Section A (42 marks)

Answer ALL questions in this Section.

1. Find the general solution of the equation

$$\sin 5\theta + \sin 3\theta = \cos\theta .$$

(5 marks)

2. It is given that

$$(1+x+ax^2)^6 = 1+6x+k_1x^2+k_2x^3 + \text{ terms involving higher powers of } x .$$

(a) Express k_1 and k_2 in terms of a.

(b) If 6, k_1 and k_2 are in A.P., find the value of a.

(6 marks)

3. E is the ellipse $\dfrac{x^2}{2}+\dfrac{y^2}{7}=1$.

(a) Let the line $y=mx+c$ be a tangent to E. Show that $c^2=2m^2+7$.

(b) Using (a), find the equations of the two tangents from the point (0, 5) to E.

(7 marks)

4. Prove by mathematical induction, that for all positive integers n,

$(2n^3+n)$ is divisible by 3 .

(6 marks)

5.

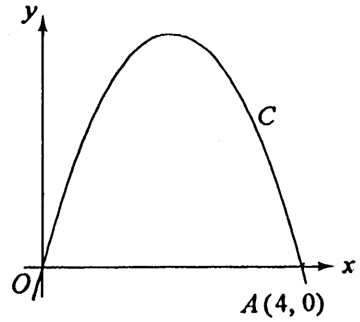

Figure 1(a)

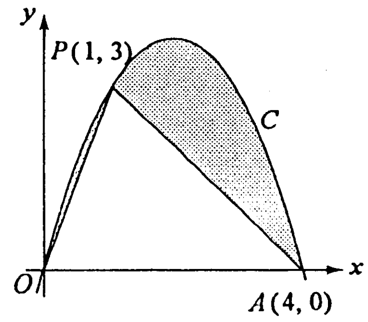

Figure 1(b)

The curve $C: y=4x-x^2$ cuts the x-axis at the origin O and the point $A(4, 0)$ as shown in Figure 1(a).

(a) Find the area of the region bounded by C and the line segment OA.

(b) In Figure 1(b), the shaded region is enclosed by the curve C and the line segements OP and PA, where P is the point (1, 3). Using (a), find the total area of the shaded region.

(6 marks)

6. The slope at any point (x, y) of a curve is given by $\dfrac{dy}{dx}=\tan^3 x \sec x$.

If the curve passes through the origin, find its equation.

[Hint : Let $u=\sec x$.]

(6 marks)

7. $P(x, y)$ is a variable point such that the distance between P and the point (4, 0) is always equal to twice the distance from P to the line $x-1=0$.

(a) Find the equation of the locus of P. State whether the locus is a circle, an ellipse, a hyperbola or a parabola.

(b) Sketch the locus of P.

(6 marks)

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Section B (48 marks)

Answer any THREE questions in this Section.
Each question carries 16 marks.

8. Given two straight lines $L_1 : 2x - y - 4 = 0$ and $L_2 : x - 2y + 4 = 0$. Let F be the family of straight lines passing through the point of intersection of L_1 and L_2.

(a) Write down the equation of F.

Hence find the equation of the line L in F which passes through the origin.

(4 marks)

(b)

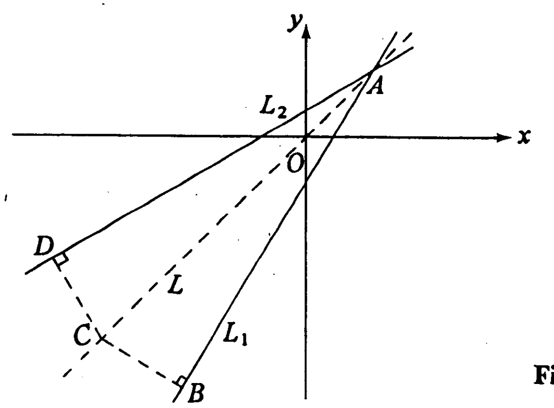

Figure 2

Let A denote the point of intersection of L_1 and L_2. C is a point in the third quadrant lying on the line L found in (a). Points B and D are the feet of perpendicular from C to L_1 and L_2 respectively. (See Figure 2.)

(i) Show that both $\tan\angle CAB$ and $\tan\angle CAD$ are equal to $\frac{1}{3}$.

(ii) If the area of quadrilateral $ABCD$ is 240, find

(1) the length of BC,

(2) the coordinates of point C.

(12 marks)

9. (a) Evaluate $\int_0^{\pi} \sin^5 x \, dx$.

[Hint : Let $t = \cos x$.]

(4 marks)

(b) Using the substitution $u = \pi - x$ and the result of (a), evaluate

$$\int_0^{\pi} x \sin^5 x \, dx.$$

(5 marks)

(c) By differentiating $y = x^2 \sin^5 x$ with respect to x and using the result of (b), evaluate

$$I_1 = \int_0^{\pi} x^2 \sin^4 x \cos x \, dx.$$

(5 marks)

(d) Let $I_2 = \int_0^{\pi} x^2 \sin^4 x \cos|x| \, dx$.

State, with a reason, whether I_2 is smaller than, equal to or larger than I_1 in (c).

(2 marks)

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10. The equation of a family of circles F is given by

$$x^2+y^2-8kx-6ky+25(k^2-1)=0,$$

where k is real.

(a) (i) Find the centre of a circle in F in terms of k.

Hence show that the centres of all circles in F lie on the line $3x-4y=0$.

(ii) Show that all circles in F have the same radius 5.

(3 marks)

(b)

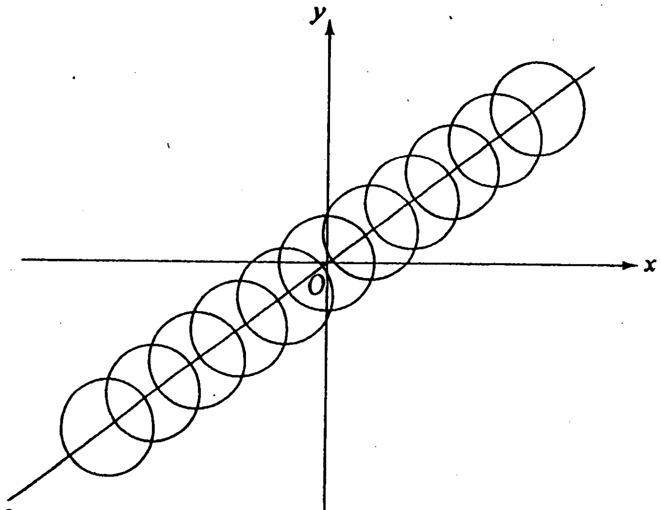

Figure 3

Figure 3 shows some circles in F. It is given that there are two parallel lines, both of which are common tangents to all circles in F.

Write down the slope of these two common tangents.

Hence find the equations of these two common tangents.

(6 marks)

(c) A circle in F cuts the x-axis at two points A and B.

Using (a) (i), write down the distance from the centre of the circle to the x-axis in terms of k.

Hence, or otherwise, find the equations of the two possible circles in F satisfying the condition $AB=8$.

(7 marks)

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11.

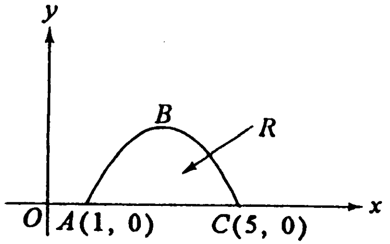

Figure 4 (a)

In Figure 4(a), the region R is enclosed by the parabola $y = 4-(x-3)^2$ and the line segment AC, where A and C are the points (1, 0) and (5, 0) respectively. B is the vertex of the parabola.

(a) Write down the coordinates of B.

(1 mark)

(b) (i) Show that the equation of the curve AB is

$$x = 3 - \sqrt{4-y}.$$

(ii) Write down the equation of the curve BC.

(2 marks)

(c)

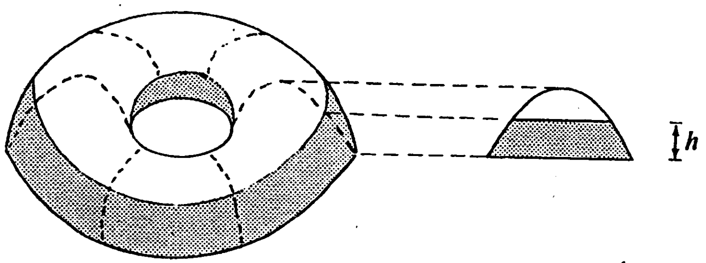

Figure 4(b)

(c) **(Continued)**

A jelly ring is in the shape of the solid of revolution of the region R in Figure 4(a) about the y-axis. Furthermore, the jelly ring contains two layers. Let h be the height of the lower layer. (See Figure 4(b) .)

(i) Show that the volume of the lower layer of the jelly ring is

$$8\pi\left[8-(4-h)^{\frac{3}{2}}\right].$$

(ii) If the two layers have equal volumes, find the value of h correct to 3 significant figures.

(9 marks)

(d)

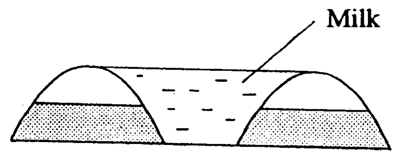

Figure 4(c)

If milk is poured into the centre of the jelly ring in (c) until it is completely filled (see Figure 4(c)), find the volume of milk required.

(4 marks)

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12.

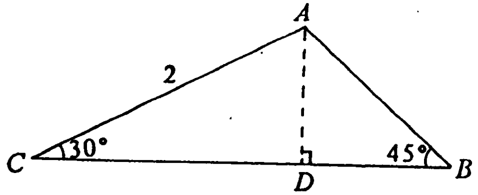

Figure 5(a)

In Figure 5(a), ABC is a triangular piece of paper such that $\angle B = 45°$, $\angle C = 30°$ and $AC = 2$. D is the foot of perpendicular from A to BC.

(a) Find AB, BD and DC. (3 marks)

(b)

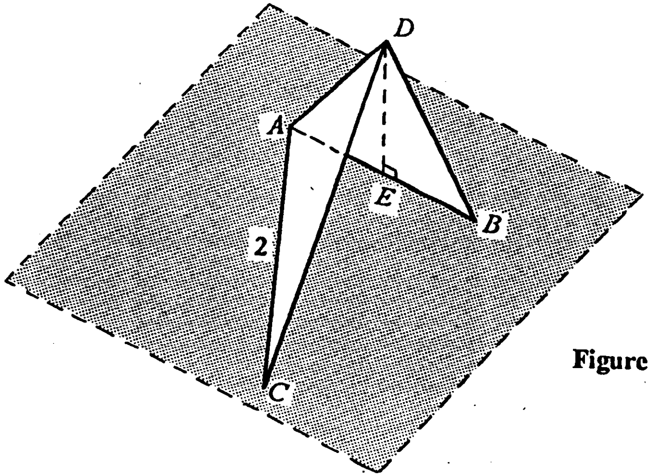

Figure 5(b)

The paper is folded along AD. It is then placed on a horizontal table such that the edges AB and AC lie on the table and the plane DAB is vertical. (See Figure 5(b).) E is the foot of perpendicular from D to AB.

(i) If θ is the angle between DC and the horizontal, show that $\sin\theta = \frac{\sqrt{6}}{6}$.

(ii) Find CE. Hence show that $\angle EAC = 45°$.

(iii) Find the angle between the two planes DAB and DAC to the nearest degree.

[Hint : You may use a ruler to tear off Figure 5(c) to help you answer part (b).] (13 marks)

Note : You need not hand in Figure 5(c).

12. (b) (Continued)

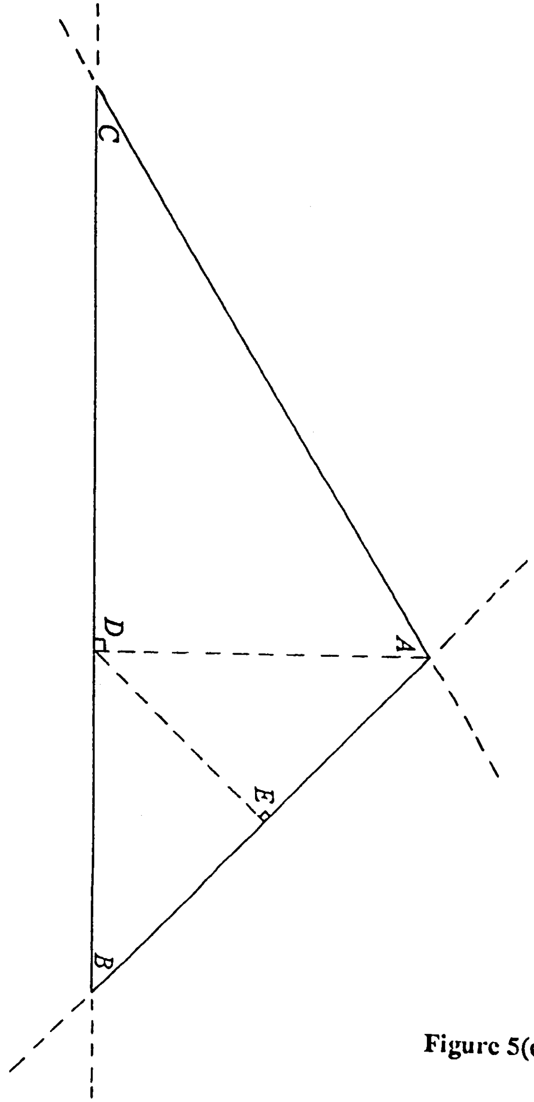

Figure 5(c)

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