, $l = \sqrt{21} + \frac{1}{2}$ $(\approx 5.08) < 2\sqrt{3} + 2$ | | |
| $\therefore \ell$ attains the greatest value at $x = 0$. | 1A | |
| | (6) | |
| (c) (i) $E = \frac{k}{(2-x)^2} + \frac{2k}{x^2+3}$ | 1A | OR $E = \frac{k(3x^2 - 8x + 11)}{(2-x)^2(x+3)}$ Accept $E = \frac{k_1}{(2-x)^2} + \frac{2k_2}{x^2+3}$ |

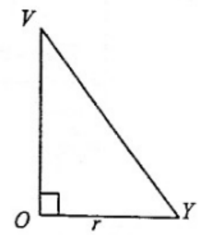
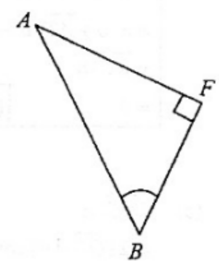
| $0 \le x < 1$ | $x = 1$ | $1 < x \le \frac{3}{2}$ |
|---|---|---|
| $\frac{dl}{dx} < 0$ | $\frac{dl}{dx} = 0$ | $\frac{dl}{dx} > 0$ |

| Solution | Marks | Remarks |
|---|---|---|
| (ii) From (b), $I$ attains the least value at $x=1$. | | |
| At $x=1$, $E=\frac{k}{1}+\frac{2k}{4}$ | 1M | |
| $=\frac{3k}{2}$ | | |
| At $x=\frac{3}{2}$, $E=\frac{8k}{21}+4k>\frac{3k}{2}$ | 1M | Accept other counter examples: e.g. $E(1.1)\approx 1.71k>\frac{3k}{2}$. |
| $\therefore$ $E$ will not attain its greatest value at $x=1$. | | |
| The student is incorrect. | 1A | |
| **Alternative solution** | | |
| $\frac{dE}{dx}=\frac{-2k(-1)}{(2-x)^3}+\frac{-2k(2x)}{(x^2+3)^2}$ | 1M | |
| $=\frac{2k}{(2-x)^3}-\frac{4kx}{(x^2+3)^2}$ | | |
| From (b), $I$ attains the least value at $x=1$. | | |
| At $x=1$, $\frac{dE}{dx}=2k-\frac{4k}{16}$ | 1M | OR solving $\frac{dE}{dx}=0$ successfully |
| $=\frac{7k}{4}>0$ | | Accept $\frac{dE}{dx}=\frac{7k}{4}\neq 0$ |
| $E$ is increasing at $x=1$ | 1A | |
| $\therefore$ $E$ will not attain its greatest value at $x=1$. The student is incorrect. | | |
| | (4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 16. (a) (i) $y=\frac{1}{3}x^2-\frac{4}{3}x+1$ | | |
| $x^2-4x+3-3y=0$ | | |
| $x=\frac{4\pm\sqrt{16-4(3-3y)}}{2}$ | | |
| $=2\pm\sqrt{3y+1}$ | | |
| As $0\le x\le 1$, the equation of $C_1$ is $x=2-\sqrt{3y+1}$. | 1 | |
| (ii) The equation of $C_2$ is $x=2+\sqrt{3y+1}$. | 1A | |
| | (2) | |
| (b) (i) (1) $V=\pi\int_0^h[(2+\sqrt{3y+1})^2-(2-\sqrt{3y+1})^2]\,\mathrm{d}y$ | 1M+1A | 1M for $V=\pi\int_a^b x^2\,\mathrm{d}y$ |
| $=\pi\int_0^h 8\sqrt{3y+1}\,\mathrm{d}y$ | 1A | |
| $=8\pi[\frac{2}{3}(\frac{1}{3})(3y+1)^{\frac{3}{2}}]_0^h$ | | |
| $=\frac{16\pi}{9}[(3h+1)^{\frac{3}{2}}-1]$ | 1 | |
| (2) $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{16\pi}{9}(\frac{3}{2})(3h+1)^{\frac{1}{2}}(3\frac{\mathrm{d}h}{\mathrm{d}t})$ | 1M | for $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\cdot\frac{\mathrm{d}h}{\mathrm{d}t}$ |
| $=8\pi(3h+1)^{\frac{1}{2}}\frac{\mathrm{d}h}{\mathrm{d}t}$ | | |
| Put $\frac{\mathrm{d}V}{\mathrm{d}t}=8\pi$: | | |
| $8\pi=8\pi(3h+1)^{\frac{1}{2}}(\frac{\mathrm{d}h}{\mathrm{d}t})$ | | |
| $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{(3h+1)^{\frac{1}{2}}}$. | 1A | |
| (ii) When $h=1$, $V=\frac{112\pi}{9}$ | | |
| When $1<h<4$, $V=\frac{112\pi}{9}+\pi(3)^2(h-1)$ | 1M | 1M for $V=$ volume of lower part $+\pi(3)^2(h-1)$ |
| $=9\pi h+\frac{31\pi}{9}$ | | |
| $\frac{\mathrm{d}V}{\mathrm{d}t}=9\pi\frac{dh}{dt}$ | | |
| $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{8}{9}$ | 1A | |

| Solution | Marks | Remarks |
|---|---|---|
| Alternative Solution | | |
| $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{\frac{\mathrm{d}V}{\mathrm{d}t}}{\begin{pmatrix}\text{cross-sectional area}\\ \text{of the cylindrical part}\end{pmatrix}}$ | | |
| $=\dfrac{8\pi}{\pi(3)^2}$ | 1M | |
| $=\dfrac{8}{9}$ | 1A | |
| (iii)   | 1A+1A | 1A for the curved shape for $0 \le t \le t_1$<br>1A for the straight line for $t_1 \le t \le t_2$ |
| | (10) | pp−1 if labeling incomplete (Must show $h$, $t$, $t_1$, $t_2$, 1, 4) |

| Solution | Marks | Remarks |
|---|---|---|
| 17. (a) (i) Using Heron's formula, | | |
| Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ | | |
| $= \sqrt{9(9-5)(9-6)(9-7)}$ | 1M | $(s = \frac{5+6+7}{2} = 9)$ |
| $= 6\sqrt{6}$ | 1A | |
| **Alternative solution** | | |
| $\cos\angle BAC = \frac{5^2+6^2-7^2}{2(5)(6)}$ | | $\cos\angle ABC = \frac{5}{7}$ |
| $= \frac{1}{5}$ | | $\cos\angle ACB = \frac{19}{35}$ |
| Area of $\Delta ABC = \frac{1}{2}(AB)(AC)\sin\angle BAC$ | 1M | |
| $= \frac{1}{2}(6)(5)(\frac{\sqrt{5^2-1}}{5})$ | | |
| $= 6\sqrt{6}$ | 1A | |
| (ii) Area of $\Delta AOB$ + Area of $\Delta BOC$ + Area of $\Delta COA$ = Area of $\Delta ABC$ | | |
| $\frac{1}{2}(6)(r)+\frac{1}{2}(7)(r)+\frac{1}{2}(5)(r) = 6\sqrt{6}$ | 1M | |
| $9r = 6\sqrt{6}$ | | |
| $r = \frac{2\sqrt{6}}{3}$ | 1 | |
| | (4) | |
| (b) (i) Let $X$ be the point of contact of $AB$ and the inscribed circle. | | |
| $\angle VXO = 60°$ | 1A |  |
| Consider $\Delta VXO$: | | |
| $\tan 60° = \frac{VO}{r}$ | | |
| $VO = \frac{2\sqrt{6}}{3}(\sqrt{3})$ | | |
| $= 2\sqrt{2}$ | | |
| Volume of $VABC = \frac{1}{3}$(area of $\Delta ABC$) $(VO)$ | | |
| $= \frac{1}{3}(6\sqrt{6})(2\sqrt{2})$ | 1M | Must have found $VO$ |
| $= 8\sqrt{3}$ | 1A | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| (ii) Let $Y$ be the point of contact of $BC$ and the inscribed circle.<br>Consider $\Delta VOY$:<br>$VY^2 = VO^2 + OY^2 = (2\sqrt{2})^2 + (\frac{2\sqrt{6}}{3})^2$<br>$VY = \frac{4\sqrt{6}}{3}$ | |  |
| **Alternative Solution**<br>$\Delta VOX \cong \Delta VOY\ (SAS)$<br>$\angle VYO = \angle VXO = 60°$<br>Consider $\Delta VOY$:<br>$\cos 60° = \frac{r}{VY}$<br>$VY = \frac{r}{\cos 60°} = (\frac{2\sqrt{6}}{3})(2) = \frac{4\sqrt{6}}{3}$ | | |
| Area of $\Delta VBC = \frac{1}{2}(BC)(VY)$<br>$= \frac{1}{2}(7)(\frac{4\sqrt{6}}{3})$ | 1M | Must have found $VY$ |
| $= \frac{14\sqrt{6}}{3}$ | 1A | |
| (iii) Let $F$ be the foot of perpendicular from $A$ to the plane $VBC$.<br>$\angle ABF$ represents the angle between $AB$ and the plane $VBC$. | 1A |  |
| $\frac{1}{3}$(area of $\Delta VBC$) $(AF)$ = volume of tetrahedron<br>$\frac{1}{3}(\frac{14\sqrt{6}}{3})(AF) = 8\sqrt{3}$ | 1M | |
| $AF = \frac{18\sqrt{2}}{7} (\approx 3.637)$<br>$\sin \angle ABF = \frac{AF}{AB}$<br>$= \frac{\frac{18\sqrt{2}}{7}}{6}$<br>$= \frac{3\sqrt{2}}{7}\ (\approx 0.606)$<br>$\angle ABF = 37°$ (correct to the nearest degree) | 1A | |
| | (8) | |

| Solution | Marks | Remarks |
|---|---|---|
| 18. (a) $\overrightarrow{OG} = \dfrac{2\overrightarrow{OM} + \overrightarrow{OB}}{2+1}$ | | |
| $= \dfrac{2(\frac{\mathbf{a}}{2}) + \mathbf{b}}{3}$ | | |
| $= \dfrac{\mathbf{a}+\mathbf{b}}{3}$ | 1A | |
| | (1) | |
| (b) $\overrightarrow{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a}$ | | |
| $= (\overrightarrow{OB} + \overrightarrow{BT}) \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a}$ | 1A | |
| $= \mathbf{b} \cdot \mathbf{a} + \overrightarrow{BT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a}$ | | |
| $= \overrightarrow{BT} \cdot \mathbf{a}$ | | |
| $= 0$ (Since $\overrightarrow{BT} \perp \mathbf{a}$) | 1 | |
| Alternative solution | | |
| $\overrightarrow{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a}$ | | $\because \overrightarrow{BT} \perp \mathbf{a}$ |
| $= (\overrightarrow{OT} - \mathbf{b}) \cdot \mathbf{a}$ | 1A | $\therefore \overrightarrow{BT} \cdot \mathbf{a} = 0$ |
| $= \overrightarrow{BT} \cdot \mathbf{a}$ | | $\therefore (\overrightarrow{OT} - \mathbf{b}) \cdot \mathbf{a} = 0$ |
| $= 0$ (Since $\overrightarrow{BT} \perp \mathbf{a}$) | 1 | i.e. $\overrightarrow{OT} \cdot \mathbf{a} - \mathbf{b} \cdot \mathbf{a} = 0$ |
| Similarly, $\overrightarrow{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b} = 0$. | 1A | |
| Alternative solution | | |
| $\overrightarrow{OT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$ | | |
| $= (\overrightarrow{OA} + \overrightarrow{AT}) \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$ | | |
| $= \mathbf{a} \cdot \mathbf{b} + \overrightarrow{AT} \cdot \mathbf{b} - \mathbf{a} \cdot \mathbf{b}$ | | |
| $= \overrightarrow{AT} \cdot \mathbf{b}$ | | |
| $= 0$ (Since $\overrightarrow{AT} \perp \mathbf{b}$) | 1A | |
| | (3) | |
| (c) $2\overrightarrow{OC} \cdot \mathbf{a}$ | | |
| $= 2\lvert\overrightarrow{OC}\rvert\lvert\mathbf{a}\rvert \cos\angle COM$ | 1A | |
| $= 2(OM)\lvert\mathbf{a}\rvert$ | | |
| $= \lvert\mathbf{a}\rvert^2$ | 1 | |
| Alternative solution | | |
| $2\overrightarrow{OC} \cdot \mathbf{a} = 2(\overrightarrow{OM} + \overrightarrow{MC}) \cdot \mathbf{a}$ | 1A | |
| $= 2\overrightarrow{OM} \cdot \mathbf{a} + 2\overrightarrow{MC} \cdot \mathbf{a}$ | | |
| $= \mathbf{a} \cdot \mathbf{a} + 0$ (Since $\overrightarrow{MC} \perp \mathbf{a}$) | | |
| $= \lvert\mathbf{a}\rvert^2$ | 1 | |
| Similarly, $\overrightarrow{OC} \cdot \mathbf{b} = \dfrac{\lvert\mathbf{b}\rvert^2}{2}$. | 1A | |
| | (3) | |

| Solution | Marks | Remarks |
|---|---|---|
| (d) (i) $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{a}$ | | |
| $=[\overrightarrow{GO}+\overrightarrow{OT}-2(\overrightarrow{CO}+\overrightarrow{OG})]\cdot\mathbf{a}$ | 1A | 1A for $\overrightarrow{GT}=\overrightarrow{GO}+\overrightarrow{OT}$ and $\overrightarrow{CG}=\overrightarrow{CO}+\overrightarrow{OG}$ |
| $=(\overrightarrow{OT}+2\overrightarrow{OC}-3\overrightarrow{OG})\cdot\mathbf{a}$ | 1A | |
| $=\overrightarrow{OT}\cdot\mathbf{a}+2\overrightarrow{OC}\cdot\mathbf{a}-3\overrightarrow{OG}\cdot\mathbf{a}$ | | |
| Since $\overrightarrow{OT}\cdot\mathbf{a}=\mathbf{b}\cdot\mathbf{a}$ (result of (b)) | | |
| $2\overrightarrow{OC}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2$ (result of (c)) | | |
| and $\overrightarrow{OG}=\dfrac{\mathbf{a}+\mathbf{b}}{3}$, (result of (a)) | | |
| $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{a}=\mathbf{b}\cdot\mathbf{a}+\lvert\mathbf{a}\rvert^2-(\lvert\mathbf{a}\rvert^2+\mathbf{b}\cdot\mathbf{a})$ | 1M | For using either one |
| $=0$ | 1A | |
| Similarly, $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{b}=0$. | 1A | Need $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{a}=0$ manipulated correctly |
| (ii) Since $(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{a}=(\overrightarrow{GT}-2\overrightarrow{CG})\cdot\mathbf{b}=0$ and $\mathbf{a}$, $\mathbf{b}$ are non-parallel, | | |
| $\overrightarrow{GT}-2\overrightarrow{CG}=\mathbf{0}$, | | |
| i.e. $\overrightarrow{GT}=2\overrightarrow{CG}$ | | |
| As $\overrightarrow{GT}=\lambda\overrightarrow{CG}$ for a scalar $\lambda$, $\overrightarrow{GT}\,//\,\overrightarrow{CG}$. | | |
| $G$, $T$ and $C$ are collinear. | 1 | Omitting vector sign / dot sign in most cases, or using the expression $v^2$ or $\overrightarrow{v}^2$, (pp−1) |
| | (5) | |