香港考試局
HONG KONG EXAMINATIONS AUTHORITY

2000年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2000

附加數學　試卷二
ADDITIONAL MATHEMATICS　PAPER 2

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。
After the examinations, marking schemes will be available for reference at the teachers' centre.

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates would use alternative methods not specified in the marking scheme. Markers should be patient in marking these alternative solutions. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol ⓟⓟ–1 should be use d to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable).

   Note the following points:

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the <u>net</u> total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ┆┄┄┄┄┄┄┆ , whereas alternative answers are enclosed by solid rectangles ☐ .

8. (a) Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

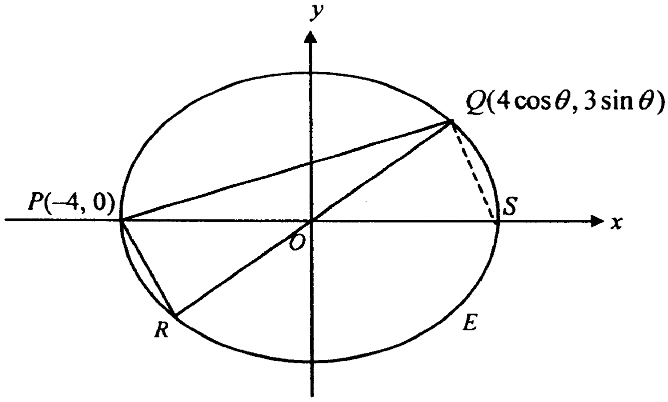
   (b) In case a certain degree of accuracy had been specified in the question, answers not accurate up to that degree should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for the first time if happened. In any case, do not deduct any marks for excess degree of accuracy in those steps where candidates could not score any marks.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

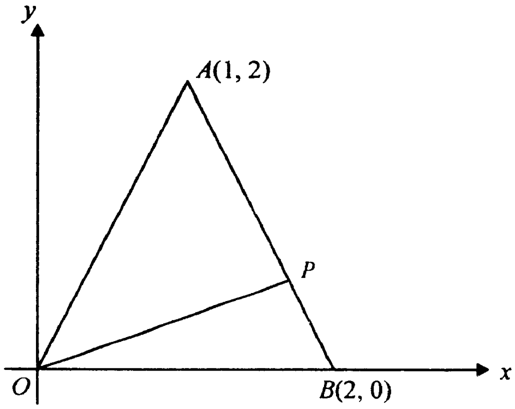
10. Unless the form of answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they were correct.

Solution	Marks	Remarks
1. Let $u = 2x + 1$. $du = 2dx$	1A	For a proper substitution
$\int \sqrt{2x+1}\,dx$ $= \int u^{\frac{1}{2}}(\frac{1}{2}du)$	1A	Omit du in most cases (pp–1)
$\boxed{= \frac{1}{2}[\frac{2}{3}u^{\frac{3}{2}}] + c}$ c is a constant	1A	Awarded even if 'c' was omitted
$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + c$	1A	Withhold this mark if 'c' was omitted
Alternative solution (1) Let $u = \sqrt{2x+1}$ · $du = \frac{1}{\sqrt{2x+1}}dx$	1A	
$\int \sqrt{2x+1}\,dx$ $= \int u(u\,du)$	1A	
$\boxed{= \frac{1}{3}u^{3} + c}$ c is a constant	1A	
$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + c$	1A	Withhold this mark if 'c' was omitted
Alternative solution (2) $\int \sqrt{2x+1}\,dx$ $\boxed{= \int \sqrt{2x+1}[\frac{1}{2}d(2x+1)]}$	1A+1A	1A for dx → d(2x+1) (can be omitted)
$\boxed{= \frac{1}{2}[\frac{2}{3}(2x+1)^{\frac{3}{2}}] + c}$ c is a constant	1A	(can be omitted)
$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + c$	1A	Withhold this mark if 'c' was omitted
	4	

Solution	Marks	Remarks
2. $(1+2x)^7 = 1 + {}_7C_1(2x) + {}_7C_2(2x)^2 + \cdots$	1A	
$= 1 + 14x + 84x^2 + \cdots$	1M	For ${}_7C_1 = 7$ and ${}_7C_2 = 21$
$(2-x)^2 = 4 - 4x + x^2$	1A	
$(1+2x)^7(2-x)^2 = (1 + 14x + 84x^2 + \cdots)(4 - 4x + x^2)$		
$= 4 - 4x + x^2 + 14x(4) + 14x(-4x) + 84x^2(4) + \cdots$	1M	
$= 4 + 52x + 281x^2 + \cdots$	1A	Omit dots in all cases (pp–1)
	5	

Solution	Marks	Remarks
3. 		
(a) The coordinates of R are $(-4\cos\theta, -3\sin\theta)$.	1A+1A	
(b) Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} -4 & 0 \\ -4\cos\theta & -3\sin\theta \\ 4\cos\theta & 3\sin\theta \\ -4 & 0 \end{vmatrix}$	1M	
$= \frac{1}{2}(12\sin\theta - 12\sin\theta\cos\theta + 12\sin\theta\cos\theta$ $+12\sin\theta)$		
$= 12\sin\theta$	1A	Accept $-12\sin\theta$
Alternative solution Area of ΔPQR $=$ Area of $\Delta OPQ +$ Area of ΔOPR $= \frac{1}{2}(4)(3\sin\theta) + \frac{1}{2}(4)(3\sin\theta)$ $= 12\sin\theta$	 1M 1A	OR $=$ area of ΔPQS $= \frac{1}{2}(8)(3\sin\theta)$
$12\sin\theta = 6$	1M	
$\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$ $\therefore$ the coordinates of Q are $(4\cos\frac{\pi}{6}, 3\sin\frac{\pi}{6})$, i.e. $(2\sqrt{3}, \frac{3}{2})$.	 1A 6	

Solution	Marks	Remarks
4. For $n = 1$, LHS $= 1^2 = 1$.		
RHS $= (-1)^{1-1}\frac{1(1+1)}{2} = 1 =$ LHS.	1	
∴ the statement is true for $n = 1$.		
Assume $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k-1}k^2 = (-1)^{k-1}\frac{k(k+1)}{2}$	1	
for some positive integer k.		
Then $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{k-1}k^2 + (-1)^k(k+1)^2$		
$= (-1)^{k-1}\frac{k(k+1)}{2} + (-1)^k(k+1)^2$	1	
$= (-1)^k[-\frac{k(k+1)}{2} + (k+1)^2]$	1M	OR $= (-1)^{k-1}[\frac{k(k+1)}{2} - (k+1)^2]$
$= (-1)^k\frac{(k+1)}{2}[-k + 2(k+1)]$		For $(-1)^{k-1}A + (-1)^k B$ $= (-1)^{k-1}(A - B)$
$= (-1)^k\frac{(k+1)(k+2)}{2}$	1	or $= (-1)^k(-A + B)$
The statement is also true for $n = k + 1$ if it is true for $n = k$. By the principle of mathematical induction,		
the statement is true for all positive integers n.	1	
	6	

Solution	Marks	Remarks
5. 		
(a) The coordinates of P are $(\frac{2+r}{1+r}, \frac{2r}{1+r})$.	1A+1A	
(b) Slope of $OP = \frac{2r}{1+r} \div (\frac{2+r}{1+r})$		
$= \frac{2r}{2+r}$	1	
(c) $\tan \angle AOP = \frac{m_{OA} - m_{OP}}{1 + m_{OA} m_{OP}}$		
$= \frac{2 - \frac{2r}{2+r}}{1 + 2(\frac{2r}{2+r})}$	1M	For $\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$ or $\frac{m_1 - m_2}{1 + m_1 m_2}$
$= \frac{2(2+r) - 2r}{2 + r + 4r}$ $= \frac{4}{5r+2}$		
$\frac{4}{5r+2} = \tan 45°$	1M	
$4 = 5r + 2$		
$r = \frac{2}{5}$	1A	
<u>Alternative solution (1)</u> Let $\angle AOB = \theta$. $\tan\theta = 2$ $\tan \angle POB = \frac{2r}{2+r}$		
$\tan(\theta - 45°) = \frac{2r}{2+r}$ $\frac{\tan\theta - \tan 45°}{1 + \tan\theta \tan 45°} = \frac{2r}{2+r}$	1M	<u>OR</u> $\tan\theta = \tan(45° + \angle POB)$ $2 = \frac{\tan 45° + \frac{2r}{2+r}}{1 - \tan 45°(\frac{2r}{2+r})}$
$\frac{2-1}{2+1} = \frac{2r}{2+r}$	1M	
$2 + r = 6r$		
$r = \frac{2}{5}$	1A	

Solution	Marks	Remarks
Alternative solution (2)		
Area of $\Delta AOB = \frac{1}{2}(2)(2) = 2$		
Area of $\Delta OBP = \frac{1}{2}(2)(\frac{2r}{2+r}) = \frac{2r}{2+r}$		
Area of $\Delta OAP = \frac{1}{2}(OA)(OP)\sin 45°$		
$= \frac{1}{2}(\sqrt{5})\sqrt{\frac{(2+r)^2+(2r)^2}{(1+r)^2}}\sin 45°$	1M	For area $= \frac{1}{2}ab\sin c$
$= \frac{\sqrt{10}}{4}(\frac{\sqrt{5r^2+4r+4}}{1+r})$		
$\therefore \frac{\sqrt{10}}{4}(\frac{\sqrt{5r^2+4r+4}}{1+r}) = 2 - \frac{2r}{1+r}$	1M	
$\sqrt{5r^2+4r+4} = \frac{8}{\sqrt{10}}$		
$25r^2+20r-12=0$		
$r = \frac{2}{5}$ or $-\frac{6}{5}$ (rejected)		
$\therefore r = \frac{2}{5}$	1A	
	6	

Solution	Marks	Remarks
6. (a) Slope of $y=-x+1$ is -1.		
At point A, $\frac{dy}{dx}=-1$.		
$2x+3=-1$	1M	
$x=-2$	1A	
Put $x=-2$, $y=-(-2)+1$		
$y=3$	1A	
$\therefore$ the coordinates of A are $(-2, 3)$.		
(b) $y=\int(2x+3)\,dx$	1M	
$=x^2+3x+c$ c is a constant	1A	Awarded even if c was omitted
Put $x=-2$, $y=3$: $3=(-2)^2+3(-2)+c$	1M	For finding c
$c=5$	1A	Withhold this mark if "$y=$" was omitted
$\therefore$ the equation of the curve is $y=x^2+3x+5$.		
Alternative solution		
6. $y=\int(2x+3)\,dx$	1M	
$=x^2+3x+c$ c is a constant	1A	Awarded even if c was omitted
$\begin{cases} y=x^2+3x+c \\ y=-x+1 \end{cases}$		
$x^2+3x+c=-x+1$	1M	
$x^2+4x+c-1=0$		
$\Delta=16-4(c-1)=0$	1M	
$c=5$	1A	Withhold this mark if "$y=$" was omitted
$\therefore$ the equation of the curve is $y=x^2+3x+5$.		
$x^2+4x+5-1=0$		
$x=-2$	1A	
$y=-x+1=3$	1A	
$\therefore$ the coordinates of A are $(-2, 3)$.		
	7	

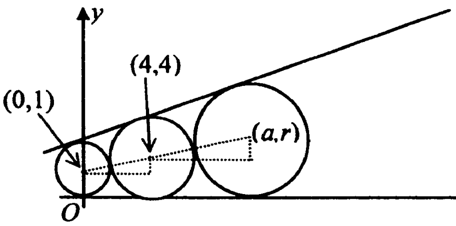
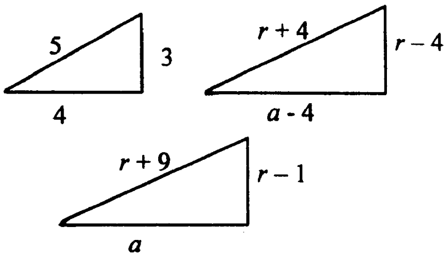
Solution	Marks	Remarks
7. (a) $\cos x-\sqrt{3}\sin x$		
$=2(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)$		$\begin{cases} r\cos\theta=1 \\ r\sin\theta=\sqrt{3} \end{cases}$ $r=2\,,\theta=\frac{\pi}{3}$
$=2(\cos\frac{\pi}{3}\cos x-\sin\frac{\pi}{3}\sin x)$		
$=2\cos(x+\frac{\pi}{3})$ OR $=2\cos(x-\frac{5\pi}{3})$	1A+1A	
$\cos x-\sqrt{3}\sin x=2$		
$2\cos(x+\frac{\pi}{3})=2$	1M	
$\cos(x+\frac{\pi}{3})=1$		
$x+\frac{\pi}{3}=2n\pi\pm 0$ n is an integer	1M	For $2n\pi\pm\alpha$
$x=2n\pi-\frac{\pi}{3}$ (OR = $360n°-60°$)	1A	$2n\pi-60°$ etc. $(u-1)$
$=\frac{(6n-1)\pi}{3}$		
Alternative solution		
$\cos x-\sqrt{3}\sin x$		
$=2(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x)$		
$=2(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x)$		
$=2\sin(\frac{\pi}{6}-x)$	1A+1A	
$\cos x-\sqrt{3}\sin x=2$		
$2\sin(\frac{\pi}{6}-x)=2$	1M	
$\sin(\frac{\pi}{6}-x)=1$		
$\frac{\pi}{6}-x=n\pi+(-1)^n\frac{\pi}{2}$	1M	For $n\pi+(-1)^n\alpha$
$x=\frac{\pi}{6}-n\pi-(-1)^n\frac{\pi}{2}.$	1A	
(b) $\begin{cases} y=\cos x \\ y=2+\sqrt{3}\sin x \end{cases}$ $\cos x=2+\sqrt{3}\sin x$ $\cos x-\sqrt{3}\sin x=2$	1M	For either one
From (a), $x=\frac{(6n-1)\pi}{3}$.		

Solution	Marks	Remarks
For $0<x<9\pi$, $0<\frac{(6n-1)\pi}{3}<9\pi$ $0<6n-1<27$ $1<6n<28$ $\frac{1}{6}<n<\frac{14}{3}$ $1\le n\le 4$	1M	For attempting to count
∴ there are 4 points of intersection.	1A	Awarded only if (a) was correct
Alternative solution $\begin{cases} y=\cos x \\ y=2+\sqrt{3}\sin x \end{cases}$ $\cos x=2+\sqrt{3}\sin x$ $\cos x-\sqrt{3}\sin x=2$ From (a), $x=\frac{(6n-1)\pi}{3}$.	1M	For either one
n x 0 $-\frac{\pi}{3}<0$ 1 $\frac{5\pi}{3}$ 2 $\frac{11\pi}{3}$ 3 $\frac{17\pi}{3}$ 4 $\frac{23\pi}{3}$ 5 $\frac{29\pi}{3}>9\pi$	1M	OR $x=2n\pi-\frac{\pi}{3}$ Since $\frac{9\pi}{2\pi}=4.5$, ---- 1M So there are 4 points of intersection ---- 1A
∴ there are 4 points of intersections.	1A	
	8	

Solution	Marks	Remarks
8. (a) $\int \cos 3x \cos x \, dx = \int \frac{1}{2}(\cos 4x + \cos 2x) \, dx$	1M+1A	
$= \frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + c$ c is a constant	1A	Withhold 1A for omitting c
	3	
(b) $\frac{\sin 5x - \sin x}{\sin x} = \frac{2\sin 2x \cos 3x}{\sin x}$	1A	
$= \frac{2(2\sin x \cos x)\cos 3x}{\sin x}$		
$= 4\cos x \cos 3x$	1	
Alternative solution		
$4\sin x \cos x \cos 3x = 2\sin 2x \cos 3x$	1A	OR $= 2\sin x(\cos 4x + \cos 2x)$ OR $= 2\cos x(\sin 4x - \sin 2x)$
$= \sin 5x - \sin x$		
$\therefore \frac{\sin 5x - \sin x}{\sin x} = 4\cos x \cos 3x$	1	
$\int \frac{\sin 5x}{\sin x} dx = \int (1 + 4\cos 3x \cos x) dx$	1M	
$= x + 4(\frac{\sin 4x}{8} + \frac{\sin 2x}{4}) + c$ c is a constant		
$= x + \frac{1}{2}\sin 4x + \sin 2x + c$	1A	
Alternative solution		
$\int \frac{\sin 5x}{\sin x} dx = \int \frac{\sin 3x \cos 2x + \sin 2x \cos 3x}{\sin x} dx$		
$= \int \frac{(3\sin x - 4\sin^3 x)\cos 2x + 2\sin x \cos x \cos 3x}{\sin x} dx$		
$= \int [(3 - 4\sin^2 x)\cos 2x + 2\cos x \cos 3x] \, dx$		
$= \int [3\cos 2x - 2(1 - \cos 2x)\cos 2x] \, dx + 2\int \cos x \cos 3x \, dx$		
$= \int \cos 2x \, dx + \int (1 + \cos 4x) dx + 2\int \cos x \cos 3x \, dx$	1M	
$= \frac{1}{2}\sin 2x + x + \frac{1}{4}\sin 4x + 2(\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x) + c$		
$= x + \frac{1}{2}\sin 4x + \sin 2x + c$	1A	
	4	

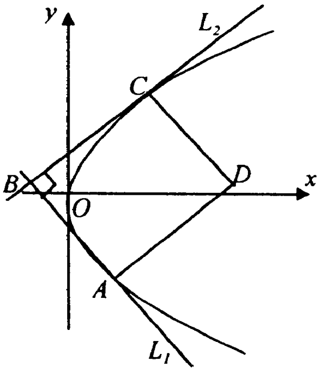
Solution	Marks	Remarks
(c) Put $x=\frac{\pi}{2}-\theta$:	1A	
$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\sin 5x}{\sin x}\mathrm{d}x=\int_{\frac{\pi}{3}}^{\frac{\pi}{4}}\frac{\sin 5(\frac{\pi}{2}-\theta)}{\sin(\frac{\pi}{2}-\theta)}(-\mathrm{d}\theta)$	1A+1A	1A for integrand, 1A for limit
$\boxed{=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos 5\theta}{\cos\theta}\mathrm{d}\theta}$		
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos 5x}{\cos x}\mathrm{d}x$	1	
	4	
(d) Area of shaded region		
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(\frac{\cos 5x}{\cos x}-\frac{\sin 5x}{\sin x})\mathrm{d}x$	1M+1A	1M for $A=\int_a^b(y_2-y_1)\mathrm{d}x$
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos 5x}{\cos x}\mathrm{d}x-\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sin 5x}{\sin x}\mathrm{d}x$		
$=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{\sin 5x}{\sin x}\mathrm{d}x-\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\sin 5x}{\sin x}\mathrm{d}x$ (using (c))	1A	For 1st term
$=[x+\frac{1}{2}\sin 4x+\sin 2x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}-[x+\frac{1}{2}\sin 4x+\sin 2x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$	1M	For using (b)
$=(\frac{\pi}{4}+1)-(\frac{\pi}{6}+\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2})-[(\frac{\pi}{3}-\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2})-(\frac{\pi}{4}+1)]$		
$=2-\sqrt{3}$	1A	
Alternative solution		
Area of shaded region		
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}(\frac{\cos 5x}{\cos x}-\frac{\sin 5x}{\sin x})\mathrm{d}x$	1M+1A	Same as above
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\cos 5x\sin x-\sin 5x\cos x}{\cos x\sin x}\mathrm{d}x$		
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{-\sin 4x}{\sin x\cos x}\mathrm{d}x$	1A	
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{-2(2\sin x\cos x)\cos 2x}{\sin x\cos x}\mathrm{d}x$		
$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}-4\cos 2x\mathrm{d}x$		
$=[-2\sin 2x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$	1M	
$=2-\sqrt{3}$	1A	
	5	

Solution	Marks	Remarks
9. (a) (i) Put $k=-1$ into F, the equation becomes $x^2+y^2+(-4+4)x+(-3+1)y-(-8+8)=0$ i.e. $x^2+y^2-2y=0$. $\therefore$ C_1 is a circle in F.	1A+1	1A for $k=-1$
<u>Alternative solution</u> Comparing coefficients of F and C_1.	1A	
$\begin{cases}4k+4=0\\3k+1=-2\\8k+8=0\end{cases}$ $k=-1$ satisfies the above 3 equations.		
$\therefore$ C_1 is a circle in F.	1	
(ii) Put $y=0$ into $x^2+y^2-2y=0$: $x^2=0$ $x=0$	1M	
i.e. there is only one intersection point with the x-axis. $\therefore$ C_1 touches the x-axis.	1	
<u>Alternative solution</u> Centre of $C_1=(0,1)$. radius of $C_1=1$	1A	
Since the y-coordinate of centre is equal to the radius, C_1 touches the x-axis.	1	
	4	
(b) (i) Put $y=0$ in F: $x^2+(4k+4)x-(8k+8)=0$	1M	For putting $y=0$
Since the circle touches the x-axis, $(4k+4)^2+4(8k+8)=0$	1M	For $\Delta=0$
$16k^2+64k+48=0$ $16(k+1)(k+3)=0$ $k=-1$ (rejected) or $k=-3$,	1A	
$\therefore$ The equation of C_2 is $x^2+y^2+[4(-3)+4]x+[3(-3)+1]y-[(-3)\times 8+8]=0$ $x^2+y^2-8x-8y+16=0$	1A	<u>OR</u> $(x-4)^2+(y-4)^2=16$

Solution	Marks	Remarks
<u>Alternative solution</u>		
$x^2+y^2+(4k+4)x+(3k+1)y-(8k+8)=0$		
$[x+(2k+2)]^2+[y+(\frac{3k+1}{2})]^2=(2k+2)^2+(\frac{3k+1}{2})^2+(8k+8)$	1M	For finding centre and radius of F
$=\frac{25k^2+70k+49}{4}$		
If C_2 touches the x-axis,		
$\left\|-(\frac{3k+1}{2})\right\|=\sqrt{\frac{25k^2+70k+49}{4}}$	1M	For equating radius = y coord. of centre Accept omitting absolute sign
$9k^2+6k+1=25k^2+70k+49$		
$16k^2+64k+48=0$		
$\boxed{k=-1\text{ (rejected)}}$ or $k=-3$	1A	
∴ The equation of C_2 is $x^2+y^2-8x-8y+16=0$.	1A	
(ii) Centre of $C_1=(0,1)$, radius = 1. Centre of $C_2=(4,4)$, radius = 4.	1M	
Distance between centres $=\sqrt{(4-0)^2+(4-1)^2}$	1M	
$=5$		
= sum of radii of C_1 and C_2 ∴ C_1 and C_2 touch externally.	1	
	7	
(c) Let radius of C_3 be r and coordinates of its centre be (a, r).	1M	For y-coord = radius
		
By similar triangles,		
$\frac{r+4}{1+4}=\frac{r-4}{4-1}$	1M	
$3r+12=5r-20$		
$r=16$		
$\frac{a-4}{4-0}=\frac{r+4}{4+1}$	1M	
$=\frac{16+4}{4+1}$		
$a=20$	1A	Awarded if either one was correct
∴ the equation of C_3 is $(x-20)^2+(y-16)^2=256$. $\boxed{\text{OR } x^2+y^2-40x-32y+400=0}$	1A	

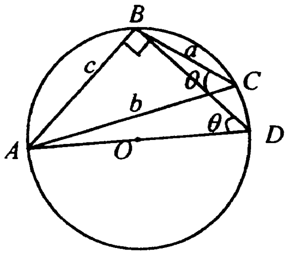
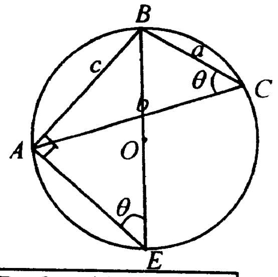
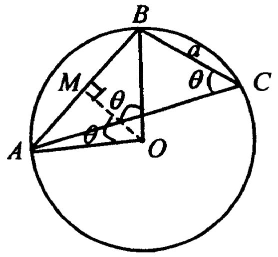
Solution	Marks	Remarks
<u>Alternative solution (1)</u> Equation of line through centres of C_1, C_2 and C_3 $\frac{y-1}{x-0}=\frac{4-1}{4-0}$	1M	
$y=\frac{3}{4}x+1$ Let coordinates of centre of C_3 be $(a\,,\frac{3}{4}a+1)$, radius of $C_3=\frac{3}{4}a+1$.	1M	For y-coord. = radius
Distance between centres of C_2 and C_3 = sum of radii $\sqrt{(a-4)^2+(\frac{3}{4}a+1-4)^2}=\frac{3}{4}a+1+4$	1M	$\sqrt{(\frac{4r-4}{3}-4)^2+(r-4)^2}=r+4$ ⋮ $r^2-17r+16=0$ $r=16$
$a^2-8a+16+\frac{9}{16}a^2-\frac{9}{2}a+9=\frac{9}{16}a^2+\frac{15}{2}a+25$ $a^2-20a=0$ $\boxed{a=0\text{ (rejected)}}$ or $a=20$	1A	
$\therefore$ centre of C_3 = (20, 16) and radius = 16. $\therefore$ the equation of C_3 is $(x-20)^2+(y-16)^2=256$.	1A	
<u>Alternative solution (2)</u> Equation of line through centres of C_1, C_2 and C_3 ⋮	1M	Same as above
$y=\frac{3}{4}x+1$ $r=\frac{3}{4}a+1$ ---- (1) Using Pythagoras' Theorem, $(a-4)^2+(r-4)^2=(r+4)^2$	1M	<u>OR</u> $a^2+(r-1)^2=(r+9)^2$ $a^2=20r+80$
$a^2-8a+16=16r$ ---- (2) Substitute (1) into (2) : $a^2-8a+16=16(\frac{3}{4}a+1)$	1M	$a^2=20(\frac{3}{4}a+1)+80$
$a^2-20a=0$ $\boxed{a=0\text{ (rejected) or}}$ $a=20$	1A	$a^2-15a-100=0$ a = 20 or −5 (rejected)
⋮ $\therefore$ The equation of C_3 is $(x-20)^2+(y-16)^2=256$.	1A	
	5	

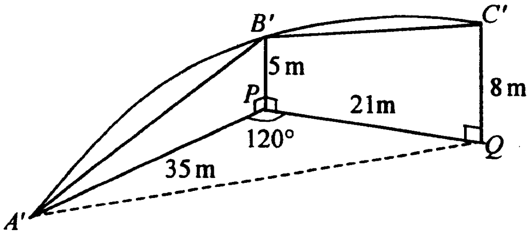
Solution	Marks	Remarks
10. (a) (i) $y^2=4x$		
$2y\frac{dy}{dx}=4$		
$\frac{dy}{dx}=\frac{2}{y}$	1M	
At point A, $\frac{dy}{dx}=\frac{2}{2t_1}=\frac{1}{t_1}$		
Equation of L_1 is		
$\frac{y-2t_1}{x-t_1^2}=\frac{1}{t_1}$		
$t_1y-2t_1^2=x-t_1^2$		
$x-t_1y+t_1^2=0$	1	
<u>Alternative solution</u>		
Using the formula $yy_1=2(x+x_1)$, the equation of L_1 is $y(2t_1)=2(x+t_1^2)$	1A	
i.e. $x-t_1y+t_1^2=0$.	1	
(ii) Equation of L_2 is $x-t_2y+t_2^2=0$.	1A	
$\begin{cases}x-t_1y+t_1^2=0 & \text{------(1)}\\ x-t_2y+t_2^2=0 & \text{------(2)}\end{cases}$		
$(1)-(2)$: $(t_2-t_1)y+(t_1^2-t_2^2)=0$	1M	For solving (1) and (2)
$y=t_1+t_2$		
$x=t_1(t_1+t_2)-t_1^2=t_1t_2$	1	
$\therefore$ the coordinates of B are (t_1t_2,t_1+t_2).		
(iii) The coordinates of M are $(\frac{t_1^2+t_2^2}{2},\frac{2t_1+2t_2}{2})$,	1M	For finding y-coord. of M
i.e. $(\frac{t_1^2+t_2^2}{2},t_1+t_2)$.		
As the y-coordinates of B and M are equal, BM is parallel to the x-axis.	1	
<u>Alternative solution</u>		
The coordinates of M are $(\frac{t_1^2+t_2^2}{2},\frac{2t_1+2t_2}{2})$.	1M	
Slope of $BM=\frac{(t_1+t_2)-(t_1+t_2)}{\frac{t_1^2+t_2^2}{2}-t_1t_2}$		
$\boxed{=\frac{0}{(t_1-t_2)}}$		
$=0$		
$\therefore$ BM is parallel to the x-axis.	1	
	7	

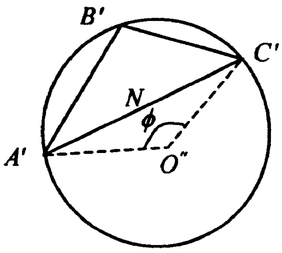
Solution	Marks	Remarks
(b) (i) (Slope of L_1) (Slope of L_2) = −1		
$(\frac{1}{t_1})(\frac{1}{t_2})=-1$	1A	
$t_1t_2=-1$	1A	
(ii) Since *ABCD* is a rectangle, mid-point of *BD* coincides with mid-point of *AC*, i.e. point *M*.		
$\frac{x+t_1t_2}{2}=\frac{t_1^2+t_2^2}{2}$	1M	
$x=t_1^2+t_2^2-t_1t_2$		
$=t_1^2+t_2^2+1$ $\because t_1t_2=-1$	1	
Since *BD* is parallel to the *x*-axis, the *y*-coordinate of *D* = *y*-coordinate of *B* = t_1+t_2.	1	OR $\frac{y+t_1+t_2}{2}=t_1+t_2$ $y=t_1+t_2$
∴ the coordinates of *D* are $(t_1^2+t_2^2+1, t_1+t_2)$.		
Alternative solution		
Equation of *AD* is		
$\frac{y-2t_1}{x-t_1^2}=-t_1$		
$t_1x+y=2t_1+t_1^3$		
Similarly, equation of *CD* is $t_2x+y=2t_2+t_2^3$.	1M	
$\begin{cases} t_1x+y=2t_1+t_1^3 \text{ ------(3)} \\ t_2x+y=2t_2+t_2^3 \text{ ------(4)} \end{cases}$		
(3)−(4) : $(t_1-t_2)x=2(t_1-t_2)+(t_1^3-t_2^3)$		
$x=2+(t_1^2+t_1t_2+t_2^2)$		
$=2+t_1^2+t_2^2-1$		
$=t_1^2+t_2^2+1$	1	
$y=-t_1(t_1^2+t_2^2+1)+2t_1+t_1^3$		
$=-t_1t_2^2+t_1$		
$=t_1+t_2$	1	
∴ the coordinates of *D* are $(t_1^2+t_2^2+1,\ t_1+t_2)$.		
(iii) Let (*x*, *y*) be the coordinates of *D*.		
$\begin{cases} x=t_1^2+t_2^2+1 \\ y=t_1+t_2 \end{cases}$	1A	
$x=(t_1+t_2)^2-2t_1t_2+1$	1M+1M	1M for using $t_1^2+t_2^2=(t_1+t_2)^2-2t_1t_2$
$=y^2-2(-1)+1$		1M for eliminating t_1, t_2
$x=y^2+3$	1A	Accept equivalent forms
∴ the equation of the locus is $x-y^2-3=0$.		
	9	

Solution	Marks	Remarks
11. (a) Volume $= \int_{-h}^{0} \pi x^2 \mathrm{d}y$	1M	1M for $\pi \int_{a}^{b} x^2 \mathrm{d}y$
$= \int_{-h}^{0} \pi(r^2 - y^2)\mathrm{d}y$	1A	
$= \pi[r^2 y - \frac{1}{3} y^3]_{-h}^{0}$	1A	For $[r^2 y - \frac{1}{3} y^3]$
$= \pi(r^2 h - \frac{1}{3} h^3)$ cubic units	1	
	4	
(b) Put $h = 1, r = \sqrt{\frac{89}{3}}$: Using (a), capacity of the mould $= \pi [\frac{89}{3}(1) - \frac{1}{3}(1)^3]$	1A	Accept omitting either one OR $= \pi \int_{-1}^{0} (\frac{89}{3} - y^2)\mathrm{d}y$
$= \frac{88\pi}{3}$ cubic units	1	
	2	
(c) (i) (1) Distance $= 4 \sin\theta$.	1A	
(2) Put $r = 4, h = 4 \sin\theta$. Using (a), amount of gold poured into the pot		
$= \pi[4^2 (4 \sin\theta) - \frac{1}{3}(4 \sin\theta)^3]$	1M	
$= \pi(64 \sin\theta - \frac{64}{3} \sin^3\theta)$	1A	
Alternative solution Amount of gold $= \frac{2}{3}\pi(4)^3 - \int_{-4}^{-4\sin\theta} \pi(16 - y^2)\mathrm{d}y$	1M	OR $= \int_{-4\sin\theta}^{0} \pi(16 - y^2)\mathrm{d}y$
$= \frac{128\pi}{3} - \pi[-64 \sin\theta + \frac{64}{3} \sin^3\theta + 64 - \frac{64}{3}]$		
$= \pi(64 \sin\theta - \frac{64}{3} \sin^3\theta)$	1A	

Solution	Marks	Remarks
(ii) When the mould is completely filled,		
$\pi(64\sin\theta - \frac{64}{3}\sin^3\theta) = \frac{88\pi}{3}$	1M	
$64\sin^3\theta - 192\sin\theta + 88 = 0$		
$8\sin^3\theta - 24\sin\theta + 11 = 0$ ----(*)	1	
Put $\sin\theta = \frac{1}{2}$: $8\sin^3\theta - 24\sin\theta + 11 = 0$. $\therefore \sin\theta = \frac{1}{2}$ is a root of (*)	1A	(can be omitted)
$(2\sin\theta - 1)(4\sin^2\theta + 2\sin\theta - 11) = 0$	1M+1A	1M for $(2\sin\theta - 1)(a\sin^2\theta + b\sin\theta + c) = 0$
$\sin\theta = \frac{1}{2}$ or $\sin\theta = \frac{-2\pm\sqrt{180}}{8}$ (rejected)	1A	
$\therefore \sin\theta = \frac{1}{2}$		
$\theta = \frac{\pi}{6}$	1A	Accept degrees
	10	

Solution	Marks	Remarks
12. (a) 		
(i) $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$	1A	
(ii) Consider ΔABD : $AD = 2r$ $\angle BDA = \angle BCA = \theta$ $\angle ABD = 90^\circ$	1A	 **OR** Consider ΔABE $BE = 2r$ $\angle BEA = \theta$ $\angle BAE = 90^\circ$
$\therefore \sin\theta = \dfrac{c}{2r}$	1A	
$r = \dfrac{c}{2\sin\theta}$	1	
Alternative solution (1) Using the theorem $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2r$, $\dfrac{c}{\sin\theta} = 2r$	2A	For either one
$r = \dfrac{c}{a\sin\theta}$	1	
Alternative solution (2) 		
$\angle AOB = 2\angle ACB$ $= 2\theta$ Consider ΔAOM (M is the mid-point of AB) :	1A	
$\sin\theta = \dfrac{AM}{AO}$		
$\sin\theta = \dfrac{\frac{1}{2}c}{r}$	1A	
$r = \dfrac{c}{2\sin\theta}$	1	

Solution	Marks	Remarks
(iii) $\sin^2\theta+\cos^2\theta=1$		
$(\frac{c}{2r})^2+(\frac{a^2+b^2-c^2}{2ab})^2=1$	1M+1A	
$\frac{c^2}{4r^2}=1-\frac{(a^2+b^2-c^2)^2}{4a^2b^2}$		
$r^2=\frac{a^2b^2c^2}{4a^2b^2-(a^2+b^2-c^2)^2}$		
$r=\frac{abc}{\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}}$	1	
Alternative solution		
$\sin^2\theta=1-\cos^2\theta$		
$=1-(\frac{a^2+b^2-c^2}{2ab})^2$		
$=\frac{4a^2b^2-(a^2+b^2-c^2)^2}{4a^2b^2}$		
$\sin\theta=\frac{\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}}{2ab}$		
$\therefore\ r=\frac{c}{2\sin\theta}$		
$=\frac{c}{2(\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}/2ab}$	1M+1A	
$=\frac{abc}{\sqrt{4a^2b^2-(a^2+b^2-c^2)^2}}$	1	
	7	
(b) 		
(i) Consider $\Delta A'B'C'$: $A'B'=\sqrt{(A'P)^2+(PB')^2}$		
$=\sqrt{35^2+5^2}$	1A	
$=\sqrt{1250}$ (≈ 35.36)		
$B'C'=\sqrt{(PQ)^2+(QC'-PB')^2}$		
$=\sqrt{21^2+(8-5)^2}$	1A	
$=\sqrt{450}$ (≈ 21.21)		

Solution	Marks	Remarks
$(A'Q)^2 = (A'P)^2 + (PQ)^2 - 2(A'P)(PQ)^2 \cos\angle A'PQ$ $= 35^2 + 21^2 - 2(35)(21)\cos 120°$ $= 2401$		
$A'C' = \sqrt{(A'Q)^2 + (QC')^2}$ $= \sqrt{2401 + 8^2}$ $= \sqrt{2465}$ (≈ 49.65)	1M	
Using (a) (iii), put $a = \sqrt{450}$, $b = \sqrt{2465}$, $c = \sqrt{1250}$:		Accept other combinations
$r = \frac{\sqrt{1250}\sqrt{450}\sqrt{2465}}{\sqrt{4(450)(2465) - (450 + 2465 - 1250)^2}}$	1M	
$\boxed{= 28.86}$		
$= 29$ m (correct to 2 sig. figures) ∴ the radius of arc $A'B'C'$ is 29 m.	1A	Omit/wrong unit $(u - 1)$
(ii) 		
Let O' be the centre of the circle passing through A', B' and C', ϕ be the angle subtended by arc $A'B'C'$ at O'.		
$\cos\angle A'B'C' = \frac{1250 + 450 - 2465}{2\sqrt{1250}\sqrt{450}}$ $= -0.51$	1M	OR $\frac{A'C'}{\sin\angle A'B'C'} = 2r$ $\sin\angle A'B'C' = \frac{\sqrt{2465}}{2(28.86)}$ ----- 1M $= 0.8602$
$\angle A'B'C' = 2.106$ $\boxed{\text{OR } 120.66°}$		
$\phi = 2\pi - 2(\angle A'B'C')$ $= 2\pi - 2(2.106)$ $= 2.07$ $\boxed{\text{OR } 118.67°}$	1M	

Solution	Marks	Remarks
<u>Alternative solution (1)</u> Consider $\Delta OA'N$ (N is the mid-point of $A'C'$) $\sin\frac{\phi}{2} = \frac{\frac{1}{2}A'C'}{r}$ $\sin\frac{\phi}{2} = \frac{\frac{1}{2}\sqrt{2465}}{28.86}$ $= 0.8602$ $\phi = 2.07$ $\boxed{\underline{\text{OR}}\ 118.67^\circ}$	2M	<u>OR</u> $(A'C')^2 = r^2 + r^2 - 2r^2\cos\phi$ $(\sqrt{2465})^2 =^2 (28.86)^2 - 2(28.86)^2\cos\phi$ $\cos\phi = -0.4798$
<u>Alternative solution (2)</u> $\sin\angle A'C'B' = \frac{A'B'}{2r}$ $= \frac{\sqrt{1250}}{2(28.86)}$ $\angle A'C'B' = 0.6592$ $\boxed{\underline{\text{OR}}\ 37.77^\circ}$	1M	
$\sin\angle B'A'C' = \frac{B'C'}{2r}$ $= \frac{\sqrt{450}}{2(28.86)}$ $\angle B'A'C' = 0.3763$ $\boxed{\underline{\text{OR}}\ 21.56^\circ}$ $\phi = \angle A'O'B' + \angle B'O'C'$ $= 2(\angle A'C'B' + \angle B'A'C')$ $= 2(0.6592 + 0.3763)$ $= 2.07$ $\boxed{\underline{\text{OR}}\ 118.67^\circ}$	1M	
Length of walkway $=$ length of $\overset{\frown}{A'B'C'} = r\phi$ $= 28.86\ (2.07)$ $\boxed{=59.77}$	1M	
$= 60$ m (correct to 2 sig. figures) $\therefore$ the length of the walkway is 60 m.	1A 9	Omit/wrong unit ($u - 1$)