**香港考試局**
**HONG KONG EXAMINATIONS AUTHORITY**

**一九九七年香港中學會考**
**HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997**

**附加數學 試卷二**
**ADDITIONAL MATHEMATICS PAPER II**





只限教師參閱 FOR TEACHERS' USE ONLY

## GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp−1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u−1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points :

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the net total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚ , whereas alternative answers are enclosed by solid rectangles ▭ .

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.



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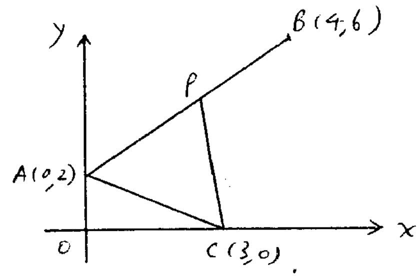
| Solution | Marks | Remarks |
|---|---|---|
| 1. $\frac{\sin 3\theta}{\sin\theta}+\frac{\cos 3\theta}{\cos\theta}$ | | |
| $=\frac{\sin 3\theta\cos\theta+\sin\theta\cos 3\theta}{\sin\theta\cos\theta}$ | | |
| $=\frac{\sin 4\theta}{\sin\theta\cos\theta}$ | 1A | For numerator only |
| $=\frac{2\sin 2\theta\cos 2\theta}{\sin\theta\cos\theta}$ | 1A | For numerator only |
| $=\frac{4\sin\theta\cos\theta\cos 2\theta}{\sin\theta\cos\theta}$ OR $=\frac{2\sin 2\theta\cos 2\theta}{\frac{1}{2}\sin 2\theta}$ | 1A | |
| $=4\cos 2\theta$ | 1 | |
| Alternative solution | | |
| $\frac{\sin 3\theta}{\sin\theta}+\frac{\cos 3\theta}{\cos\theta}$ | | |
| $=\frac{3\sin\theta-4\sin^3\theta}{\sin\theta}+\frac{4\cos^3\theta-3\cos\theta}{\cos\theta}$ | 1A+1A | For numerator only |
| $=3-4\sin^2\theta+4\cos^2\theta-3$ | | |
| $=4(\cos^2\theta-\sin^2\theta)$ | 1A | |
| $=4\cos 2\theta$ | 1 | |
| | 4 | |
| 2. Let $u=x-1, du=dx$ | | |
| $\int x\sqrt{x-1}\,dx$ | | |
| $=\int (u+1)u^{\frac{1}{2}}\,du$ | 1A | Omit $du$ (pp−1) |
| $=\int (u^{\frac{3}{2}}+u^{\frac{1}{2}})\,du$ | 1A | |
| $=\frac{2}{5}u^{\frac{5}{2}}+\frac{2}{3}u^{\frac{3}{2}}+c$   $c$ is a constant | 1A | For primitive function only |
| $=\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+c$ OR $=\frac{2}{15}(3x+2)(x-1)^{\frac{3}{2}}+c$ | 1A | no mark if '$c$' is omitted |
| Alternative solution | | |
| Let $u=\sqrt{x-1}, du=\frac{1}{2\sqrt{x-1}}dx$ | | OR Let $u^2=x-1$. |
| $\int x\sqrt{x-1}\,dx$ | | |
| $=\int (u^2+1)u(2u\,du)$ | 1A | |
| $=\int (2u^4+2u^2)\,du$ | 1A | |
| $=\frac{2}{5}u^5+\frac{2}{3}u^3+c$   $c$ is a constant | 1A | |
| $=\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+c$ | 1A | |
| | 4 | |

| Solution | Marks | Remarks |
|---|---|---|
| 3. (a) The coordinates of $P$ are $\left(\frac{4\lambda}{1+\lambda}, \frac{6\lambda+2}{1+\lambda}\right)$. | 1A+1A | |
| (b) $\frac{1}{2}\begin{vmatrix} 0 & 2 \\ 3 & 0 \\ \frac{4\lambda}{1+\lambda} & \frac{6\lambda+2}{1+\lambda} \\ 0 & 2 \end{vmatrix} = 6$ OR $\frac{1}{2}\begin{vmatrix} 0 & 2 \\ 3 & 0 \\ \frac{4\lambda}{1+\lambda} & \frac{6\lambda+2}{1+\lambda} \\ 0 & 2 \end{vmatrix} = \pm 6$ | 1M+1A | 1M for LHS   |
| $\frac{1}{2}\left(\frac{18\lambda+6}{1+\lambda}+\frac{8\lambda}{1+\lambda}-6\right)=6$ OR $\frac{1}{2}\left(\frac{18\lambda+6}{1+\lambda}+\frac{8\lambda}{1+\lambda}-6\right)=\pm 6$ | | |
| $\lambda=\frac{3}{2}$ or $-\frac{3}{8}$ (rejected) | | |
| $\lambda=\frac{3}{2}$ ; $\lambda=\frac{3}{2}$ | 1A | |
| (b) Alternative solution (1) | | |
| Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 2 \\ 3 & 0 \\ 4 & 6 \\ 0 & 2 \end{vmatrix} = 10$ | | |
| Area of $\Delta PAC = \frac{\lambda}{\lambda+1}$(Area of $\Delta ABC$) | 1M | |
| $6 = \frac{\lambda}{\lambda+1}(10)$ | 1A | |
| $\lambda=\frac{3}{2}$ | 1A | |
| Alternative solution (2) | | |
| $AC = \sqrt{(0-3)^2+(2-0)^2} = \sqrt{13}$ | | |
| Equation of $AC$ is $\frac{x}{3}+\frac{y}{2}=1$ <br> $2x+3y-6=0$ | | |
| Distance from $P$ to $AC$ <br> $=\frac{1}{\sqrt{13}}\left\lvert 2\left(\frac{4\lambda}{1+\lambda}\right)+3\left(\frac{6\lambda+2}{1+\lambda}\right)-6\right\rvert$ <br> $=\frac{20\lambda}{\sqrt{13}(1+\lambda)}$ | | |
| $\therefore \frac{1}{2}\left(\frac{20\lambda}{\sqrt{13}(1+\lambda)}\right)(\sqrt{13})=6$ | 1M+1A | 1M for LHS |
| $\frac{10\lambda}{1+\lambda}=6$ | | |
| $\lambda=\frac{3}{2}$ | 1A | |
| | 5 | |



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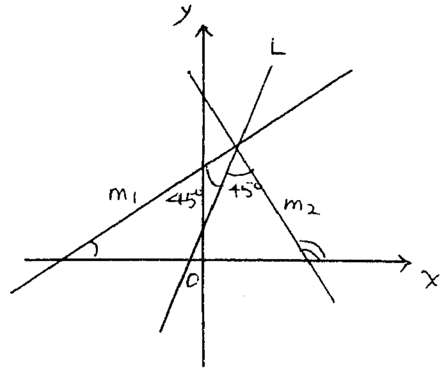
| Solution | Marks | Remarks |
|---|---|---|
| 4. $6\sin x+8\cos x$ | | |
| $=10(\frac{6}{10}\sin x+\frac{8}{10}\cos x)$ | 1M | $\begin{cases} r\cos\alpha=6 \\ r\sin\alpha=8 \end{cases}$ $r=10, \alpha=53.13°$ |
| $\approx 10\sin(x+53.13°)$ | 1A+1A | Accept $\alpha=53°$ |
| OR $=10\sin(x+\alpha)$ where $\cos\alpha=\frac{6}{10}$ | | |
| $6\sin x+8\cos x=5$ | | |
| $10\sin(x+\alpha)=5$ | | |
| $x+\alpha=180n°+(-1)^n 30°$   $n$ is an integer | 1M | For $180n°+(-1)^n\theta$ |
| $x=180n°+(-1)^n 30°-53°$ correct to the nearest degree | 1A | $n\pi+(-1)^n(\frac{\pi}{6})-53°$ etc. (u−1) |
| | 5 | |
| 5. $\frac{dy}{dx}=6x+\frac{1}{x^2}$ | | |
| $y=\int(6x+\frac{1}{x^2})dx$ | 1M | Withhold this mark if "$y=$" is omitted<br>Omit 'dx' (pp−1) |
| $y=3x^2-\frac{1}{x}+c$   $c$ is a constant | 1A+1A | 1A for $3x^2$, 1A for $-\frac{1}{x}$<br>Withhold 1 mark if $c$ is omitted. |
| Put $x=1, y=0$   $0=3-1+c$<br>$c=-2$ | 1M | |
| ∴ The equation of the curve is $y=3x^2-\frac{1}{x}-2$. | 1A | |
| | 5 | |

| Solution | Marks | Remarks |
|---|---|---|
| 6. (a) $\left\lvert \dfrac{m-2}{1+2m} \right\rvert = \tan 45°$ | 1A | (pp−1) if absolute sign is omitted |
| $\dfrac{m-2}{1+2m} = \pm 1$ | |   |
| $m = \dfrac{1}{3}$ or $-3$ | 1A+1A | |
| Alternative solution | | |
| $\dfrac{1+m_1}{1-m_1} = 2$ | 1A | |
| $m_1 = \dfrac{1}{3}$ | 1A | |
| $m_2 = -3$ ($\because$ The 2 lines are perpendicular.) | 1A | OR $\dfrac{1+2}{1-2} = m_2$, $m_2 = -3$ |
| (b) $2x-3y+2+k(x-y-1)=0$ | | |
| $(2+k)x-(3+k)y+(2-k)=0$ | | |
| Slope $= \dfrac{2+k}{3+k}$ | 1A | |
| From (a), $\dfrac{2+k}{3+k} = \dfrac{1}{3}$ | 1M | |
| $k = -\dfrac{3}{2}$ | | |
| $\therefore$ The equation of the line is $2x-3y+2-\dfrac{3}{2}(x-y-1)=0$ | | |
| $x-3y+7=0$ | 1A | |
| Alterantive solution | | |
| $\begin{cases} 2x-3y+2=0 \\ x-y-1=0 \end{cases}$ | 1A | |
| Solving the two equations, $x=5$, $y=4$. | | |
| From (a), the equation of the line is | | |
| $\dfrac{y-4}{x-5} = \dfrac{1}{3}$ | 1M | |
| $x-3y+7=0$ | 1A | |
| | 6 | |



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| Solution | Marks | Remarks |
|---|---|---|
| 7. For $n=1$, $T_1=(1+1)(1!)=2$ | | |
| $=1[(1+1)!]$ | | |
| ∴ The statement is true for $n=1$. | 1 | |
| Assume $T_1+T_2+\ldots+T_k=k[(k+1)!]$ for some positive integer $k$. | 1 | |
| Then $T_1+T_2+\ldots+T_k+T_{k+1}$ | | |
| $=k[(k+1)!]+[(k+1)^2+1][(k+1)!]$ | 1 | |
| $=(k+1)![k+k^2+2k+2]$ | 1 | |
| $=(k+1)!(k^2+3k+2)$ | | |
| $=(k+1)!(k+1)(k+2)$ | | |
| $=(k+1)[(k+2)!]$ | 1 | |
| ∴ The statement is also true for $n=k+1$ if it is true for $n=k$. By the principle of mathematical induction, the statement is true for all positive integers $n$. | 1 | |
| | 6 | |
| 8. $(1+x)^n(1-2x)^4$ | | |
| $=(1+{}_nC_1x+{}_nC_2x^2+\ldots)[1+{}_4C_1(-2x)+{}_4C_2(-2x)^2+\ldots]$ | 1A+1A | |
| $=(1+{}_nC_1x+{}_nC_2x^2+\ldots)(1-8x+24x^2+\ldots)$ | | |
| $=1+({}_nC_1-8)x+({}_nC_2-8{}_nC_1+24)x^2+\ldots$ | 1M+1A | (pp–1) for omitting dots |
| OR $=1+(n-8)x+[\frac{n(n-1)}{2}-8n+24]x^2+\ldots$ | | |
| If the coefficient of $x^2=54$, | | |
| ${}_nC_2-8{}_nC_1+24=54$ | 1M | |
| $\frac{n(n-1)}{2}-8n+24=54$ | | |
| $n^2-17n-60=0$ | | |
| $(n+3)(n-20)=0$ | | |
| $n=20$    $n=-3$ is rejected | 1A | |
| Coefficient of $x={}_{20}C_1-8$ | | |
| $=12$ | 1A | |
| | 7 | |



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| Solution | Marks | Remarks |
|---|---|---|
| 9. (a) $\sqrt{(x-1)^2+y^2}=\lvert x+1 \rvert$ **OR** $(x-1)^2+y^2=(x+1)^2$ | 1A+1A | 1A for LHS, 1A for RHS |
| $(x^2-2x+1)+y^2=x^2+2x+1$ | | (pp–1) if absolute sign is omitted |
| $y^2=4x$ | 1 | |
| | 3 | |
| (b) $y^2=4x$ | | |
| $2y\frac{dy}{dx}=4$ | | |
| At the point $(t^2, 2t)$, $\frac{dy}{dx}=\frac{4}{2(2t)}=\frac{1}{t}$ | | |
| Equation of tangent is $\frac{y-2t}{x-t^2}=\frac{1}{t}$ | 1M | |
| $x-ty+t^2=0$ | 1 | |
| Alternative solution | | |
| Using the formula $yy_1=\frac{1}{2}\cdot 4(x+x_1)$, the equation of the tangent is | | |
| $y(2t)=2(x+t^2)$ | 1A | |
| $x-ty+t^2=0$ | 1 | |
| Equation of normal | | |
| $\frac{y-2t}{x-t^2}=-t$ | | |
| $tx+y-2t-t^3=0$ | 1A | |
| | 3 | |
| (c) (i) Since $P$ and $C$ have common tangent at $R$, the normal to $P$ at $R$ passes through $Q(k, 0)$. | 1M | (can be omitted) |
| $0=-t(k)+2t+t^3$ | 1M | |
| $t^3=kt-2t$ | | |
| $t^2=k-2$ $\quad \because t\neq 0$ | 1 | |
| Alternative solution (1) | | |
| $RQ$ is perpendicular to the tangent to $P$ at $R$. | 1M | (can be omitted) |
| Slope of $RQ=\frac{2t}{t^2-k}$ | | |
| Slope of tangent to $P$ at $R=\frac{1}{t}$ | | |
| $\frac{2t}{t^2-k}(\frac{1}{t})=-1$ | 1M | |
| $t^2=k-2$ | 1 | |



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| Solution | Marks | Remarks |
|---|---|---|
| Alterantive solution (2) | | |
| Equation of $C$ is $(x-k)^2+y^2=(t^2-k)^2+4t^2$ — (1) | | |
| Equation of tangent to $P$ at $R$ is $x-ty+t^2=0$ — (2) | | |
| Substitute (2) into (1), | | |
| $(ty-t^2-k)^2+y^2=(t^2-k)^2+4t^2$ | 1M | |
| $(t^2+1)y^2-2t(t^2+k)y+4t^2(k-1)=0$ | | |
| Discriminant $=4t^2(t^2+k)^2-4(t^2+1)4t^2(k-1)=0$ | 1M | |
| $t^4+(4-2k)t^2+(k-2)^2=0$ | | |
| $[t^2-(k-2)]^2=0$ | | |
| $t^2=k-2$ | 1 | |
| Alternative solution (3) | | |
| Equation of $C$ is $(x-k)^2+y^2=(t^2-k)^2+4t^2$. | | |
| $2(x-k)+2y\dfrac{dy}{dx}=0$ | | |
| $\dfrac{dy}{dx}=-\dfrac{(x-k)}{y}$ | | |
| At point $R$, $\dfrac{dy}{dx}=-\dfrac{(t^2-k)}{2t}$ | | |
| Equation of tangent to $C$ at $R$ | | |
| $\dfrac{y-2t}{x-t^2}=-\dfrac{(t^2-k)}{2t}$ | 1M | |
| $(k-t^2)x-2ty+(t^4+4t^2-t^2k)=0$ | | |
| Compare with the equation of tangent to $R$ at $R$ $(x-ty+t^2=0)$ | | |
| $\dfrac{k-t^2}{2}=1$ OR $\dfrac{t^4+4t^2-t^2k}{2}=t^2$, $t^4+2t^2=t^2k$ | 1M | |
| $t^2=k-2$ OR $t^2=k-2$ | 1 | |
| Alternative solution (4) | | |
| Distance between $R$ and $Q$ $=\sqrt{(t^2-k)^2+(2t)^2}$ | | |
| Distance from $Q$ to the tangent to $P$ at $R$ $=\left\|\dfrac{k+t^2}{\sqrt{1+t^2}}\right\|$ | 1M | |
| Since the circle touches $P$ at $R$, | | |
| $\sqrt{(t^2-k)^2+4t^2}=\left\|\dfrac{k+t^2}{\sqrt{1+t^2}}\right\|$ | 1M | |
| $[(t^2-k)^2+4t^2](1+t^2)=(t^2+k)^2$ | | |
| $t^2(t^4-2kt^2+4t^2+k^2-4k+4)=0$ | | |
| $t^4+(4-2t)t^2+(k-2)^2=0$ | | |
| $[t^2-(k-2)]^2=0$ | | |
| $t^2=k-2$ | 1 | |



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|---|---|---|
| (ii) (1) Put (0, 2) into $x - ty + t^2 = 0$. | | |
| $0 - 2t + t^2 = 0$ | 1A | |
| $t = 2$ **OR** $t = 0$ or $2$ | 1A | |
| At point $S$, $t = -2$. | 1A | **OR** The point $S$ is (4, −4). |
| The equation of the tangent to $P$ at $S$ is | | |
| $x - (-2)y + (-2)^2 = 0$ | 1M | |
| $x + 2y + 4 = 0$ | 1A | |
| Alternative solution<br>.<br>.<br>.<br>$t = 2$ | 1A+1A | Same as above |
| Equation of tangent to $P$ at $R$ is<br>$x - 2y + 4 = 0$ | 1A | **OR** slope of tangent at $R = \frac{1}{2}$ |
| By symmetry, slope of tangent to $P$ at $S$<br>= −(slope of tangent to $P$ at $R$)<br>$= -\frac{1}{2}$ | 1M | |
| $\therefore$ Equation of tangent to $P$ at $S$ is<br>$x + 2y + 4 = 0$. | 1A | |
| (2) $k = t^2 + 2 = 6$ | 1M | |
| Point $Q$ is (6, 0). | | |
| radius $= \sqrt{(6-4)^2 + (0-4)^2} = \sqrt{20}$ | | |
| $\therefore$ The equation of $C$ is $(x-6)^2 + y^2 = 20$. | 1A | $x^2 + y^2 - 12x + 16 = 0$ |
| | 10 | |



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| Solution | Marks | Remarks |
|---|---|---|
| 10. (a) $\sin x = \sin 2x$ <br> $\sin x = 2\sin x\cos x$ <br> $\cos x = \frac{1}{2}$ <br> $x = \frac{\pi}{3}$ | 1M | |
| Put $x = \frac{\pi}{3}$, $y = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$. <br> $\therefore$ The coordinates of $A$ are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. | 1 | |
| Alternative solution <br> Put $x = \frac{\pi}{3}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ <br> $\sin 2(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ | 1M | |
| $\therefore$ The coordinates of $A$ are $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. | 1 | |
| | 2 | |
| (b) Area $= \int_0^{\pi} \lvert\sin 2x - \sin x\rvert dx$ | | |
| $= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx + \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x) dx$ | 1M+1A+1A | 1M for Area $= \int_a^b (y_2 - y_1) dx$, <br> omit 'dx' (pp−1) <br> 1A for each expression |
| $= \left[-\frac{1}{2}\cos 2x + \cos x\right]_0^{\frac{\pi}{3}} + \left[-\cos x + \frac{1}{2}\cos 2x\right]_{\frac{\pi}{3}}^{\pi}$ | 1A | For primitive function, awarded if one of them is correct |
| $= \left[\frac{1}{4} + \frac{1}{2} - (-\frac{1}{2} + 1)\right] + \left[1 + \frac{1}{2} - (-\frac{1}{2} - \frac{1}{4})\right]$ <br> $= \frac{1}{4} + 2\frac{1}{4}$ | | |
| $= 2\frac{1}{2}$ | 1A | |
| Alternative solution <br> Area $A_1$ between $x = 0$ to $\frac{\pi}{3}$ | | |
| $A_1 = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx$ OR $= \left\lvert\int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx\right\rvert$ | 1M+1A | 1M for Area $= \int_a^b (y_2 - y_1) dx$, |
| $= \left[-\frac{\cos 2x}{2} + \cos x\right]_0^{\frac{\pi}{3}}$ | 1A | For primitive function |
| $= \left[\frac{1}{4} + \frac{1}{2} - (-\frac{1}{2} + 1)\right]$ <br> $= \frac{1}{4}$ | | |



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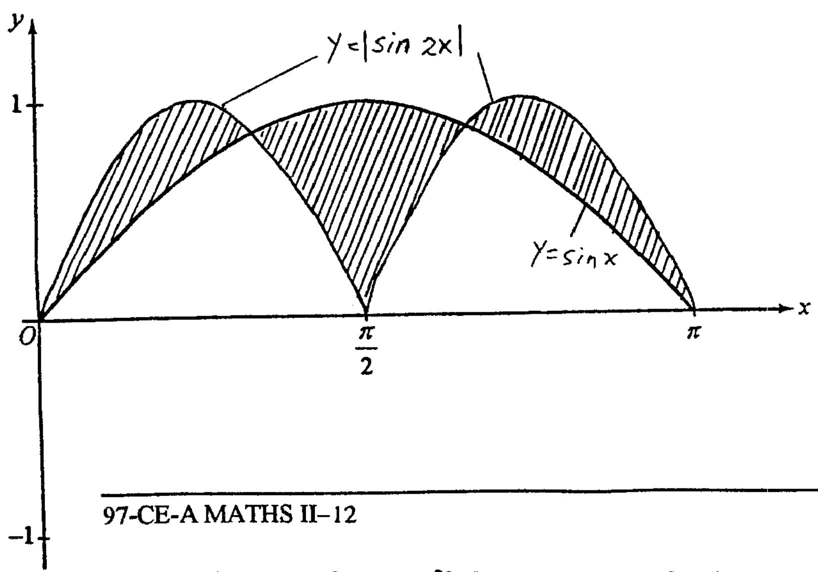
| Solution | Marks | Remarks |
|---|---|---|
| Area $A_2$ between $x=\frac{\pi}{3}$ to $\pi$ | | |
| $A_2=\int_{\frac{\pi}{3}}^{\pi}(\sin x-\sin 2x)\,dx$ OR $=\left\|\int_{\frac{\pi}{3}}^{\pi}(\sin 2x-\sin x)\,dx\right\|$ | 1A | |
| $=\left[-\cos x+\frac{1}{2}\cos 2x\right]_{\frac{\pi}{3}}^{\pi}$ | | |
| $=\left[1+\frac{1}{2}-(-\frac{1}{2}-\frac{1}{4})\right]$ | | |
| $=2\frac{1}{4}$ | | |
| $\therefore$ Total area $=A_1+A_2$ | | |
| $=2\frac{1}{2}$ | 1A | |
| | 5 | |
| (c) Volume $=\pi\int_0^{\frac{\pi}{3}}\sin^2 x\,dx+\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\sin^2 2x\,dx$ | 1M+1A | 1M for $V=\pi\int_a^b y^2\,dx$ |
| $=\pi\int_0^{\frac{\pi}{3}}\frac{1}{2}(1-\cos 2x)\,dx+\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{1}{2}(1-\cos 4x)\,dx$ | 1M | |
| $=\frac{\pi}{2}\left[x-\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{3}}+\frac{\pi}{2}\left[x-\frac{1}{4}\sin 4x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ | 1A | For either of the 2 primitive functions |
| $=\frac{\pi}{2}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}-0\right]+\frac{\pi}{2}\left[\frac{\pi}{2}-0-(\frac{\pi}{3}+\frac{\sqrt{3}}{8})\right]$ | | |
| $=(\frac{\pi^2}{6}-\frac{\sqrt{3}\pi}{8})+(\frac{\pi^2}{12}-\frac{\sqrt{3}\pi}{16})$ | | |
| $=\frac{\pi}{16}(4\pi-3\sqrt{3})$ | 1A | |
| | 5 | |
| (d) | 2A | For the graph |
| | 2A | For shading the area |
| | 4 | |



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| Solution | Marks | Remarks |
|---|---|---|
| 11. (a) $du = -\csc^2\theta d\theta$ | 1A | |
| $\int \cot^n\theta\csc^2\theta d\theta = -\int u^n du$ | 1A | |
| $= -\frac{u^{n+1}}{n+1} + c$ (c is a constant) | | |
| $= -\frac{\cot^{n+1}\theta}{n+1} + c$ | 1A | no mark if 'c' is omitted |
| | 3 | |
| (b) $\int \cot^{n+2}\theta d\theta = \int \cot^n\theta\cot^2\theta d\theta$ | | |
| $= \int \cot^n\theta(\csc^2\theta - 1)d\theta$ | 2A | |
| $= \int \cot^n\theta\csc^2\theta d\theta - \int \cot^n\theta d\theta$ | 1M | For separating into 2 terms |
| $= -\frac{\cot^{n+1}\theta}{n+1} - \int \cot^n\theta d\theta$ | 1 | |
| | 4 | |
| (c) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta d\theta$ | | |
| $= [-\cot\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta$ | 1A | Omitting limits (pp−1) |
| $= [-\cot\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - [\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ | 1A | |
| $= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$ | 1 | |
| **Alternative solution** | | |
| $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\csc^2\theta - 1)d\theta$ | 1A | |
| $= [-\cot\theta - \theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ | 1A | |
| $= 1 - \frac{\sqrt{3}}{3} - \frac{\pi}{12}$ | 1 | |
| | 3 | |



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| Solution | Marks | Remarks |
|---|---|---|
| (d) $dx = \sec\theta\tan\theta d\theta$ | 1A | |
| $\int_{\sqrt{2}}^{2} \frac{dx}{x\sqrt{(x^2-1)^5}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta d\theta}{\sec\theta\sqrt{(\sec^2\theta-1)^5}}$ | 1A+1A | 1A for integrand, 1A for limits |
| $= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^4\theta d\theta$ | 1A | |
| $= \left[-\frac{\cot^3\theta}{3}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cot^2\theta d\theta$ | 1M | For using (b) |
| $= -\frac{1}{3}\left[\frac{1}{(\sqrt{3})^3} - 1\right] - (1 - \frac{1}{\sqrt{3}} - \frac{\pi}{12})$ | | |
| $= -\frac{2}{3} + \frac{8\sqrt{3}}{27} + \frac{\pi}{12}$ | 1A | |
| | 6 | Omit $du$, $d\theta$ etc. (pp−1) |

| Solution | Marks | Remarks |
|---|---|---|
| 12. (a) (i) By Cosine Law, | | |
| $(AC)^2=(AB)^2+(BC)^2-2(AB)(BC)\cos\angle ABC$ | | |
| $=(3a)^2+(2a)^2-2(3a)(2a)\cos 120°$ | 1A | |
| $=19a^2$ | | |
| $AC=\sqrt{19}\,a$ | 1A | |
| (ii) $MC=\frac{1}{2}AC=\frac{\sqrt{19}}{2}a$ | | |
| $\tan\angle HMC=\frac{HC}{MC}$ | 1M | |
| $=\frac{a}{\frac{1}{2}\sqrt{19}\,a}$ | | |
| $\angle HMC\approx 25°$ | 1A | |
| $\therefore$ The angle of elevation is 25° correct to the nearest degree. | | |
| | 4 | |
| (b) (i) $(BD)^2=(3a)^2+(2a)^2-2(3a)(2a)\cos 60°$ | | |
| $=7a^2$ | | |
| $BD=\sqrt{7}\,a$ | 1A | |
| Consider the area of $\Delta BCD$. | | |
| $\frac{1}{2}(BD)(CE)=\frac{1}{2}(BC)(CD)\sin 60°$ | 1M | |
| $\frac{1}{2}(\sqrt{7}a)(CE)=\frac{1}{2}(2a)(3a)\frac{\sqrt{3}}{2}$ | 1M | For substitution |
| $CE=\frac{3\sqrt{21}}{7}a$ | 1A | Accept $\frac{3\sqrt{3}}{\sqrt{7}}a$ |
| Alternative solution | | |
| $BD=\sqrt{7}\,a$ | 1A | |
| By Sine Law, $\frac{BD}{\sin 60°}=\frac{BC}{\sin\angle BDC}$ | | |
| $\sin\angle BDC=\frac{(2a)\sin 60°}{\sqrt{7}\,a}$ | 1M | $\sin\angle DBC=\frac{(3a)\sin 60°}{\sqrt{7}\,a}$ |
| $=\sqrt{\frac{3}{7}}$ | | $=\frac{3\sqrt{3}}{2\sqrt{7}}$ |
| $CE=CD\sin\angle CDE$ | 1M | $CE=BC\sin\angle DBC$ |
| $=3a(\sqrt{\frac{3}{7}})$ | | $=2a(\frac{3\sqrt{3}}{2\sqrt{7}})$ |
| $=\frac{3\sqrt{21}}{7}a$ | 1A | $=\frac{3\sqrt{21}}{7}a$ |



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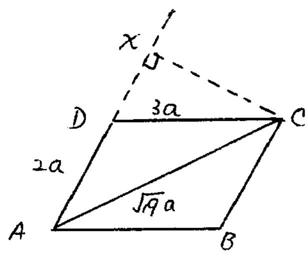
| Solution | Marks | Remarks |
|---|---|---|
| (ii) $(HE)^2 = a^2 + (CE)^2$ $\quad = a^2 + (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{34}{7}a^2$ | | |
| $(EB)^2 = (BC)^2 - (CE)^2$ $\quad = (2a)^2 - (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{1}{7}a^2$ | 1M | |
| $(HE)^2 + (EB)^2 = [a^2 + (CE)^2] + [(BC)^2 - (CE)^2]$ $\quad = \frac{34}{7}a^2 + \frac{1}{7}a^2$ | 1M | |
| $= a^2 + (BC)^2$ $\quad = a^2 + (2a)^2$ | | |
| $= (HB)^2$ | | |
| $\therefore$ $HE$ is perpendicular to $BD$ (Converse of Pythagora's Theorem). | 1 | |
| **Alternative solution** | | |
| $(HE)^2 = a^2 + (CE)^2$ $\quad = a^2 + (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{34}{7}a^2$ | 1M | |
| $(DE)^2 = (CD)^2 - (CE)^2$ $\quad = (3a)^2 - (\frac{3\sqrt{3}}{\sqrt{7}}a)^2 = \frac{36}{7}a^2$ | | |
| $(HE)^2 + (DE)^2 = [a^2 + (CE)^2] + [(CD)^2 - (CE)^2]$ $\quad = \frac{34}{7}a^2 + \frac{36}{7}a^2$ | 1M | |
| $= a^2 + (CD)^2$ $\quad = a^2 + (3a)^2$ | | |
| $= (DH)^2$ | | |
| $\therefore$ $HE$ is perpendicular to $BD$ (Converse of Pythagora's Theorem). | 1 | |
| The angle between the planes $HBD$ and $ABCD$ is $\angle HEC$. | | |
| $\tan\angle HEC = \frac{HC}{EC}$ | 1M | |
| $= \frac{a}{3\sqrt{21}a/7}$ | | |
| $= \frac{7}{3\sqrt{21}}$ | | |
| $\angle HEC \approx 27°$ correct to the nearest degree. | 1A | |
| | 9 | |
| (c) Point $X$ lies on $AD$ produced such that $CX \perp AX$. | 1A |  |
| $AX = AD + DX$ | 1M | |
| $= 2a + CD\cos 60°$ | | |
| $= 2a + \frac{3a}{2}$ | | |
| $= \frac{7a}{2}$ | 1A | |
| **Alternative solution (1)** | | |
| Point $X$ lies on $AD$ produced such that $CX \perp AX$. | 1A | |
| $\cos\angle CAD = \frac{(2a)^2 + (\sqrt{19}a)^2 - (3a)^2}{2(2a)(\sqrt{19}a)}$ | | |
| $= \frac{7}{2\sqrt{19}}$ | | |
| $AX = AC\cos\angle CAD$ | 1M | |
| $= \sqrt{19}a(\frac{7}{2\sqrt{19}})$ | | |
| $= \frac{7a}{2}$ | 1A | |



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| Solution | Marks | Remarks |
|---|---|---|
| Alternative solution (2) | | |
| Point $X$ lies on $AD$ produced such that $CX \perp AX$. | 1A | |
| $AC^2 - AX^2 = DC^2 - DX^2$ | | |
| Let $AX = d$. | | |
| $(\sqrt{19}a)^2 - d^2 = (3a)^2 - (d-2a)^2$ | 1M | |
| $14a^2 = 4ad$ | | |
| $d = \dfrac{7a}{2}$ | 1A | |
| | 3 | |



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| Solution | Marks | Remarks |
|---|---|---|
| 13. (a) $x^2+y^2-6x-2y+k(2x-4y+3)=0$ | | |
| $x^2+y^2-(6-2k)x-(2+4k)y+3k=0$ | 1M | For collecting terms |
| The centre is $(3-k, 1+2k)$. | 1A | |
| Radius $=\sqrt{(3-k)^2+(1+2k)^2-3k}$ | 1M | $\frac{1}{2}\sqrt{(2k-6)^2+(4k+2)^2-4(3k)}$ |
| $=\sqrt{5k^2-5k+10}$ | | |
| $=\sqrt{5(k^2-k+2)}$ | 1 / 4 | |
| (b) Radius $=\sqrt{5(k^2-k+\frac{1}{4})-\frac{5}{4}+10}$ | | |
| $=\sqrt{5(k-\frac{1}{2})^2+\frac{35}{4}}$ | 1M+1A | |
| $\therefore$ Radius of smallest circle in $F=\frac{\sqrt{35}}{2}$. | 1A | |
| Since $AB$ is a diameter of the smallest circle in $F$, | | |
| $AB=2\times\frac{\sqrt{35}}{2}=\sqrt{35}$. | 1A | |
| Alternative solution (1) | | |
| Let $r=\sqrt{5k^2-5k+10}$ | | |
| $\frac{dr}{dk}=\frac{10k-5}{2\sqrt{5k^2-5k+10}}=\frac{5(2k-1)}{2\sqrt{5k^2-5k+10}}$ | 1M | OR differentiating $(5k^2-5k+10)$ |
| $\frac{dr}{dk}=0$ when $k=\frac{1}{2}$. | 1A | |
| Since $\frac{dr}{dk}>0$ when $k>\frac{1}{2}$ and $\frac{dr}{dk}<0$ when $k<\frac{1}{2}$, $r$ is smallest when $k=\frac{1}{2}$ | | OR using 2nd derivative test |
| Smallest radius $=\sqrt{5(\frac{1}{2})^2-5(\frac{1}{2})+10}$ | | |
| $=\frac{\sqrt{35}}{2}$ | 1A | |
| Since $AB$ is a diameter of the smallest circle in $F$, | | |
| $AB=2\times\frac{\sqrt{35}}{2}=\sqrt{35}$. | 1A | |
| Alternative solution (2) | | |
| The circle is smallest when the centre lies on $AB$. | | |
| Put $(3-k, 1+2k)$ into $2x-4y+3=0$, | | |
| $2(3-k)-4(1+2k)+3=0$ | 1M | |
| $k=\frac{1}{2}$ | 1A | |
| $\therefore$ Smallest radius $=\sqrt{5(\frac{1}{2})^2-5(\frac{1}{2})+10}$ | | |
| $=\frac{\sqrt{35}}{2}$ | 1A | |
| Since $AB$ is a diameter of the smallest circle in $F$, | | |
| $\therefore$ $AB=2\times\frac{\sqrt{35}}{2}=\sqrt{35}$. | 1A | |



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| Solution | Marks | Remarks |
|---|---|---|
| Alternative solution (3) | | |
| $\begin{cases} x^2+y^2-6x-2y=0 \\ 2x-4y+3=0 \end{cases}$ | | |
| $(\frac{4y-3}{2})^2+y^2-6(\frac{4y-3}{2})-2y=0$ | 1M | |
| $20y^2-80y+45=0$ | 1A | OR $4x^2-20x-3=0$ |
| $y=\frac{4+\sqrt{7}}{2}$ or $\frac{4-\sqrt{7}}{2}$ | | |
| $x=\frac{5+2\sqrt{7}}{2}$ $\quad \frac{5-2\sqrt{7}}{2}$ | | |
| $\therefore$ The coordinates of $A$ and $B$ are $(\frac{5+2\sqrt{7}}{2},\frac{4+\sqrt{7}}{2})$ and $(\frac{5-2\sqrt{7}}{2},\frac{4-\sqrt{7}}{2})$. | 1A | |
| Length of $AB$ $=\sqrt{[(\frac{5+2\sqrt{7}}{2})-(\frac{5-2\sqrt{7}}{2})]^2+[(\frac{4+\sqrt{7}}{2})-(\frac{4-\sqrt{7}}{2})]^2}$ | | |
| $=\sqrt{35}$ | 1A | |
| | 4 | |
| (c) (i) Distance $=\left\lvert\frac{4(3-k)+2(1+2k)-9}{\sqrt{4^2+2^2}}\right\rvert$ | 1M | Accept omitting absolute sign |
| $=\frac{\sqrt{5}}{2}$ which is a constant. | 1 | |
| The locus of the centres of the circles in $F$ is parallel to the line $L$. | 1A | |
| OR The locus of the centres of the circles in $F$ and the line has no intersection point. | | |
| Alternative solution | | |
| $M_{AB}=\frac{1}{2}$, $M_L=-2$ | | |
| $M_{AB}\cdot M_L=-1$ $\quad\therefore$ $AB$ and $L$ are perpendicular. | 1 | |
| As the locus of the centres of circles in $F$ is the perpendicular bisector of $AB$, so the locus of the centre of circles in $F$ and $L$ are parallel. | | |
| So the distance from the centre of a circle in $F$ to $L$ is always a constant. | 1 | |
| The locus of the centres of circles in $F$ and $L$ are parallel. | 1A | |
| (ii) $(\frac{CD}{2})^2+(\frac{\sqrt{5}}{2})^2=(\text{radius})^2$ | 1M | |
| $(\frac{\sqrt{35}}{2})^2+(\frac{\sqrt{5}}{2})^2=(\sqrt{5k^2-5k+10})^2$ | 1M | |
| $5k^2-5k=0$ | 1A | |
| $k=0$ or $1$ | | |
| $\therefore$ The equations of the two circles are | | |
| $x^2+y^2-6x-2y=0$ | 1A | $(x-3)^2+(y-1)^2=10$ |
| and $x^2+y^2-4x-6y+3=0$. | 1A | $(x-2)^2+(y-3)^2=10$ |



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| Solution | Marks | Remarks |
|---|---|---|
| **Alternative solution** | | |
| Distance from centre to $AB$ = Distance from centre to $CD$ | 1M | |
| $\left\lvert \dfrac{2(3-k)-4(1+2k)+3}{\sqrt{4^2+2^2}} \right\rvert = \dfrac{\sqrt{5}}{2}$ | 1M | Omit absolute sign (pp–1) |
| $\dfrac{5-10k}{\sqrt{20}} = \pm\dfrac{\sqrt{5}}{2}$ | 1A | For LHS |
| $k=0$ or $1$ | | |
| ∴ The equations of the two circles are | | |
| $x^2+y^2-6x-2y=0$ | 1A | |
| and $x^2+y^2-4x-6y+3=0$. | 1A | |
| | 8 | |