香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九七年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997

附加數學 試卷一
ADDITIONAL MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任敎會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/敎師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代敎育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/敎師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於敎師中心，供敎師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol (pp–1) should be used to denote marks deducted for poor presentation (p.p.). Note the following points:

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those steps where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However, the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. The symbol (u–1) should be used to denote marks deducted for wrong/no units in the final answers (if applicable). Note the following points :

   (a) At most deduct 1 mark for wrong/no units for the whole paper.

   (b) Do not deduct any marks for wrong/no units in case candidate's answer was already wrong.

6. Marks entered in the Page Total Box should be the <u>net</u> total score on that page.

7. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ┆┈┈┈┈┆ , whereas alternative answers are enclosed by solid rectangles ▭ .

8. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

9. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

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Solution	Marks	Remarks
1. $f(x)=\sqrt{3+x^2}$		
$f'(x)=\frac{1}{2}(3+x^2)^{-\frac{1}{2}}(2x)$	1M+1A	1M for chain rule
$=\frac{x}{\sqrt{3+x^2}}$		
$f'(-1)=\frac{-1}{\sqrt{3+(-1)^2}}$		
$=-\frac{1}{2}$	1A	
	3	
2. $y^2+\sqrt[3]{x}\,y-3=0$		
Differentiate with respect to x,		
$2y\frac{dy}{dx}+\sqrt[3]{x}\frac{dy}{dx}+\frac{1}{3}x^{-\frac{2}{3}}y=0$	1A+1A	1A for $\frac{d}{dx}(\sqrt[3]{x}\,y)$, 1A for $\frac{d}{dx}(y^2-3=0)$
$\frac{dy}{dx}=\frac{-y}{3x^{\frac{2}{3}}(2y+x^{\frac{1}{3}})}$		
At $P(8, 1)$,		
$2(1)\frac{dy}{dx}+(8)^{\frac{1}{3}}\frac{dy}{dx}+\frac{1}{3}(8)^{-\frac{2}{3}}(1)=0$		
$\frac{dy}{dx}=\frac{-1}{48}$	1A	
Alternative solution		
$y^2+\sqrt[3]{x}\,y-3=0$		
$y=\frac{-x^{\frac{1}{3}}\pm\sqrt{x^{\frac{2}{3}}+12}}{2}$		
At $P(8, 1)$ lies on the curve $y=\frac{-x^{\frac{1}{3}}+\sqrt{x^{\frac{2}{3}}+12}}{2}$.	1A+1A	
$\frac{dy}{dx}=\frac{1}{2}\left[-\frac{1}{3}x^{-\frac{2}{3}}+\frac{\frac{2}{3}x^{-\frac{1}{3}}}{2\sqrt{x^{\frac{2}{3}}+12}}\right]$		
At $P(8, 1)$, $\frac{dy}{dx}=-\frac{1}{48}$	1A	
	3	

Solution	Marks	Remarks
3. (a) $\frac{1+i}{1-i}=\frac{1+i}{1-i}(\frac{1+i}{1+i})$	1M	
$=i$	1A	
(b) $(1+i)^{2n}=(1-i)^{2n}$		
$(\frac{1+i}{1-i})^{2n}=1$	1A	
$i^{2n}=1$	1M	
$n=2k$, where k is a positive integer. (OR $n=2,4,6,\ldots$; OR n is an even positive integer.)	1A	k not defined – no marks Include 0 – no marks
Alternative solution		
(b) $(1+i)^{2n}=[\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})]^{2n}=(\sqrt{2})^{2n}(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2})$ $(1-i)^{2n}=(\sqrt{2})^{2n}[\cos(-\frac{n\pi}{2})+i\sin(-\frac{n\pi}{2})]=(\sqrt{2})^{2n}(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2})$ $(1+i)^{2n}=(1-i)^{2n}$ $(\sqrt{2})^{2n}(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2})=(\sqrt{2})^{2n}(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2})$	1M	
$\sin\frac{n\pi}{2}=0$	1A	
$n=2k$, where k is a positive integer.	1A	
	5	
4. $h=30\tan\theta$	1A	
Differentiate with respect to time t,		
$\frac{dh}{dt}=30\sec^2\theta\frac{d\theta}{dt}$	1M	For chain rule
when $h=30\sqrt{3}$, $\tan\theta=\sqrt{3}$		
$\theta=\frac{\pi}{3}$ (OR $60°$)	1A	OR $\sec^2\theta=1+(\sqrt{3})^2=4$
$15=30\sec^2(\frac{\pi}{3})\frac{d\theta}{dt}$	1M	For substitution
$\frac{d\theta}{dt}=\frac{1}{80}\text{s}^{-1}$ (OR rad s^{-1})	1A	OR $0.0125\ \text{s}^{-1}$
$\therefore$ The rate of change of θ is $\frac{1}{80}\text{s}^{-1}$.		
	5	

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Solution	Marks	Remarks
5. $\|3x-4\|<2$		
$-2<3x-4<2$	1A	
$\frac{2}{3}<x<2$	1A	
Alternative soltuion		
$\|3x-4\|<2$		
$(3x-4)^2<4$	1A	
$3x^2-8x+4<0$		
$(3x-2)(x-2)<0$		
$\frac{2}{3}<x<2$	1A	
$\frac{1}{2x-1}\leq 1$		
$\frac{1}{2x-1}-1\leq 0$	1M	
$\frac{2-2x}{2x-1}\leq 0$	1A	
$x\geq 1$ or $x<\frac{1}{2}$	1A	
Alternative solution (1)		
$\frac{1}{2x-1}\leq 1$		
$\frac{1}{2x-1}(2x-1)^2\leq(2x-1)^2$ and $x\neq\frac{1}{2}$	1M	
$2x-1\leq 4x^2-4x+1$ and $x\neq\frac{1}{2}$		
$2x^2-3x+1\geq 0$ … …	1A	
$(2x-1)(x-1)\geq 0$ … …		
$x\geq 1$ or $x<\frac{1}{2}$	1A	

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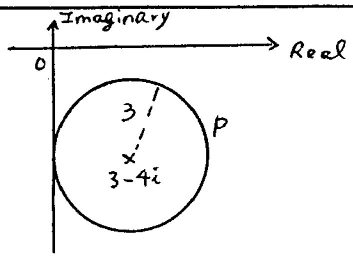
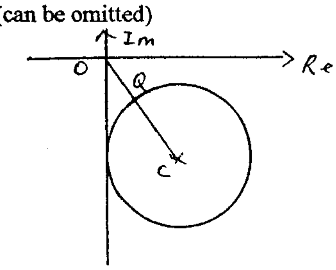
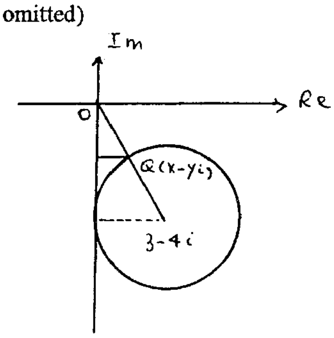
Solution	Marks	Remarks
Alternative solution (2)		
Consider the following cases (i) $x > \frac{1}{2}$, (ii) $x < \frac{1}{2}$.	1M	Awarded even if equality sign is included.
Case 1 : $x > \frac{1}{2}$		
$1 \le 2x - 1$		
$x \ge 1$		
Since $x > \frac{1}{2}$, $\therefore x \ge 1$.	1A (for Case 1 and Case 2)	
Case 2 : $x < \frac{1}{2}$		
$1 \ge 2x - 1$		
$x \le 1$		
Since $x < \frac{1}{2}$, $\therefore x < \frac{1}{2}$.		
Combining the 2 cases, $x \ge 1$ or $x < \frac{1}{2}$.	1A	
Alternative solution (3)		
$\frac{1}{2x-1} \le 1$		
$2x - 1 \ge 1$ or $2x - 1 < 0$	2A	
$x \ge 1$ or $x < \frac{1}{2}$	1A	
Combining the two solutions, $1 \le x < 2$.	1A	
	6	

Solution	Marks	Remarks
6. (a) 	1A 1A 1A	For a circle For centre at $(3-4i)$ For radius = 3 (pp–1) for not labelling the axes
(b) The point Q and the centre of the circle C lie on the same line through O.	1A	(can be omitted) 
$OC=\sqrt{3^2+(-4)^2}=5$		
$OQ=5-3=2$		
$\therefore OQ:QC=2:3$		
The complex number represented by Q $=\frac{2}{5}(3-4i)$	1M	
$=\frac{6}{5}-\frac{8}{5}i$	1A	
Alternative solution (1) The point Q and the centre of the circle lie on the same line through O.	1A	(can be omitted) 
Let the point Q represent the complex number $x-yi$. By similar triangles, $\frac{x}{3}=\frac{\sqrt{3^2+(-4)^2}-3}{\sqrt{3^2+(-4)^2}}$ $x=\frac{6}{5}$. Similarly, $\frac{y}{4}=\frac{\sqrt{3^2+(-4)^2}-3}{\sqrt{3^2+(-4)^2}}$ $y=\frac{8}{5}$.	1M	
$\therefore$ The complex number represented by Q is $\frac{6}{5}-\frac{8}{5}i$.	1A	
Alternative solution (2) The point Q and the centre of the circle lie on the same line through O.	1A	(can be omitted)
Modulus of $Q=\sqrt{3^2+(-4)^2}-3$ $=2$ Let θ be the argument of Q. $\tan\theta=-\frac{4}{3}$	1M	
The complex number represented by Q $=2(\cos\theta+i\sin\theta)$ $=2(\frac{3}{5}-\frac{4}{5}i)$ $=\frac{6}{5}-\frac{8}{5}i$	1A	

Solution	Marks	Remarks
Alternative solution (3)		
The point Q and the centre of the circle lie on the same line through O.	1A	(can be omitted)
Equation of circle is $(x-3)^2+(y+4)^2=3^2$ --- (1) Equation of line through O and Q is $y=-\frac{4}{3}x$ --- (2) Solving (1) and (2), $(x-3)^2+(-\frac{4}{3}x+4)^2=3^2$	1M	
$25x^2-150x+144=0$ $x=\frac{6}{5}$ or $\frac{24}{5}$ (rejected) when $x=\frac{6}{5}$, $y=-\frac{8}{5}$. $\therefore$ The complex number represented by Q is $\frac{6}{5}-\frac{8}{5}i$.	1A	
	6	

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Solution	Marks	Remarks
7. (a) $\lvert\vec{a}\rvert = \sqrt{2^2+4^2} = 2\sqrt{5}$	1A	Accept $\sqrt{20}$
(b) $\vec{a}\cdot\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$		
$= 2\sqrt{5}(\sqrt{5})(\frac{4}{5})$	1M	
$= 8$	1A	
(c) $\vec{a}\cdot\vec{b} = (2\vec{i}+4\vec{j})\cdot(m\vec{i}+n\vec{j})$		
$= 2m+4n$		
$\therefore\ 2m+4n = 8$ — (1)	1M	
As $\lvert\vec{b}\rvert=5$, $\sqrt{m^2+n^2}=\sqrt{5}$		
$m^2+n^2=5$ — (2)	1A	
Substitute (1) into (2),		
$(4-2n)^2+n^2=5$		$m^2+(\frac{4-m}{2})^2=5$
$5n^2-16n+11=0$		$5m^2-8m-4=0$
$n=1$ or $\frac{11}{5}$	1A	$m=2$ or $-\frac{2}{5}$
When $n=1$, $m=2$.	1A (for both)	
$n=\frac{11}{5}$, $m=-\frac{2}{5}$.		Omitting vector sign or dot sign in most cases (pp–1).
	7	
8. (a) Discriminant $=(k+2)^2-4(2)(k-1)$	1A	
$=k^2-4k+12$		
$=(k-2)^2+8>0$	1M+1 (for both)	
$\therefore\ \alpha$ and β are real and distinct.		
<u>Alternative solution (1)</u>		
Discriminant $=(k+2)^2-4(2)(k-1)$	1A	
$=k^2-4k+12$		
As discriminant of $k^2-4k+12=0$ is $(4)^2-4(12)$, which is less than zero, so the graph of $y=k^2-4k+12$ always lies above the x-axis and $k^2-4k+12$ is always positive.	1M+1 (for both)	
$\therefore\ \alpha$ and β are real and distinct.		
<u>Alternative solution (2)</u>		
Discriminant $=(k+2)^2-4(2)(k-1)$	1A	
$=k^2-4k+12$		
Let $f(k)=k^2-4k+12$.		
$f'(k)=2k-4=0$ when $k=2$	1M+1 (for all)	
$f''(k)=2>0$		
$\therefore$ Minimum value of $f(k)=2^2-4(2)+12$ $=8>0$		
$\therefore\ f(k)>0$ for all values of k.		
$\therefore\ \alpha$ and β are real and distinct.		

Solution	Marks	Remarks
(b) $\lvert\alpha-\beta\rvert>3$		
$(\alpha-\beta)^2>9$	1A	OR $\alpha-\beta>3$ or $\alpha-\beta<-3$
$(\alpha+\beta)^2-4\alpha\beta>9$	1M	For $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$
$(k+2)^2-4(2)(k-1)>9$		
$k^2-4k+3>0$	1A	
$(k-1)(k-3)>0$		
$k>3$ or $k<1$	1A	
	7	

Solution	Marks	Remarks
9. (a) (i) $\vec{AF}=\vec{AB}+\vec{BF}$	1M	Can be awarded in (ii)
$=\vec{a}+2\vec{b}$	1A	
(ii) $\vec{DP}=\vec{DA}+\vec{AP}$		
$=(m+1)\vec{a}-\vec{b}$	1A	
	3	
(b) (i) $\vec{AF}\cdot\vec{DP}=0$		
$(\vec{a}+2\vec{b})\cdot[(m+1)\vec{a}-\vec{b}]=0$	1M	
$(m+1)\vec{a}\cdot\vec{a}-\vec{a}\cdot\vec{b}+2(m+1)\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{b}=0$	1M	For distribution law
OR $(m+1)\vec{a}\cdot\vec{a}-2\vec{b}\cdot\vec{b}=0$		
$(m+1)-2(4)=0$	1A	For $\vec{a}\cdot\vec{a}=1\cdot\vec{a}\cdot\vec{b}=0,\vec{b}\cdot\vec{b}=4$
$m=7$	1	
(ii) (1) $\vec{AE}=\frac{1}{r+1}\vec{AF}$		
$=\frac{1}{r+1}(\vec{a}+2\vec{b})$	1A	
(2) $\vec{AE}=\frac{\vec{AP}+k\vec{AD}}{k+1}$	1M	
$=\frac{(m+1)\vec{a}+k\vec{b}}{k+1}$		
$=\frac{8\vec{a}+k\vec{b}}{k+1}$	1A	
Alterantive solution		
(2) $\vec{AE}=\vec{AD}+\vec{DE}$		
$=\vec{b}+\frac{1}{k+1}\vec{DP}$	1M	
$=\vec{b}+\frac{1}{k+1}[(m+1)\vec{a}-\vec{b}]$		
$=\frac{8\vec{a}+k\vec{b}}{k+1}$	1A	
Comparing the two expressions,		
$\begin{cases}\frac{1}{r+1}=\frac{8}{k+1}\text{ ----(1)}\\ \frac{2}{r+1}=\frac{k}{k+1}\text{ ----(2)}\end{cases}$	1M	
$\therefore \frac{1}{2}=\frac{8}{k}$		
$k=16$	1A	
$\frac{1}{r+1}=\frac{8}{16+1}$		
$r=\frac{9}{8}$	1A	
	10	

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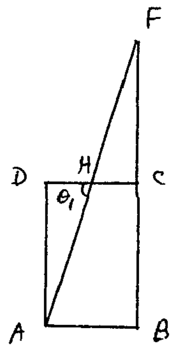
Solution	Marks	Remarks
(c) As m tends to infinity, the limiting position of E lies on DC.	1A	(can be omitted) 
Let this point be H. $\tan\theta_1 = \dfrac{AD}{DH}$	1M	
$= \dfrac{2}{1/2} = 4$		
$\therefore \theta_1 = 76°$ correct to the nearest degree.	1A	
Alternative solution (1)		
As m tends to infinity, the limiting position of E lies on DC.	1A	(can be omitted)
$\vec{AF}\cdot\vec{DC} = (\vec{a}+2\vec{b})\cdot\vec{a}$		
$\lvert\vec{AF}\rvert\lvert\vec{DC}\rvert\cos\theta_1 = (\vec{a}+2\vec{b})\cdot\vec{a}$	1M	
$\sqrt{4^2+1^2}(1)\cos\theta_1 = 1$		
$\cos\theta_1 = \dfrac{1}{\sqrt{17}}$		
$\therefore \theta_1 = 76°$ correct to the nearest degree.	1A	
Alternative solution (2)		
$\vec{AF}\cdot\vec{DP} = \lvert\vec{AF}\rvert\lvert\vec{DP}\rvert\cos\theta$		
$(\vec{a}+2\vec{b})\cdot[(m+1)\vec{a}-\vec{b}] = \sqrt{17}\sqrt{(m+1)^2+2^2}\cos\theta$		
$\cos\theta = \dfrac{m-7}{\sqrt{17}\sqrt{m^2+2m+5}}$	1A	
$= \dfrac{1-\frac{7}{m}}{\sqrt{17}\sqrt{1+\frac{2}{m}+\frac{5}{m^2}}}$		
As $m\to\infty$, $\cos\theta \to \dfrac{1}{\sqrt{17}}$.	1M	For taking limits
$\therefore \theta_1 = 76°$ correct to the nearest degree.	1A	
		Omitting vector sign or dot sign in most cases (pp−1)
	3	

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Solution	Marks	Remarks
10. (a) $f'(x)=\dfrac{(x^2+1)(2x+k)-(x^2+kx+9)(2x)}{(x^2+1)^2}$	1M	For quotient rule
$=\dfrac{-kx^2-16x+k}{(x^2+1)^2}$	1A	
$f'(3)=0$		
$\dfrac{-k(3)^2-16(3)+k}{(3^2+1)^2}=0$	1M	
$k=-6$	1	
	4	
(b) (i) $f(x)=\dfrac{x^2-6x+9}{x^2+1}$		
$f(0)=9$ ∴ The y-intercept is 9.	1A	(pp–1) for giving (0, 9)
$f(x)=0$ at $x=3$ ∴ The x-intercept is 3.	1A	(pp–1) for giving (3, 0)
(ii) $f'(x)=\dfrac{-kx^2-16x+k}{(x^2+1)^2}=\dfrac{6x^2-16x-6}{(x^2+1)^2}$		
$f'(x)=0$		
$\dfrac{6x^2-16x-6}{(x^2+1)^2}=0$	1M	
$6x^2-16x-6=0$		
$(3x+1)(x-3)=0$		
$x=-\dfrac{1}{3}$ or 3	1A	
∴ The turning points are (3, 0) and $(-\dfrac{1}{3},10)$.		
$f''(x)=\dfrac{(x^2+1)^2(12x-16)-(6x^2-16x-6)2(x^2+1)2x}{(x^2+1)^4}$	1M	For checking
$=\dfrac{4(-3x^3+12x^2+9x-4)}{(x^2+1)^3}$		
$f''(3)=\dfrac{1}{5}>0$		
∴ (3, 0) is a minimum point.	1A	no marks if checking is omitted
$f''(-\dfrac{1}{3})=-\dfrac{81}{5}<0$		
∴ $(-\dfrac{1}{3},10)$ is a maximum point.	1A	

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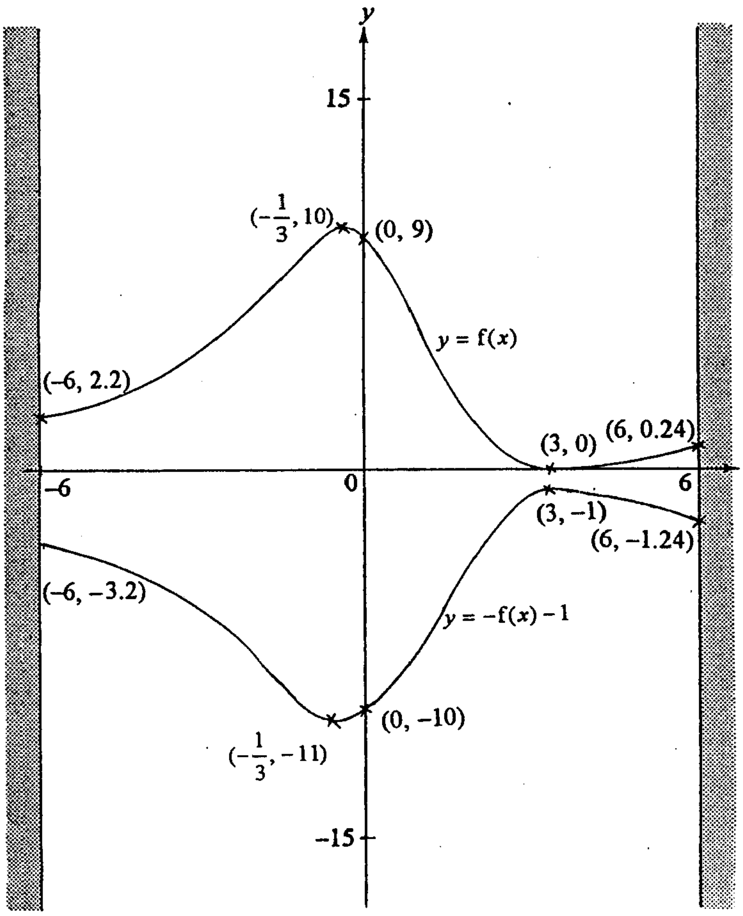
Solution	Marks	Remarks
<u>Alternative solution for checking</u> $f'(x)=\frac{2(3x+1)(x-3)}{(x^2+1)^2}$ When $x>3$, $f'(x)>0$. When $-\frac{1}{3}<x<3$, $f'(x)<0$.	1M	For checking
$\therefore$ (3, 0) is a minimum point.	1A	
When $-\frac{1}{3}<x<3$, $f'(x)<0$. When $x<-\frac{1}{3}$, $f'(x)>0$. $\therefore$ $(-\frac{1}{3}, 10)$ is a maximum point.	1A	
	7	
(c)	1A+1A+1A	<u>$y=f(x)$</u> (awarded even if checking was omitted in (b)) 1A for shape 1A for intercepts and turning points 1A for end-points
	1M +1A	<u>$y=-f(x)-1$</u> – 1M for shape – 1A if the curve is correctly drawn (At least one point on the curve should be labelled)
Figure 3		
	5	

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Solution	Marks	Remarks
11. (a) $\cos 5\theta = 0$ $5\theta = 360n° \pm 90°$ (n is an integer) OR $5\theta = 90°, 270°, 450°, 630°, 810°$		OR $(2n+1)90°$, $90n°$ (n is odd)
$\theta = 18°, 54°, 90°, 126°, 162°$ OR $\theta = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}$	2A	All correct – 2A 2-4 correct – 1A only ≤ 1 correct – no mark
	2	
(b) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$	1A	For using De Moivre's Theorem
$(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$ $+10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$	1A	For using Binomial Theorem
Comparing the real parts, $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	1M	
$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	1	
	4	
(c) (i) $f(\cos\theta) = 16\cos^4\theta - 20\cos^2\theta + 5$ $= \frac{\cos 5\theta}{\cos\theta}$ (using (b)) From (a) $18°, 54°, 126°, 162°$ are the roots of $\frac{\cos 5\theta}{\cos\theta} = 0$. ($90°$ is rejected $\because \cos\theta \neq 0$.)	2M	Award 1M if partially correct
$\therefore$ $\cos 18°, \cos 54°, \cos 126°, \cos 162°$ are the roots of $f(x) = 0$. $\therefore$ The 4 values of θ are $18°, 54°, 126°$ and $162°$.	1A	
Alternative solution (i) From (b), $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ Using (a), $18°, 54°, 90°, 126°$ & $162°$ are the roots of $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta = 0$ Since $16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ $= \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5)$, $\therefore$ $18°, 54°, 126°$ & $162°$ are the roots of $16\cos^4\theta - 20\cos^2\theta + 5 = 0$. ($\theta = 90°$ is a root of $\cos\theta = 0$.)	2M	Award 1M if partially correct
As $f(\cos\theta) = 16\cos^4\theta - 20\cos^2\theta + 5$, the 4 values of θ for which $\cos\theta$ is a root of $f(x) = 0$ are $18°, 54°, 126°$ and $162°$.	1A	
$f(x) = 16(x - \cos 18°)(x - \cos 54°)(x - \cos 126°)(x - \cos 162°)$	1M	
$= 16(x - \cos 18°)(x - \cos 54°)(x + \cos 54°)(x + \cos 18°)$ $= 16(x^2 - \cos^2 18°)(x^2 - \cos^2 54°)$	1	
(ii) From (*), $\cos^2 18°$ and $\cos^2 54°$ are the roots of the equation $16(x^2)^2 - 20x^2 + 5 = 0$ in x^2.		
$\cos^2 18° + \cos^2 54° = \frac{5}{4}$	1A	
$\cos^2 18° \cos^2 54° = \frac{5}{16}$	1A	

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Solution	Marks	Remarks
$\sin^2 18°+\sin^2 54° = (1-\cos^2 18°)+(1-\cos^2 54°)$	1M	For expressing $\sin^2\theta$ in terms of $\cos^2\theta$ (can be awarded in either step)
$= 2-\frac{5}{4}$		
$= \frac{3}{4}$		
$\sin^2 18°\sin^2 54° = (1-\cos^2 18°)(1-\cos^2 54°)$	↑	
$= 1-(\cos^2 18°+\cos^2 54°)+\cos^2 18°\cos^2 54°$	1M	For expressing in terms of sum and product
$= 1-\frac{5}{4}+\frac{5}{16}$		
$= \frac{1}{16}$		
$\therefore$ The equation is $x^2-\frac{3}{4}x+\frac{1}{16}=0$		
$16x^2-12x+1=0$	1A	
Alternative solution (1)		
From (*), $16x^4-20x^2+5=16(x^2-\cos^2 18°)(x^2-\cos^2 54°)$		
Put $x=1$: $16-20+5=16(1-\cos^2 18°)(1-\cos^2 54°)$	1M	
$\sin^2 18°\sin^2 54° = \frac{1}{16}$	1A	
Put $x=0$: $5=16\cos^2 18°\cos^2 54°$	1M	
$= 16(1-\sin^2 18°)(1-\sin^2 54°)$		
$\frac{5}{16} = 1-(\sin^2 18°+\sin^2 54°)+\sin^2 18°\sin^2 54°$		
$\sin^2 18°+\sin^2 54° = \frac{3}{4}$	1A	
$\therefore$ The equation is $x^2-\frac{3}{4}x+\frac{1}{16}=0$		
$16x^2-12x+1=0$.	1A	
Alternative solution (2)		
$16x^4-20x^2+5=0$		
$x^2=\frac{20\pm\sqrt{20^2-4(16)(5)}}{2\times 16}$		
$=\frac{5\pm\sqrt{5}}{8}$	1A	
From (*), $16x^4-20x^2+5=16(x^2-\cos^2 18°)(x^2-\cos^2 54°)$		
$\therefore \cos^2 18°=\frac{5+\sqrt{5}}{8}$ and $\cos^2 54°=\frac{5-\sqrt{5}}{8}$	1M	
$\sin^2 18°=1-\frac{5+\sqrt{5}}{8}$ $\sin^2 54°=1-\frac{5-\sqrt{5}}{8}$		
$=\frac{3-\sqrt{5}}{8}$ $=\frac{3+\sqrt{5}}{8}$	1A	
$\therefore$ The equation is $(x-\frac{3-\sqrt{5}}{8})(x-\frac{3+\sqrt{5}}{8})=0$	1M	
$x^2-(\frac{3-\sqrt{5}}{8}+\frac{3+\sqrt{5}}{8})x+\frac{(3-\sqrt{5})(3+\sqrt{5})}{64}=0$		

Solution	Marks	Remarks
$x^2 - \frac{3}{4}x + \frac{1}{16} = 0$		
$16x^2 - 12x + 1 = 0$.	1A	
Alternative solution (3)		
From (*), $\cos^2 18°$ and $\cos^2 54°$ are the roots of the equation $16(x^2)^2 - 20x^2 + 5 = 0$ in x^2.		
Put $y = 1 - x^2$.		
$16(1-y)^2 - 20(1-y) + 5 = 0$	2M	
$16(1-2y+y^2) - 20(1-y) + 5 = 0$		
$16y^2 - 12y + 1 = 0$ which is the equation with roots $\sin^2 18°$ and $\sin^2 54°$.	3A	
	10	

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Solution	Marks	Remarks
12. (a) $\sin\theta = \dfrac{GE}{OE}$	1A	
$= \dfrac{s}{1-s}$		
$\sin\theta - s\sin\theta = s$		
$s = \dfrac{\sin\theta}{1+\sin\theta}$	1	
$\dfrac{ds}{d\theta} = \dfrac{(1+\sin\theta)\cos\theta - \sin\theta\cos\theta}{(1+\sin\theta)^2}$	1M	For quotient rule
$= \dfrac{\cos\theta}{(1+\sin\theta)^2}$	1A	
	4	
(b) $\dfrac{r}{s} = \dfrac{OF}{OE}$	1A	
$\dfrac{r}{s} = \dfrac{1-2s-r}{1-s}$		
$r - rs = s - 2s^2 - rs$		
$r = s - 2s^2$	1A	
$\dfrac{dr}{d\theta} = \dfrac{dr}{ds}\dfrac{ds}{d\theta}$	1M	For chain rule
$= (1-4s)\dfrac{ds}{d\theta}$	1A	
$= (1 - 4\dfrac{\sin\theta}{1+\sin\theta})\dfrac{\cos\theta}{(1+\sin\theta)^2}$		
$= \dfrac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3}$	1	
Alterantive solution for 2nd part		
$r = s - 2s^2$		
$= \dfrac{\sin\theta}{1+\sin\theta} - 2\dfrac{\sin^2\theta}{(1+\sin\theta)^2}$		
$= \dfrac{\sin\theta - \sin^2\theta}{(1+\sin\theta)^2}$	1A	
$\dfrac{dr}{d\theta} = \dfrac{(1+\sin\theta)^2(\cos\theta - 2\sin\theta\cos\theta) - (\sin\theta - \sin^2\theta)2(1+\sin\theta)\cos\theta}{(1+\sin\theta)^4}$	1M	For expressing r in terms of θ and finding $\dfrac{dr}{d\theta}$
$= \dfrac{\cos\theta(1+\sin\theta)(1-2\sin\theta) - 2\cos\theta(\sin\theta - \sin^2\theta)}{(1+\sin\theta)^3}$		
$= \dfrac{\cos\theta(1-2\sin\theta+\sin\theta-2\sin^2\theta-2\sin\theta+2\sin^2\theta)}{(1+\sin\theta)^3}$		
$= \dfrac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3}$	1	
	5	

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Solution	Marks	Remarks
(c) (i) r is increasing when $\frac{dr}{d\theta} \geq 0$.		OR $\frac{dr}{d\theta} > 0$
$\frac{\cos\theta(1-3\sin\theta)}{(1+\sin\theta)^3} \geq 0$	1M	
$1-3\sin\theta \geq 0 \quad (\because 0<\theta<\frac{\pi}{2})$		
$\sin\theta \leq \frac{1}{3}$	1A	OR $\sin\theta < \frac{1}{3}$
$(0<)\theta \leq 0.340$		Accept degrees (19.5°)
(ii) r is decreasing when $\frac{dr}{d\theta} \leq 0$.		OR $\frac{dr}{d\theta} < 0$
$1-3\sin\theta \leq 0$		
$\sin\theta \geq \frac{1}{3}$	1A	OR $\sin\theta > \frac{1}{3}$
$(\frac{\pi}{2}>)\theta \geq 0.340$		Accept degrees (19.5°)
$\therefore$ r attains a maximum at $\theta = 0.340$.		
Maximum area of C_2 $= \pi r^2$ $= \pi[s(1-2s)]^2$		
$= \pi[\frac{\frac{1}{3}}{1+\frac{1}{3}}(1-\frac{2(\frac{1}{3})}{1+\frac{1}{3}})]^2$	1M	
$= \frac{\pi}{64}$ (OR 0.0491)	1A	Exact value or correct to at least 3 sig. figures.
	5	
(d) At $\sin\theta = \frac{1}{3}$, $\frac{ds}{d\theta} (= \frac{\sqrt{8}/3}{(1+\frac{1}{3})^2}) \neq 0$.		
Since $\frac{ds}{d\theta} \neq 0$ at $\sin\theta = \frac{1}{3}$, s (and hence the area of C_1) does not attain a minimum when the area of C_2 attains its maximum.	2	
Alterantive solution From (a), $\frac{ds}{d\theta} = \frac{\cos\theta}{(1+\sin\theta)^2} > 0 \;\forall\; 0<\theta<\frac{\pi}{2}$. $\therefore$ s increases with θ. $\therefore$ The area of C_1 does not attain a minimum at $\sin\theta = \frac{1}{3}$.	2	
	2	

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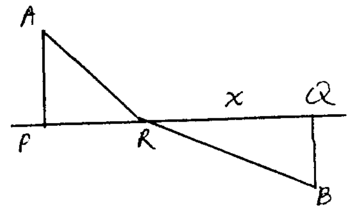
Solution	Marks	Remarks
13. (a) (i) Let $QR = x$.		
$AR = RB$		
$\sqrt{4^2+(15-x)^2} = \sqrt{2^2+x^2}$	1M	
$16 + x^2 - 30x + 225 = 4 + x^2$		
$x = 7.9$ km	1A	
Checking : Put $x = 7.9$ L.H.S. = R.H.S. = $\sqrt{66.41}$		
(ii) Since $AR = RB$ and they walk with equal speeds, they reach R at the same time. If they meet at other points, either one of them have to walk a longer distance than AR (OR RB) and the other has to wait for him/her. Hence the total time for them to meet must be longer.	2	Award 1 mark if the answer was partially correct
Shortest time $= \frac{AR}{4}$ (OR $= \frac{RB}{4}$)		
$= \frac{\sqrt{4^2+(15-7.9)^2}}{4}$	1M	
$= 2.04$ hr (OR 2 hr 2.24 min, 2 hr 2 min 14.3 s)	1A	
	6	
(b) (i) Let S be the meeting point and $QS = y$. To meet within the shortest time, they should arrive S at the same time.		
$\frac{\sqrt{4^2+(15-y)^2}}{4} = \frac{\sqrt{2^2+y^2}}{8}$	1M+1A	
$64 + 4(y^2 - 30y + 225) = 4 + y^2$		
$3y^2 - 120y + 960 = 0$	1A	
$y = \frac{120 \pm \sqrt{120^2 - 4(3)(960)}}{6}$		
≈ 28.9 or 11.1	1A	(can be omitted)
Checking : Put $y = 28.9$ $(15-y) < 0$ $\therefore y = 28.9$ is rejected Put $y = 11.1$ L.H.S. = R.H.S. = 1.404		
$\therefore$ They should meet at a distance 11.1 km from Q.	1A	
Alternative solution		
Let $PS = y$.		
$\frac{\sqrt{4^2+y^2}}{4} = \frac{\sqrt{2^2+(15-y)^2}}{8}$	1M+1A	
$64 + 4y^2 = 4 + 225 - 30y + y^2$		
$3y^2 + 30y - 165 = 0$	1A	
$y = 3.94$ or -13.9 (rejected)	1A	
$\therefore$ They should meet at a distance 3.94 km from P.	1A	
(ii) Let T be the meeting point where $QT = t$.		
$\frac{\sqrt{2^2+t^2}}{16} = \frac{\sqrt{4^2+(15-t)^2}}{4}$	1A	
$4 + t^2 = 256 + 16(t^2 - 30t + 225)$		
$15t^2 - 480t + 3852 = 0$	1A	
$\Delta = (480)^2 - 4(15)(3852)$ $= -720 < 0$		
$\therefore$ There are no real roots.	1A	

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Solution	Marks	Remarks
OR Let $PT = s$.		
$\dfrac{\sqrt{4+s^2}}{4} = \dfrac{\sqrt{2^2+(15-s)^2}}{16}$	1A	
$16(16+s^2) = 4+225-30s+s^2$		
$15s^2+30s+27=0$	1A	
$\Delta = -720 < 0$		
$\therefore$ There are no real roots.	1A	
i.e. Billy has already reached P before Amy reaches P.		
$\therefore$ They should meet at point P.	2A	
Alternative solution		
Time for Amy to reach $P = \dfrac{4}{4} = 1\text{ hr}$	1M+1A +1A	
Time for Billy to reach $P = \dfrac{\sqrt{2^2+(15)^2}}{16}$ $= 0.95\text{ hr}$		
$\therefore$ Billy has already reach P before Amy reaches P.		
So they should meet at point P.	2A	
	10	

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