香港考試局
HONG KONG EXAMINATIONS AUTHORITY

2001年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2001

附加數學
ADDITIONAL MATHEMATICS

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp−1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u−1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable).

   Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the <u>net</u> total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚ ,

   whereas alternative answers are enclosed by solid rectangles ▭ .

8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

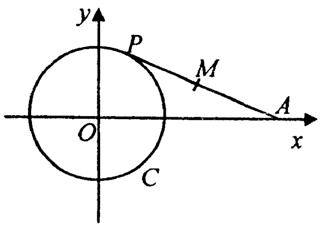
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. $\frac{d}{dx}(\frac{x^2}{2x+1})$		
$=\frac{(2x+1)\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(2x+1)}{(2x+1)^2}$		
$=\frac{(2x+1)(2x)-x^2(2)}{(2x+1)^2}$	1M+1A	1M for quotient rule $(\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2})$
$=\frac{2x(x+1)}{(2x+1)^2}$	1A	Accept equivalent forms
Alternative Solution		
$\frac{d}{dx}(\frac{x^2}{2x+1})$		
$=\frac{d}{dx}x^2(2x+1)^{-1}$		
$=2x(2x+1)^{-1}+x^2(-1)(2x+1)^{-2}(2)$	1M+1A	1M for product rule
$=\frac{2x}{2x+1}-\frac{2x^2}{(2x+1)^2}$	1A	
$=\frac{2x(x+1)}{(2x+1)^2}$		
	3	

Solution	Marks	Remarks
2. Let $u=3x^2+1$.		
$du=6xdx$	1A	
$\int\frac{x}{\sqrt{3x^2+1}}dx=\int\frac{1}{u^{\frac{1}{2}}}(\frac{du}{6})$	1A	Omit du in most cases (pp–1)
$=\int\frac{1}{6}u^{-\frac{1}{2}}du$		
$=\frac{1}{6}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$ $\boxed{c\text{ is a constant}}$	1M	For $\int u^n du=\frac{1}{n+1}u^{n+1}$ Awarded even if 'c' was omitted
$=\frac{1}{3}u^{\frac{1}{2}}+c$		
$=\frac{1}{3}(3x^2+1)^{\frac{1}{2}}+c$	1A	Withhold this mark if 'c' was omitted
Alternative Solution (1)		
Let $v=\sqrt{3x^2+1}$.		
$dv=\frac{3x}{\sqrt{3x^2+1}}dx$	1A	
$\int\frac{x}{\sqrt{3x^2+1}}dx=\int\frac{1}{3}dv$	1A	
$=\frac{1}{3}v+c$ $\boxed{c\text{ is a constant}}$	1M	
$=\frac{1}{3}(3x^2+1)^{\frac{1}{2}}+c$	1A	
Alternative Solution (2)		
$\int\frac{x}{\sqrt{3x^2+1}}dx$		
$=\int\frac{1}{\sqrt{3x^2+1}}\left[\frac{1}{6}d(3x^2+1)\right]$	1A+1A	1A for dx → d $(3x^2+1)$
$=\frac{1}{6}\frac{(3x^2+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$ $\boxed{c\text{ is a constant}}$	1M	
$=\frac{1}{3}(3x^2+1)^{\frac{1}{2}}+c$	1A	
	4	

Solution	Marks	Remarks
3. (a) The coordinates of M are $\left(\dfrac{h+4}{2}, \dfrac{k}{2}\right)$.	1A	
(b) Let the coordinates of M be (x, y). $\begin{cases} x = \dfrac{h+4}{2} \\ y = \dfrac{k}{2} \end{cases}$		
$\begin{cases} h = 2x - 4 \\ k = 2y \end{cases}$	1M	For expressing in terms of x, y
Since (h, k) is a point on C, $(2x-4)^2 + (2y)^2 = 4$	1M	For substituting into C
$4x^2 - 16x + 16 + 4y^2 = 4$		
$x^2 + y^2 - 4x + 3 = 0$	1A	Accept equivalent forms
	4	

Solution	Marks	Remarks
4. General term $= {}_8C_r(2x^3)^{8-r}(\frac{1}{x})^r$	2A	OR $= {}_8C_r(\frac{1}{x})^{8-r}(2x^3)^r$
$= {}_8C_r(2^{8-r})\,x^{24-4r}$		$= {}_8C_r(2^r)\,x^{4r-8}$
$24-4r=0$	1M	$4r-8=0$
$r=6$		$r=2$
Constant term $= {}_8C_6(2^{8-6})$		
$=28(4)$		
$=112$	1A	
Alternative Solution (1)		
$(2x^3+\frac{1}{x})^8=(2x^3)^8+{}_8C_1(2x^3)^7(\frac{1}{x})+{}_8C_2(2x^3)^6(\frac{1}{x})^2+$	1A+1A	1A for ${}_8C_6(2x^3)^2(\frac{1}{x})^6$
${}_8C_3(2x^3)^5(\frac{1}{x})^3+{}_8C_4(2x^3)^4(\frac{1}{x})^4+$		1A for other terms (can be omitted)
${}_8C_5(2x^3)^3(\frac{1}{x})^5+\boxed{{}_8C_6(2x^3)^2(\frac{1}{x})^6}+\cdots$		
$=(2x^3)^8+8(2x^3)^7(\frac{1}{x})+28(2x^3)^6(\frac{1}{x})^2+$		
$56(2x^3)^5(\frac{1}{x})^3+70(2x^3)^4(\frac{1}{x})^4+$		
$56(2x^3)^3(\frac{1}{x})^5+\boxed{28(2x^3)^2(\frac{1}{x})^6}+\cdots$		
Constant term $= {}_8C_6(2)^2$	1M	For choosing the correct term
$=28(4)$		
$=112$	1A	
Alternative Solution (2)		
$(2x^3+\frac{1}{x})^8=(\frac{2x^4+1}{x})^8$		
$=\frac{(1+2x^4)^8}{x^8}$		
$=\frac{1}{x^8}[1+{}_8C_1(2x^4)+{}_8C_2(2x^4)^2+\cdots]$	1A+1A	1A for $\frac{1}{x^8}{}_8C_2(2x^4)^2$ 1A for other terms (can be omitted)
Constant term $= {}_8C_2(2)^2$	1M	For choosing the correct term
$=28(4)$		
$=112$	1A	
	4	

Solution	Marks	Remarks
5. (a) $\frac{1+i}{1-i}=\frac{1+i}{1-i}(\frac{1+i}{1+i})$	1A	
$=\frac{1+2i+i^2}{2}$		
$=i$ $\boxed{=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\text{ (Accept degrees)}}$	1A	
Alternative Solution		
$\frac{1+i}{1-i}=\frac{\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}{\sqrt{2}[\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4})]}$	1A	
$=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$	1A	
(b) $\frac{(1+i)^{16}}{(1-i)^{15}}=(\frac{1+i}{1-i})^{16}(1-i)$	1A	
$=i^{16}(1-i)$	1M	For using (a)
$=1-i$	1A	
Alternative Solution (1)		
$\frac{(1+i)^{16}}{(1-i)^{15}}=(\frac{1+i}{1-i})^{15}(1+i)$	1A	
$=i^{15}(1+i)$	1M	For using (a)
$\boxed{=i^3(1+i)}$		
$=-i(1+i)$		
$=1-i$	1A	
Alternative Solution (2)		
$\frac{(1+i)^{16}}{(1-i)^{15}}=\frac{[\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})]^{16}}{[\sqrt{2}(\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4}))]^{15}}$	1A	
$=\frac{(\sqrt{2})^{16}(\cos 4\pi+i\sin 4\pi)}{(\sqrt{2})^{15}(\cos(-\frac{15\pi}{4})+i\sin(-\frac{15\pi}{4}))}$		
$=\sqrt{2}[\cos(4\pi+\frac{15\pi}{4})+i\sin(4\pi+\frac{15\pi}{4})]$	1M	
$=\sqrt{2}[\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4})]$	1A	Accept $\sqrt{2}(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4})$, $\sqrt{2}(\cos(\frac{7\pi}{4}+i\sin\frac{7\pi}{4})$
	5	

Solution	Marks	Remarks
6. (a) $\sin x+\cos x=\sqrt{2}(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x)$		OR $\begin{cases} r\cos\theta=1 \\ r\sin\theta=-1 \end{cases}$
$=\sqrt{2}(\sin x\sin\frac{\pi}{4}+\cos x\cos\frac{\pi}{4})$		
$=\sqrt{2}\cos(x-\frac{\pi}{4})$		
$\therefore\ r=\sqrt{2}\ ,\ \theta=-\frac{\pi}{4}\ (-45^\circ)$	1A+1A	
(b) $\sin x+\cos x=\sqrt{2}$		
$\sqrt{2}\cos(x-\frac{\pi}{4})=\sqrt{2}$	1M	
$\cos(x-\frac{\pi}{4})=1$		
$x-\frac{\pi}{4}=2n\pi$ $\boxed{n \text{ is an integer}}$	1M	For $x=2n\pi\pm\alpha$
$x=2n\pi+\frac{\pi}{4}$	1A	Accept degrees $x=2n\pi+45^\circ$ etc (u–1)
Alternative Solution (1)		
$\sin x+\cos x=\sqrt{2}$		
$\sqrt{2}\sin(x+\frac{\pi}{4})=\sqrt{2}$	1M	
$\sin(x+\frac{\pi}{4})=1$		
$x+\frac{\pi}{4}=2n\pi+\frac{\pi}{2}$ $\boxed{n \text{ is an integer}}$	1M	OR $x+\frac{\pi}{4}=n\pi+(-1)^n(\frac{\pi}{2})$ $x=(n-\frac{1}{4})\pi+(-1)^n(\frac{\pi}{2})$
$x=2n\pi+\frac{\pi}{4}$	1A	
Alternative Solution (2)		
$\sin x+\cos x=\sqrt{2}$		
$(\sin x+\cos x)^2=2$		
$\sin^2 x+2\sin x\cos x+\cos^2 x=2$		
$\sin 2x=1$	1M	For $\sin^2 x+\cos^2 x=1$ & $2\sin x\cos x=\sin 2x$
$2x=2n\pi+\frac{\pi}{2}$	1M	OR $2x=n\pi+(-1)^n(\frac{\pi}{2})$
Since $\sin x+\cos x$ is positive,		
$x=n\pi+\frac{\pi}{4}$, where n is even.	1A	$x=\frac{n\pi}{2}+(-1)^n(\frac{\pi}{4})$ (n is multiples of 4)
	5	

Solution	Marks	Remarks
7. $x-(1+\sin y)^5=1$		
Differentiate with respect to x:		
$1-5(1+\sin y)^4\cos y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	1A+2A	1A for $\frac{\mathrm{d}}{\mathrm{d}x}(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}(1)$, 2A for $\frac{\mathrm{d}}{\mathrm{d}x}[-(1+\sin y)^5]$
Alternative Solution (1)		
$x-(1+\sin y)^5=1$		
Differentiate with respect to y:		
$\frac{\mathrm{d}x}{\mathrm{d}y}-5(1+\sin y)^4\cos y=0$	1A+2A	1A for $\frac{\mathrm{d}}{\mathrm{d}y}(x)$ and $\frac{\mathrm{d}}{\mathrm{d}y}(1)$, 2A for $\frac{\mathrm{d}}{\mathrm{d}y}[-(1+\sin y)^5]$
Alternative Solution (2)		
$\sin y=(x-1)^{\frac{1}{5}}-1$	1A	
Differentiate with respect to x:		
$\cos y\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}(x-1)^{-4/5}$	1A+1A	1A for LHS, 1A for RHS
Put $x=2$, $y=0$:		
$1-5\frac{\mathrm{d}y}{\mathrm{d}x}=0$	1M	
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{5}$		
Equation of tangent is		
$\frac{y-0}{x-2}=\frac{1}{5}$		
$x-5y-2=0$	1A	Accept equivalent forms
	5	
8. (a) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$		
$=4(3)\cos 60°$	1A	
$=6$	1A	
(b) $(\vec{a}+k\vec{b})\cdot(\vec{a}-2\vec{b})=0$	1M	
$\vec{a}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+k\vec{b}\cdot\vec{a}-2k\vec{b}\cdot\vec{b}=0$		
$4^2+(k-2)(6)-2k(3)^2=0$	1A+1M	1A For $\vec{a}\cdot\vec{a}=16$ or $\vec{b}\cdot\vec{b}=9$ 1M For $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ = Ans. in (a)
$4-12k=0$		
$k=\frac{1}{3}$	1A	Omit vector sign in most cases (pp–1) Omit dot product sign more than once (pp–1)
	6	$\vec{a}^2$, $\frac{\vec{a}}{\vec{b}}$ etc. (pp–1)

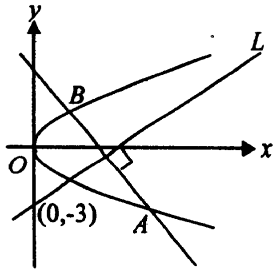
Solution	Marks	Remarks
9. $p=\dfrac{x^2+2x+8}{x-2}$		
$px-2p=x^2+2x+8$		
$x^2+(2-p)x+(8+2p)=0$	1M+1A	1M for transforming into an quadratic equation in x
Since x is real, discriminant $=(2-p)^2-4(8+2p)\ge 0$	1M	$\Delta>0$ – no mark
$p^2-12p-28\ge 0$		
$(p+2)(p-14)\ge 0$		
$p\ge 14$ or $p\le -2$.	1A	
$\therefore\ \dfrac{x^2+2x+8}{x-2}\ge 14$ or $\dfrac{x^2+2x+8}{x-2}\le -2$		
$\boxed{\lvert p\rvert\ge 14 \text{ or } \lvert p\rvert\ge 2}$	1M	
$\therefore\ \left\lvert\dfrac{x^2+2x+8}{x-2}\right\rvert\ge 2$	1A	
	6	

Solution	Marks	Remarks
10. Let the equation of the family of straight lines passing through A be		
$(x+y-5)+k(2x-3y)=0$ $\boxed{k \text{ is real}}$	1M	OR $k_1\ell_1+k_2\ell_2=0$,
$(2k+1)x+(1-3k)y-5=0$ ----- (*)	1M	$k_1\ell_1+\ell_2=0$.
Distance from the origin to the line		
$=\left\lvert\frac{(2k+1)(0)+(1-3k)(0)-5}{\sqrt{(2k+1)^2+(1-3k)^2}}\right\rvert$	1M	Omitting absolute value signs (pp–1)
$=\left\lvert\frac{-5}{\sqrt{(2k+1)^2+(1-3k)^2}}\right\rvert$		
$\left\lvert\frac{-5}{\sqrt{(2k+1)^2+(1-3k)^2}}\right\rvert=2$	1A	
$25=4(4k^2+4k+1+1-6k+9k^2)$		
$52k^2-8k-17=0$		
$\boxed{(26k-17)(2k+1)=0}$		
$k=\frac{17}{26}$ or $-\frac{1}{2}$		
Put $k=\frac{17}{26}$ and $-\frac{1}{2}$ into (*):		
the equation of the two lines are		
$12x-5y-26=0$	1A	Accept equivalent forms
and $y-2=0$.	1A	
OR $(2x-3y)+\lambda(x+y-5)=0$		
$(2+\lambda)x+(\lambda-3)y-5\lambda=0$		
$\left\lvert\frac{-5\lambda}{\sqrt{(2+\lambda)^2+(\lambda-3)^2}}\right\rvert=2$		
$17\lambda^2+8\lambda-52=0$		
$\lambda=\frac{26}{17}$ or -2		
$\vdots$		

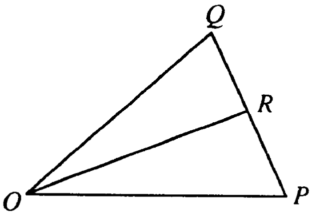
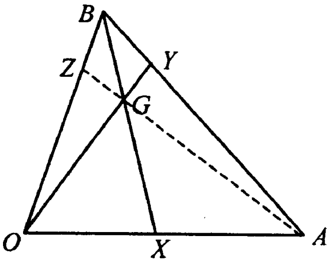
Solution	Marks	Remarks
Alternative Solution (1)		
$\begin{cases} x+y-5=0 \\ 2x-3y=0 \end{cases}$		
Solve the 2 equations : $x=3$, $y=2$.	1M	
Let the slope of the line be m.		
Equation of a line passing through A is $\frac{y-2}{x-3}=m$		
$mx-y+(2-3m)=0$	1M	
Distance from the origin to the line		
$=\left\lvert\frac{m(0)-0+(2-3m)}{\sqrt{m^2+1}}\right\rvert$	1M	Omitting absolute value sign (pp−1)
$=\left\lvert\frac{2-3m}{\sqrt{m^2+1}}\right\rvert$		
$\left\lvert\frac{2-3m}{\sqrt{m^2+1}}\right\rvert=2$	1A	
$(2-3m)^2=4(m^2+1)$		
$9m^2-12m+4=4m^2+4$		
$5m^2-12m=0$		
$m=0$ or $\frac{12}{5}$		
The equations of the two lines are		
$12x-5y-26=0$	1A	Accept equivalent forms
and $y-2=0$.	1A	
Alternative Solution (2)		
$x=3$, $y=2$	1M	Same as above
Let the equation of the line be $Ax+By+C=0$.		
$\begin{cases} 3A+2B+C=0 \\ \left\lvert\frac{C}{\sqrt{A^2+B^2}}\right\rvert=2 \end{cases}$	1M 1M	
$\left\lvert\frac{-3A-2B}{\sqrt{A^2+B^2}}\right\rvert=2$	1A	For a correct equation in 2 unknowns
$9A^2+12AB+4B^2=4(A^2+B^2)$		
$A(5A+12B)=0$		
$A=0$ or $A=-\frac{12B}{5}$		
⋮		
The equation of the two lines are		
$12x-5y-26=0$ and $y-2=0$.	1A+1A	
	6	

Solution	Marks	Remarks
11. $\frac{y}{y-2}\le 2$		
$\frac{y}{y-2}-2\le 0$	1M	
$\frac{-y+4}{y-2}\le 0$	1A	
$\frac{y-4}{y-2}\ge 0$		
$(y-4)(y-2)\ge 0$, where $y\ne 2$		
$y\ge 4$ or $y<2$	1A	$y\ge 4$ or $y\le 2$ – no mark
Alternative Solution (1)		
$\frac{y}{y-2}\le 2$		
Multiplying both sides by $(y-2)^2$,		
$y(y-2)\le 2(y-2)^2$ where $y\ne 2$	1M	
$y^2-2y\le 2y^2-8y+8$		
$y^2-6y+8\ge 0$	1A	
$(y-2)(y-4)\ge 0$		
$y\ge 4$ or $y<2$	1A	
Alternative Solution (2)		
Consider the two cases : (i) $y>2$, (ii) $y<2$.	1M	Accepting including '=' sign
Case 1 : $y>2$		
The inequality becomes		
$y\le 2(y-2)$		
$y\ge 4$		
Since $y>2$, $\therefore y\ge 4$.		
Case 2 : $y<2$		
The inequality becomes		
$y\ge 2(y-2)$	1A	Both were correct
$y\le 4$		
Since $y<2$, $\therefore y<2$.		
Combining the two cases, $y\ge 4$ or $y<2$.	1A	
$\frac{2^x}{2^x-2}\le 2$		
Put $2^x=y$.		
Using the result in (a),		
$2^x\ge 4$ or $2^x<2$	1M	
$x\ge 2$ or $x<1$	1A+1A	
	6	

Solution	Marks	Remarks
12. For $n=1$, LHS $=1(2)=2$		
RHS $=\frac{1}{3}(1)(1+1)(1+2)=2=$ LHS	1	
$\therefore$ the statement is true for $n=1$.		
Assume $1\times 2+2\times 3+3\times 4+\cdots+k(k+1)=\frac{1}{3}k(k+1)(k+2)$	1	
for some positive integer k.		
Then $1\times 2+2\times 3+3\times 4+\cdots+k(k+1)+(k+1)[(k+1)+1]$		
$=\frac{1}{3}k(k+1)(k+2)+(k+1)(k+2)$	1	
$=\frac{1}{3}(k+1)(k+2)(k+3)$	1	
The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers n.	1	Not awarded if any one of the above marks was withheld
$1\times 3+2\times 4+3\times 5+\cdots+50\times 52$		
$=1\times(2+1)+2\times(3+1)+3\times(4+1)+\cdots+50\times(51+1)$		
$=(1\times 2+2\times 3+3\times 4+\cdots+50\times 51)+(1+2+3+\cdots+50)$	1A	
$=\frac{1}{3}(50)(51)(52)+\frac{1}{2}(1+50)(50)$	1M	For using the above result
$=44200+1275$		
$=45475$	1A	
Alternative Solution		
$1\times 3+2\times 4+3\times 5+\cdots+50\times 52$		
$=(2-1)\times 3+(3-1)\times 4+(4-1)\times 5+\cdots+(51-1)\times 52$		
$=(2\times 3+3\times 4+4\times 5+\cdots+51\times 52)-(3+4+5+\cdots+52)$	1A	
$=[\frac{1}{3}(51)(52)(53)-1\times 2]-\frac{1}{2}(3+52)(50)$	1M	For using the above result
$=46850-1375$		
$=45475$	1A	
	8	

Solution	Marks	Remarks
13. (a) $\begin{cases} y=mx+2 \\ y^2=x \end{cases}$		
$(mx+2)^2=x$	1M	$\underline{\text{OR}}$ $y=my^2+2$ $my^2-y+2=0$
$m^2x^2+(4m-1)x+4=0$ ------ (*)		
Since the line intersects the parabola at two distinct points,		
discriminant $\Delta=(4m-1)^2-4m^2(4)>0$	1M	$\Delta=(-1)^2-4m(2)>0$
$16m^2-8m+1-16m^2>0$		
$m<\frac{1}{8}$	1	
	3	
(b) Let the coordinates of A and B be (x_1,y_1) and (x_2,y_2) respectively.		
From (*), $x_1+x_2=\frac{-(4m-1)}{m^2}$	1M	$y_1+y_2=\frac{1}{m}$
$\boxed{\underline{\text{OR}}\ \ x=\frac{(1-4m)\pm\sqrt{\Delta}}{2m^2}}$		$\boxed{\underline{\text{OR}}\ y=\frac{1\pm\sqrt{\Delta}}{2m}}$
$\therefore$ x coordinate of mid-point $=\frac{x_1+x_2}{2}$		y coord. $=\frac{y_1+y_2}{2}$
$=\frac{1-4m}{2m^2}$	1A	$=\frac{1}{2m}$
y coordinate of mid-point $=mx+2$		
$=m(\frac{1-4m}{2m^2})+2$		
$=\frac{1}{2m}$	1A	
$\therefore$ the coordinates of the mid-point are $(\frac{1-4m}{2m^2},\frac{1}{2m})$.		
	3	
(c) Slope of the perpendicular bisector of $AB=-\frac{1}{m}$		
$\dfrac{-3-\frac{1}{2m}}{0-\frac{1-4m}{2m^2}}=-\frac{1}{m}$	1M	
$-3-\frac{1}{2m}=\frac{1}{m}(\frac{1-4m}{2m^2})$		
$-3(2m^3)-m^2=1-4m$		
$6m^3+m^2-4m+1=0$	1	

Solution	Marks	Remarks
Alternative Solution		
Slope of $L = -\frac{1}{m}$		
Equation of L is		
$\frac{y+3}{x} = -\frac{1}{m}$	1M	$\frac{y-\frac{1}{2m}}{x-\frac{1-4m}{2m^2}} = -\frac{1}{m}$
$x+my+3m=0$		
Substitute $x=\frac{1-4m}{2m^2}$, $y=\frac{1}{2m}$:		Substitute $x=0$, $y=-3$.
$\frac{1-4m}{2m^2}+m(\frac{1}{2m})+3m=0$		
$(1-4m)+m^2+3(2m^3)=0$		
$6m^3+m^2-4m+1=0$	1	
$(m+1)(6m^2-5m+1)=0$	1A	For finding one factor
$(m+1)(3m-1)(2m-1)=0$		
$m=-1, \frac{1}{3}$ or $\frac{1}{2}$.	1A	
From (a), $m<\frac{1}{8}$ $\therefore m=-1$	1M	For using (a)
The equation of L is		
$y-\frac{1}{2(-1)} = \frac{-1}{(-1)}[x-\frac{1-4(-1)}{2(-1)^2}]$		$\frac{y-(-3)}{x-0} = -\frac{1}{(-1)}$
$y=x-3$.	1A	
	6	

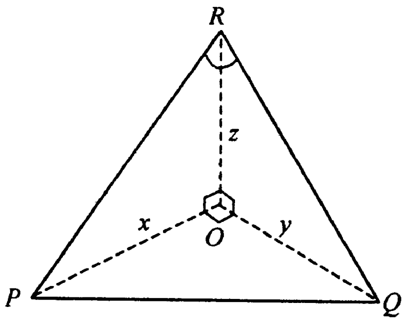
Solution	Marks	Remarks
14. (a) $\overrightarrow{OR}=\dfrac{s\overrightarrow{OP}+r\overrightarrow{OQ}}{r+s}$	1A	
If $\overrightarrow{OR}=m\overrightarrow{OP}+n\overrightarrow{OQ}$, comparing with the above expression, $m=\dfrac{s}{r+s}$ and $n=\dfrac{r}{r+s}$	1M	
$m+n=\dfrac{s}{r+s}+\dfrac{r}{r+s}$ $=1$	1	
	3	
(b) (i) $\overrightarrow{OG}=\dfrac{\frac{1}{2}\vec{a}+3\vec{b}}{4}$ $=\dfrac{1}{8}\vec{a}+\dfrac{3}{4}\vec{b}$	1A	
(ii) $\overrightarrow{OY}=k\overrightarrow{OG}$ $=\dfrac{k}{8}\vec{a}+\dfrac{3k}{4}\vec{b}$		
Using (a), $\dfrac{k}{8}+\dfrac{3k}{4}=1$ $k=\dfrac{8}{7}$	2M	
$\therefore\ \overrightarrow{OY}=\dfrac{1}{8}(\dfrac{8}{7})\vec{a}+\dfrac{3}{4}(\dfrac{8}{7})\vec{b}$ $=\dfrac{1}{7}\vec{a}+\dfrac{6}{7}\vec{b}$	1A	
(iii) (1) From (b) (i), $\overrightarrow{OG}=\dfrac{1}{8}\vec{a}+\dfrac{3}{4}\vec{b}$		
$=\dfrac{1}{8}\vec{a}+\dfrac{3}{4h}\overrightarrow{OZ}$	1M	For expressing $\vec{b}$ in terms of h, $\overrightarrow{OZ}$.
Using (a), $\dfrac{1}{8}+\dfrac{3}{4h}=1$	1M	
$h=\dfrac{6}{7}$	1A	

Solution	Marks	Remarks
Alternative Solution (1)		
Let $\overrightarrow{AZ}=\lambda\overrightarrow{AG}$.		
$\overrightarrow{OZ}=\overrightarrow{OA}+\overrightarrow{AZ}$		
$=\vec{a}+\lambda(\frac{1}{8}\vec{a}+\frac{3}{4}\vec{b}-\vec{a})$		
$=(1-\frac{7\lambda}{8})\vec{a}+\frac{3\lambda}{4}\vec{b}$	1M	
Since $\overrightarrow{OZ}=h\vec{b}$,		
$\begin{cases}1-\frac{7\lambda}{8}=0\\ \frac{3\lambda}{4}=h\end{cases}$	1M	
$\lambda=\frac{8}{7}$		
$\therefore\ h=\frac{3}{4}(\frac{8}{7})=(\frac{6}{7})$	1A	
Alternative Solution (2)		
$\overrightarrow{AG}=\frac{3\overrightarrow{AB}+\overrightarrow{AX}}{4}$		
$=\frac{3}{4}\overrightarrow{AB}+\frac{1}{8}\overrightarrow{AO}$		
Let $\overrightarrow{AZ}=\lambda\overrightarrow{AG}$.		
$\overrightarrow{AZ}=\frac{3\lambda}{4}\overrightarrow{AB}+\frac{\lambda}{8}\overrightarrow{AO}$	1M	
Using (a), $\frac{3\lambda}{4}+\frac{\lambda}{8}=1$	1M	
$\lambda=\frac{8}{7}$		
$\therefore\ \overrightarrow{AZ}=\frac{6}{7}\overrightarrow{AB}+\frac{1}{7}\overrightarrow{AO}$		
$\therefore\ OZ:ZB=6:1$		
i.e. $h=\frac{6}{7}$	1A	
(2) $\overrightarrow{ZY}=\overrightarrow{OY}-\overrightarrow{OZ}$		
$=\frac{1}{7}\vec{a}+\frac{6}{7}\vec{b}-\frac{6}{7}\vec{b}$	1M	
$=\frac{1}{7}\vec{a}$		
As $\overrightarrow{ZY}=\lambda\overrightarrow{OA}$, so ZY is parallel to OA.	1	Omit vector sign in most cases (pp–1)

Solution	Marks	Remarks
Alternative Solution In ΔBZY and ΔBOA: $\angle ZBY = \angle OBA$		
$\frac{BY}{BA} = \frac{1}{7}$ (Since $\overrightarrow{OY} = \frac{1}{7}\vec{a} + \frac{6}{7}\vec{b}$) $\frac{BZ}{BO} = \frac{1}{7}$ (Since $h = \frac{6}{7}$)	1M	$\frac{BY}{YA} = \frac{BE}{ZO} = \frac{1}{6}$ So $ZY \parallel OA$
$\therefore \Delta BZY \sim \Delta BOA$ (ratio of 2 sides, inc. L)		(converse of equal ratios)
So ZY is parallel to OA. (Corr. sides of similar Δs)	1	
	9	

Solution	Marks	Remarks

15. (a)



(i) $RP = \sqrt{x^2+z^2}$, $PQ = \sqrt{x^2+y^2}$, $QR = \sqrt{y^2+z^2}$

$$\cos \angle PRQ = \frac{RP^2+QR^2-PQ^2}{2(RP)(QR)}$$

$$= \frac{(x^2+z^2)+(y^2+z^2)-(x^2+y^2)}{2\sqrt{x^2+z^2}\,\sqrt{y^2+z^2}}$$ 1M

$$= \frac{z^2}{\sqrt{x^2+z^2}\,\sqrt{y^2+z^2}}$$ 1A

(ii) $S_1 = \frac{1}{2}xz$, $S_2 = \frac{1}{2}xy$, $S_3 = \frac{1}{2}yz$

$$\sin \angle PRQ = \frac{\sqrt{(x^2+z^2)(y^2+z^2)-z^4}}{\sqrt{x^2+z^2}\,\sqrt{y^2+z^2}}$$ 1M

$$S_4 = \frac{1}{2}(QR)(RP)\sin \angle PRQ$$

$$= \frac{1}{2}\sqrt{y^2+z^2}\sqrt{x^2+z^2}\,\frac{\sqrt{(x^2+z^2)(y^2+z^2)-z^4}}{\sqrt{x^2+z^2}\,\sqrt{y^2+z^2}}$$ 1M

$$= \frac{1}{2}\sqrt{(x^2+z^2)(y^2+z^2)-z^4}$$ 1A

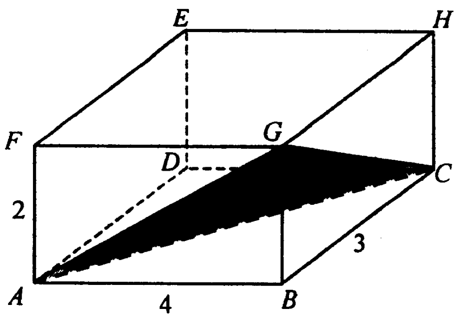
$$S_4{}^2 = \frac{1}{4}(x^2y^2+y^2z^2+z^2x^2)$$

$$= (\frac{xy}{2})^2 + (\frac{yz}{2})^2 + (\frac{zx}{2})^2$$

$$= S_1{}^2 + S_2{}^2 + S_3{}^2$$ 1

6

(b)



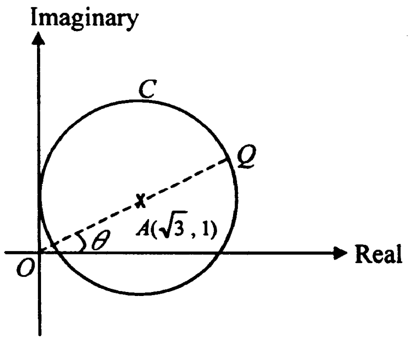
(i) Volume of $ABCG = \frac{1}{3} \times \text{base area} \times \text{height}$

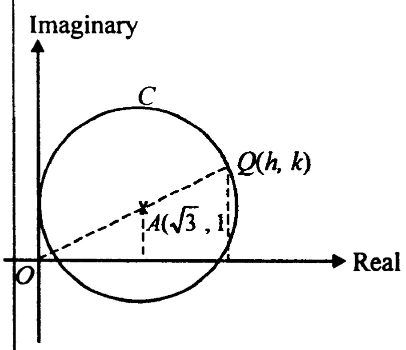
$$= \frac{1}{3} \times (\frac{1}{2} \times 4 \times 3) \times 2$$

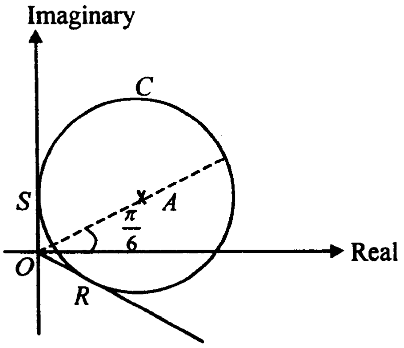
$$= 4$$ 1A

Solution	Marks	Remarks
(ii) Using (a), area of $\Delta GAC = \sqrt{(\frac{4\times 2}{2})^2 + (\frac{2\times 3}{2})^2 + (\frac{4\times 3}{2})^2}$	1M	For using (a) (ii)
$= \sqrt{61}$		
Let h be the perpendicular distance from B to plane GAC. Considering the volume of $ABCG$,		
$\frac{1}{3}(\sqrt{61})h = 4$	1M	
$h = \frac{12}{\sqrt{61}}$		
Let θ be the angle between AB and plane GAC.	1A	For identifying the angle (can be omitted)
$\sin\theta = \frac{h}{AB}$		
$= \frac{12/\sqrt{61}}{4}$	1M	
$\theta = 23°$ (correct to the nearest degree)	1A	
	6	

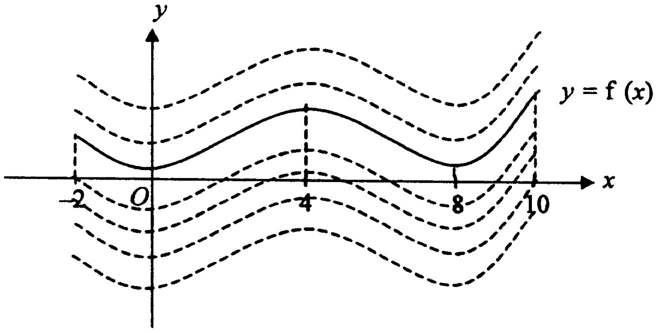
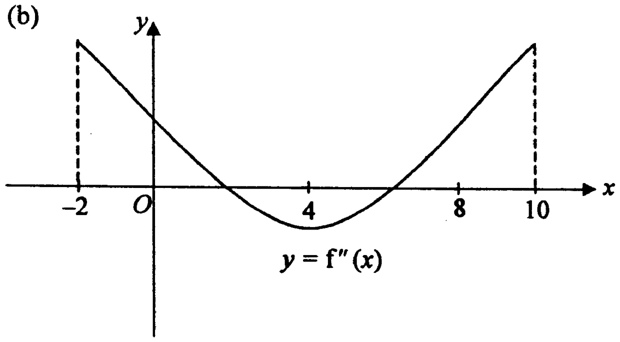
Solution	Marks	Remarks
16. (a) Area $=\int_{-5}^{5} y\,dx$ $\boxed{=2\int_{0}^{5} y\,dx}$		
$=\int_{-5}^{5} 3\sqrt{1-\frac{x^2}{25}}\,dx$	1M	For $A=\int_{a}^{b} y\,dx$
Put $x=5\sin\theta$. $dx=5\cos\theta d\theta$		
Area $=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3\sqrt{1-\frac{25\sin^2\theta}{25}}(5\cos\theta d\theta)$	1A	For integrand only
$=15\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta d\theta$		
$=15\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2\theta)d\theta$	1A	For $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$ only
$=\frac{15}{2}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$	1A	For $[\theta+\frac{1}{2}\sin 2\theta]$ only
$=\frac{15}{2}\left[\frac{\pi}{2}+\frac{1}{2}\sin\pi+\frac{\pi}{2}-\frac{1}{2}\sin(-\pi)\right]$		
$=\frac{15\pi}{2}$	1A	
	5	
(b) Volume $=\int_{-5}^{5} \pi y^2\,dx$ $\boxed{=2\int_{0}^{5} \pi y^2\,dx}$		
$=\pi\int_{-5}^{5} 9(1-\frac{x^2}{25})dx$	1M +1A	1M for $V=\int_{a}^{b} \pi y^2\,dx$, 1A – for integrand only
$=9\pi\left[x-\frac{x^3}{75}\right]_{-5}^{5}$	1A	For primitive function only
$=9\pi\left[5-\frac{125}{75}+5-\frac{125}{75}\right]$		
$=60\pi$	1A	
	4	
(c) Capacity of the can = Base area × height		
$=(\frac{15\pi}{2}\times 2)\times(6\times 4)$	1M+1A	1M for base area = 2 × Area in (a) 1A for height
$=360\pi$	1A	
	3	

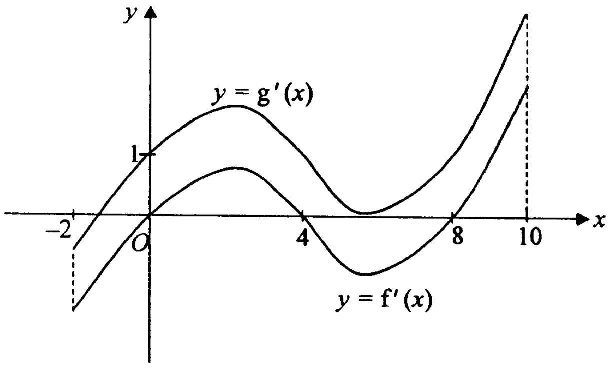
Solution	Marks	Remarks
17. (a) The centre is $(\sqrt{3}, 1)$ (OR $\sqrt{3} + i$).	1A	
The radius is $\sqrt{3}$.	1A	
	2	
(b) The position of Q is shown below :		
	1A	For identifying position of Q
Let A denote the centre of C and θ be the angle between OA and the real axis.		
$OA = \sqrt{(\sqrt{3})^2 + 1^2} = 2$		
$OQ = OA + AQ$		
$= 2 + \sqrt{3}$	1M	For evaluating OQ
$\tan\theta = \frac{1}{\sqrt{3}}$	1M	For evaluating argument
$\theta = \frac{\pi}{6}$		
$\therefore$ the complex number represented by Q is		
$(2+\sqrt{3})(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})\boxed{= (\sqrt{3} + \frac{3}{2}) + (1 + \frac{\sqrt{3}}{2})\, i}$	1M+1A	Accept degrees

Solution	Marks	Remarks
Alternative Solution (1) 	1A	For identifying position of Q
$OA=\sqrt{(\sqrt{3})^2+1^2}=2$ $OQ=2+\sqrt{3}$	1M	
Let the coordinates of Q be (h, k). By similar triangles, $\frac{OA}{OQ}=\frac{\sqrt{3}}{h}=\frac{1}{k}$ $\frac{2}{2+\sqrt{3}}=\frac{\sqrt{3}}{h}$ and $\frac{2}{2+\sqrt{3}}=\frac{1}{k}$	1M+1M	
$h=\sqrt{3}+\frac{3}{2}$, $k=1+\frac{\sqrt{3}}{2}$ $\therefore$ the complex number represented by Q is $(\sqrt{3}+\frac{3}{2})+(1+\frac{\sqrt{3}}{2})i$.	1A	
Alternative Solution (2) Position of Q	1A	Same as above
Equation of $C:(x-\sqrt{3})^2+(y-1)^2=3$ Equation of OA (OR OQ): $y=\frac{1}{\sqrt{3}}x$	1M	For setting up the equations of C and OA
$\begin{cases}(x-\sqrt{3})^2+(y-1)^2=3\\ y=\frac{1}{\sqrt{3}}x\end{cases}$ $(x-\sqrt{3})^2+(\frac{1}{\sqrt{3}}x-1)^2=3$	1M	For solving x
$4x^2-8\sqrt{3}x+3=0$ $x=\frac{8\sqrt{3}\pm\sqrt{144}}{8}$ $=\sqrt{3}+\frac{3}{2}$ or $\sqrt{3}-\frac{3}{2}$ (rejected)		
$y=\frac{1}{\sqrt{3}}(\sqrt{3}+\frac{3}{2})$ $=1+\frac{\sqrt{3}}{2}$	1M	For finding y
$\therefore$ the complex number represented by Q is $(\sqrt{3}+\frac{3}{2})+(1+\frac{\sqrt{3}}{2})i$.	1A	

Solution	Marks	Remarks
Alternative Solution (3)		
Position of Q	1A	Same as above
Since O, A, Q lie on the same line let the complex number represented by Q be $r(\sqrt{3}+i)$.	1M	
$\left\| r(\sqrt{3}+i)-(\sqrt{3}+i) \right\| = \sqrt{3}$	1M	For substituting into the equation
$\left\| (r-1)(\sqrt{3}+i) \right\| = \sqrt{3}$		
$(r-1)^2\ (4)=3$	1M	
$r=1+\frac{\sqrt{3}}{2}$ or $1-\frac{\sqrt{3}}{2}$ (rejected) $\therefore$ the complex number represented by Q is $(1+\frac{\sqrt{3}}{2})(\sqrt{3}+i)$.	1A	
	5	
(c) The position of R is shown below (OR is a tangent to C) : 	1A	For identifying position of R
Let S be the point of contact between C and the imaginary axis. $OR=OS=1$	1M	For evaluating OR
$\angle AOR=\angle AOS=\frac{\pi}{2}-\frac{\pi}{6}$ $=\frac{\pi}{3}$	1M	
Angle between OR and the real axis $=\frac{\pi}{3}-\frac{\pi}{6}$		
$=\frac{\pi}{6}$	1A	
$\therefore$ the complex number represented by R is $\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$ $\boxed{=\frac{\sqrt{3}}{2}-\frac{1}{2}i}$	1A	

Solution	Marks	Remarks
Alternative Solution Position of R Let the equation of OR be $y = mx$.	1A	Same as above
$\begin{cases}(x-\sqrt{3})^2+(y-1)^2=3\\ y=mx\end{cases}$ $(x-\sqrt{3})^2+(mx-1)^2=3$ $(1+m^2)x^2-(2m+2\sqrt{3})x+1=0$ - - - - - (*) Since OR is a tangent to C, $\Delta=(2m+2\sqrt{3})^2-4(1+m^2)=0$ $4m^2+8\sqrt{3}m+12-4-4m^2=0$	1M	
$m=-\frac{1}{\sqrt{3}}$	1A	
$\tan\alpha=-\frac{1}{\sqrt{3}}$ $\alpha=-\frac{\pi}{6}$		**OR** Substitute $m=-\frac{1}{\sqrt{3}}$ into (*)
$OR = OS = 1$	1M	$\frac{4}{3}x^2-\frac{4\sqrt{3}}{3}x+1=0$ 1M $x=\frac{\sqrt{3}}{2}$, $y=-\frac{1}{\sqrt{3}}(\frac{\sqrt{3}}{2})=-\frac{1}{2}$
$\therefore$ the complex number represented by R is $\cos(-\frac{\pi}{6})+i\sin(-\frac{\pi}{6})$.	1A	
	5	

Solution	Marks	Remarks
18. (a) (i) $f(x)$ is increasing when $0 \le x \le 4$ and $8 \le x \boxed{\le 10}$.	2A	OR $0 < x < 4$ or $8 < x < 10$ Any one correct – 1A All correct – 2A
(ii) From Figure 5 (a), $f'(x) = 0$ at $x = 0$, 4 and 8. As $f'(x)$ changes from –ve to +ve as x increases through 0 and 8, $f(x)$ attains a minimum at $x = 0$ and 8. As $f'(x)$ changes from +ve to –ve as x increases through 4, $f(x)$ attains a maximum at $x = 4$.	1A+1	1A for the 3 x coordinates 1 for the explanation
(iii) 	1A+1A	1A for the 3 turning points 1A for shape
	6	
(b) 	1A+1A	1A for $f''(4) < 0$ and $f''(0)$ & $f''(8) > 0$ 1A for shape
	2	

Solution	Marks	Remarks
(c) (i) $g(x)=f(x)+x$. $g'(x)=f'(x)+1$. - - - - - (1)  	1M+1A	1M for translating vertically upwards
(ii) Differentiate (1) with respect to x : $g''(x)=f''(x)$. Since $f''(x)=g''(x)$, the graphs of $y=f''(x)$ and $y=g''(x)$ are identical. The student is incorrect.	2A	
	4	