A. Maths

1996 I

GENERAL INSTRUCTIONS TO MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method is specified in the question.

2. In the marking scheme, marks are classified as follows :

   'M' marks – awarded for knowing a correct method of solution and attempting to apply it;

   'A' marks – awarded for the accuracy of the answer;

   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answer should **NOT** be awarded. Unless otherwise specified, no marks in the marking scheme are subdivisible.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. The symbol pp–1 should be used to denote marks deducted for poor presentation (p.p.). Marks entered in the Page Total Box should be the net total score on that page. Note the following points :

   (a) At most deduct 1 mark for p.p. in each question, up to a maximum of 3 marks for the whole paper.

   (b) For similar p.p., deduct only 1 mark for the first time that it occurs, i.e. do not penalise candidates twice in the whole paper for the same p.p.

   (c) In any case, do not deduct any marks for p.p. in those parts where candidates could not score any marks.

   (d) Some cases in which marks should be deducted for p.p. are specified in the marking scheme. However the lists are by no means exhaustive. Markers should exercise their professional judgement to give p.p.s in other situations.

5. In the Marking Scheme, steps which can be omitted are enclosed by dotted rectangles ⬚, whereas alternative answers are enclosed by solid rectangles ▭.

6. Unless otherwise specified in the question, numerical answers not given in exact values should not be accepted.

7. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

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Solution	Marks	Remarks
1. $f'(x)=3\sin^2 x\cos x$	1A	
$f''(x)=6\sin x\cos^2 x-3\sin^3 x$	1M+1A	Accept equivalent forms
	3	
2. $\frac{d}{dx}(x^2)=\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x}$	1A	Withhold this mark if $\lim_{\Delta x\to 0}$ is omitted.
$=\lim_{\Delta x\to 0}\frac{2x\Delta x+(\Delta x)^2}{\Delta x}$	1A	For simplification
$=\lim_{\Delta x\to 0}(2x+\Delta x)$	1A	For simplification
$=2x$	1A	
	4	
3. $\frac{2x-3}{x+1}\le 1$		
$\frac{2x-3}{x+1}-1\le 0$	1M	
$\frac{x-4}{x+1}\le 0$	1A	
$-1<x\le 4$	2A	1A only for $-1\le x\le 4$
	4	
Alternative Solution (1)		
Consider the following cases (i) $x>-1$, (ii) $x<-1$.	1M	Awarded even if equality sign is included
Case 1 : $x>-1$		
$2x-3\le x+1$		
$x\le 4$		
Since $x>-1$, $\therefore$ $-1<x\le 4$.	1A (for both $2x-3\le x+1$ and $2x-3\ge x+1$)	
Case 2 : $x<-1$		
$2x-3\ge x+1$		
$x\ge 4$		
Since $x<-1$, $\therefore$ there is no solution.		
Combining the 2 cases, $-1<x\le 4$.	2A	1A only for $-1\le x\le 4$

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Solution	Marks	Remarks
Alternative Solution (2)		
$\dfrac{2x-3}{x+1} \le 1$		
$(2x-3)(x+1) \le (x+1)^2$ and $x \neq -1$	1M	
$x^2-3x-4 \le 0$ and $x \neq -1$		
$(x+1)(x-4) \le 0$ and $x \neq -1$	1A	1A only for $-1 \le x \le 4$
$-1 < x \le 4$	2A	
4. (a) $x^2-6x+11 \equiv (x^2-6x+9)+2$	1M	For using method of completing square
$= (x-3)^2+2$		
$\therefore a=-3,\ b=2$	1A	
Alternative Solution (1)		
$x^2-6x+11 \equiv (x+a)^2+b$		
$= x^2+2ax+a^2+b$		
Comparing coefficients,		
$\begin{cases} 2a=-6 \\ a^2+b=11 \end{cases}$	1M	For comparing coefficients
$\therefore a=-3,\ b=2$	1A	
Alternative Solution (2)		
$x^2-6x+11 \equiv (x+a)^2+b$		
Put $x=0$, $a^2+b=11$		
$x=1$ $(a+1)^2+b=6$	1M	For substituting any two real values of x
Solving the 2 equations, $a=-3$, $b=2$.	1A	
The least value of $x^2-6x+11$ is 2.	1A	

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Solution	Marks	Remarks
(b) The range of possible values of $\frac{1}{x^2-6x+11}$ is		
$0<\frac{1}{x^2-6x+11}\le\frac{1}{2}$.	1M+1A	1M for $\le\frac{1}{\text{least value fond in (a)}}$
	5	1A if all correct
Alternative solution Let $\frac{1}{x^2-6x+11}=r$ $rx^2-6rx+(11r-1)=0$ $\Delta=36r^2-4r(11r-1)\ge 0$	1M	
$2r^2-r\le 0$ $r(2r-1)\le 0$ $0<r\le\frac{1}{2}$ Since $r\neq 0$	1A	
5. (a) $\frac{2+4i}{1-i}=\frac{2+4i}{1-i}(\frac{1+i}{1+i})$	1M	
$=\frac{2+2i+4i-4}{2}$ $=-1+3i$	1A	
(b) $p+qi=\frac{2+4i}{1-i}(q+i)$ $p+qi=(-1+3i)(q+i)$ $=-q-3+(3q-1)i$	1M	For simplifying to the form $a+bi=c+di$
Comparing coefficients, $\begin{cases}p=-q-3\\q=3q-1\end{cases}$	1M	
Alternative solution $p+qi=\frac{2+4i}{1-i}(q+i)$ $(p+qi)(1-i)=(2+4i)(q+i)$ $(p+q)+(-p+q)i=(2q-4)+(2+4q)i$	1M	For simplifying to the form $a+bi=c+di$
Comparing coefficients, $\begin{cases}p+q=2q-4\\-p+q=2+4q\end{cases}$	1M	
$\therefore q=\frac{1}{2}$	1A	
$p=\frac{-7}{2}$	1A	
	6	

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Solution	Marks	Remarks
6. $\frac{dy}{dx}=\frac{-6}{(x+1)^2}$	1A	
$\frac{-6}{(x+1)^2}=-\frac{1}{6}$	1M+1A	1A for slope of line $=-\frac{1}{6}$
$(x+1)^2=36$		
$x=5$ or -7		
The points of contact are $(5, 1)$ and $(-7, -1)$.	1A+1A	
Equations of tangents are		
$\frac{y-1}{x-5}=-\frac{1}{6}$ and $\frac{y+1}{x+7}=-\frac{1}{6}$	1M	For point-slope form
$x+6y-11=0$ and $x+6y+13=0$.	1A	
Alternative Solution		
Let the equation of the tangent be $x+6y+c=0$	1A	
Put $x=-(6y+c)$ into C.		
$y=\frac{6}{-6y+(1-c)}$	1M	
$6y^2+(c-1)y+6=0$	1A	
$\Delta=(c-1)^2-144=0$	1M	
$c=13$ or -11	1A+1A	
$\therefore$ The equations of the tangents are $x+6y+13=0$ and $x+6y-11=0$.	1A	
	7	
7. (a) Unit vector $=\frac{1}{\sqrt{4^2+3^2}}(4\vec{i}+3\vec{j})$	1M	Accept $\frac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert}$
$=\frac{4}{5}\vec{i}+\frac{3}{5}\vec{j}$	1A	
$\overrightarrow{OC}=\frac{16}{5}(\frac{4}{5}\vec{i}+\frac{3}{5}\vec{j})$ $=\frac{64}{25}\vec{i}+\frac{48}{25}\vec{j}$	1A	OR $\frac{16}{25}(4\vec{i}+3\vec{j})$

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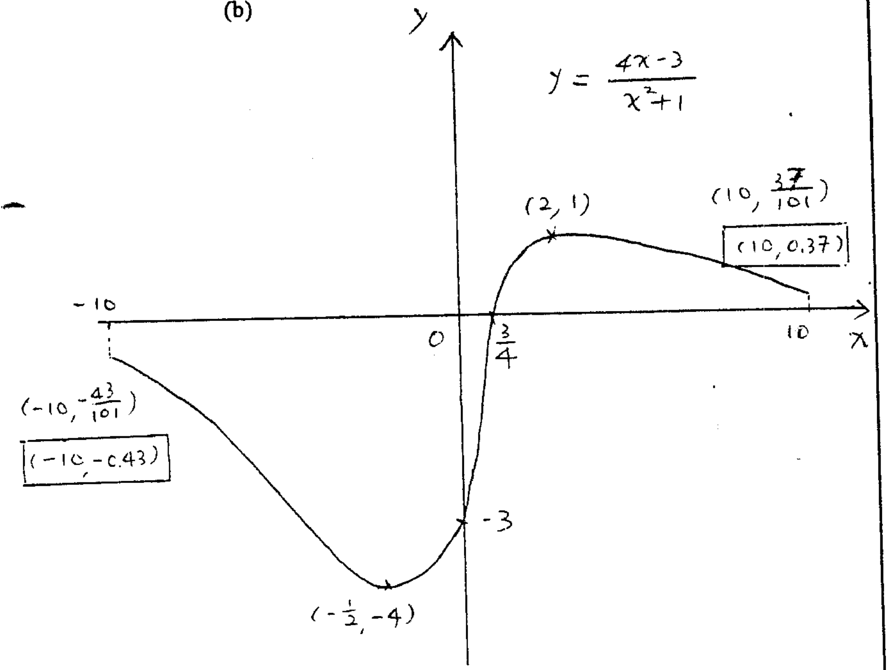
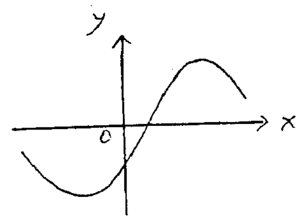
Solution	Marks	Remarks
(b) $\vec{BC}=\vec{OC}-\vec{OB}$		
$=(\frac{64}{25}\vec{i}+\frac{48}{25}\vec{j})-(\vec{i}+4\vec{j})$	1M	
$=\frac{39}{25}\vec{i}-\frac{52}{25}\vec{j}$		
$\vec{BC}\cdot\vec{OA}=(\frac{39}{25}\vec{i}-\frac{52}{25}\vec{j})\cdot(4\vec{i}+3\vec{j})$		
$=\frac{39}{25}(4)-\frac{52}{25}(3)$	1M	For $\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=1$ and $\vec{i}\cdot\vec{j}=0$
$=0$		
$\therefore$ BC is perpendicular to OA.	1	Omitting vector sign generally (pp–1) Omitting dot sign generally (pp–1) (pp–1) for writing $\vec{i}^2$ or $\frac{\vec{a}}{\vec{b}}=\frac{1}{2}$ etc.
	6	
Alternative solution		
(b) $\lvert\vec{OB}\rvert=\sqrt{1^2+4^2}=\sqrt{17}$		
$\lvert\vec{OC}\rvert=\sqrt{(\frac{64}{25})^2+(\frac{48}{25})^2}=\frac{16}{5}$		
$\vec{BC}=\vec{OC}-\vec{OB}$		
$=(\frac{64}{25}\vec{i}+\frac{48}{25}\vec{j})-(\vec{i}+4\vec{j})$	1M	
$=\frac{39}{25}\vec{i}-\frac{52}{25}\vec{j}$		
$\lvert\vec{BC}\rvert=\sqrt{(\frac{39}{25})^2+(\frac{-52}{25})^2}=\frac{13}{5}$		
$\lvert\vec{OC}\rvert^2+\lvert\vec{BC}\rvert^2=(\frac{16}{5})^2+(\frac{13}{5})^2$		
$=(\frac{16}{5})^2+(\frac{13}{5})^2$	1M	For using Pythagoras Theorem
$=17=OB^2$		
$\therefore$ BC is perpendicular to OA.	1	
8. (a) $(k-2)^2-4(k+1)>0$	1M	No mark for ≥ 0
$k^2-8k>0$	1A	For simplifying L.H.S.
$k(k-8)>0$		
$k>8$ or $k<0$	1A	
(b) $\alpha+\beta=k-2$	1A	
$\lvert k-2\rvert<5$		
$-5<k-2<5$	1M	For $-5<\alpha+\beta<5$ OR $(\alpha+\beta)^2<25$
$-3<k<7$	1A	
Combining with (a), $-3<k<0$.	1A	
	7	

Solution	Marks	Remarks
9. (a) (i) The x-intercept is $\frac{3}{4}$.	1A	Accept $(\frac{3}{4}, 0)$
The y-intercept is -3.	1A	Accept $(0, -3)$
(ii) $\frac{dy}{dx}=\frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2}$	1M+1A	1M for quotient rule. Awarded even if included in (iii)
$\frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2}\le 0$	1M	Accept <0
$-4x^2+6x+4\le 0$		
$(2x+1)(x-2)\ge 0$		
$x\ge 2$ or $x\le -\frac{1}{2}$	1A	Accept $x>2$ or $x<-\frac{1}{2}$
(iii) $\frac{dy}{dx}=\frac{4(x^2+1)-2x(4x-3)}{(x^2+1)^2}$		
$\frac{dy}{dx}=0$ at $x=2$ or $-\frac{1}{2}$	1M	
The turning points are $(2, 1)$ and $(-\frac{1}{2}, -4)$.		
$(2, 1)$ is a maximum point.	1A	Awarded independent from the answer in (ii).
$(-\frac{1}{2}, -4)$ is a minimum point.	1A	
	9	
(b) 	1A	Shape Accept 
	1A	Labelling the end-points
	1A	Labelling the intercepts and turning points
	3	

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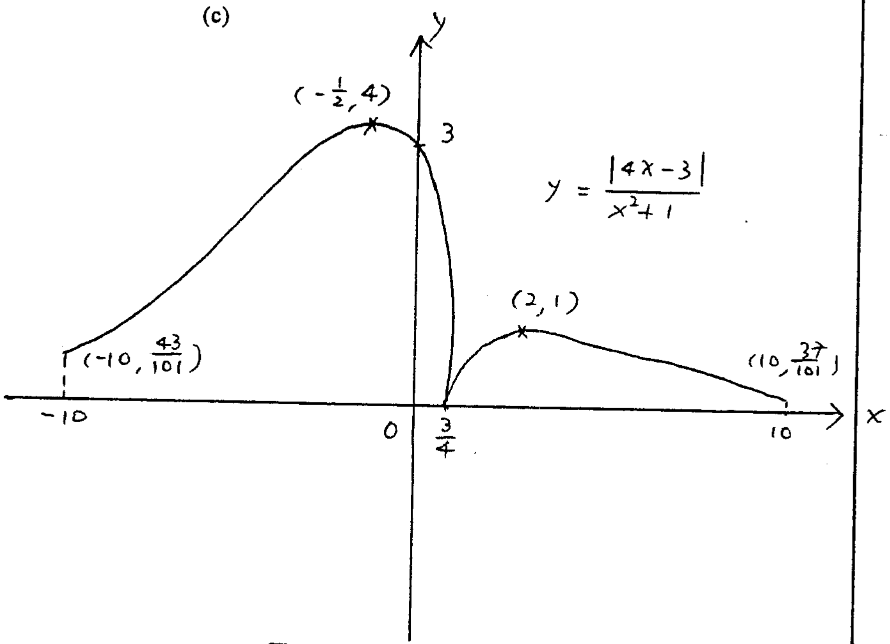
Solution	Marks	Remarks
(c) 	2M	For reflection (Accept no labelling)
The greatest value is 4.	1A	
The least value is 0.	1A	
	4	

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Solution	Marks	Remarks
10. (a) (i) $\overrightarrow{AE}=\dfrac{2\vec{a}+t\vec{b}}{1+t}$	1A	
(ii) $\overrightarrow{AE}=\dfrac{7\vec{a}+\overrightarrow{AF}}{8}$	2A	
Alternative Solution		
$\overrightarrow{AE}=\overrightarrow{AD}+\overrightarrow{DE}$	1A	
$=\vec{a}+\frac{1}{8}(\overrightarrow{DF})$		
$=\vec{a}+\frac{1}{8}(\overrightarrow{AF}-\vec{a})$		
$=\frac{7}{8}\vec{a}+\frac{1}{8}\overrightarrow{AF}$	1A	
$\dfrac{2\vec{a}+t\vec{b}}{1+t}=\dfrac{7\vec{a}+\overrightarrow{AF}}{8}$	1M	
$\overrightarrow{AF}=8(\dfrac{2\vec{a}+t\vec{b}}{1+t})-7\vec{a}$		
$=\dfrac{9-7t}{1+t}\vec{a}+\dfrac{8t}{1+t}\vec{b}$	1	
Alternative Solution		
$\overrightarrow{AF}=\overrightarrow{AD}+\overrightarrow{DF}$	1M	
$=\overrightarrow{AD}+8\overrightarrow{DE}$		
$=\vec{a}+8(\overrightarrow{AE}-\overrightarrow{AD})$		
$=\vec{a}+8(\dfrac{2\vec{a}+t\vec{b}}{1+t}-\vec{a})$		
$=\dfrac{9-7t}{1+t}\vec{a}+\dfrac{8t}{1+t}\vec{b}$	1	
	5	
(b) (i) Since A, B, F are collinear,		
$\dfrac{9-7t}{1+t}=0$	1M	
$t=\dfrac{9}{7}$	1A	
(ii) (1) $\vec{a}\cdot\vec{b}=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\angle BAC$		
$=3(2)(\frac{1}{3})$		
$=2$	1A	
(2) $\overrightarrow{AB}\cdot\overrightarrow{BC}=\vec{b}\cdot(\overrightarrow{AC}-\overrightarrow{AB})$	1M	For $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$
$=\vec{b}\cdot(2\vec{a}-\vec{b})$		
$=2\vec{b}\cdot\vec{a}-\vec{b}\cdot\vec{b}$	1M	For distributive law
$=2(2)-4$	1A	For $\vec{b}\cdot\vec{b}=4$
$=0$	1A	

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Solution	Marks	Remarks
$\overrightarrow{AD}\cdot\overrightarrow{DE}=\overrightarrow{AD}\cdot(\overrightarrow{AE}-\overrightarrow{AD})$		
$=\vec{a}\cdot\left(\dfrac{2\vec{a}+\frac{9}{7}\vec{b}}{1+\frac{9}{7}}-\vec{a}\right)$	1M	For substituting t to find $\overrightarrow{DE}$
$=\vec{a}\cdot\left(\dfrac{-2\vec{a}+9\vec{b}}{16}\right)$		
$=-\dfrac{2}{16}(\vec{a}\cdot\vec{a})+\dfrac{9}{16}(\vec{b}\cdot\vec{a})$		
$=-\dfrac{2}{16}(3)^2+\dfrac{9}{16}(2)$		
$=0$	1A	
(3) Since $\overrightarrow{AB}\cdot\overrightarrow{BC}=\overrightarrow{AD}\cdot\overrightarrow{DE}=0$, $\therefore\ \angle CBF=\angle CDF$ $\left(=\dfrac{\pi}{2}\right)$	1M	
$\therefore$ The points B, C, D and F are concyclic. (Converse of $\angle$s in the same segment.)		
$\therefore$ The circle also passes through F.	1A	Omitting vector sign generally (pp–1) Omitting dot sign generally (pp–1) (pp–1) for writing $\vec{i}^{\,2}$ or $\dfrac{\vec{a}}{\vec{b}}=\dfrac{1}{2}$ etc.
	11	

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Solution	Marks	Remarks
11. (a) $V=(\frac{1}{2}x^2\sin 60°)\ell$		
$=\frac{\sqrt{3}}{4}x^2\ell$	1A	
Since $V=24, \frac{\sqrt{3}}{4}x^2\ell=24$	1M	For establishing a relation in x and ℓ
$x^2\ell=32\sqrt{3}$		
$S=2(\frac{1}{2}x^2\sin 60°)+2x\ell$	1A	
$=\frac{\sqrt{3}}{2}x^2+2x(\frac{32\sqrt{3}}{x^2})$		
$S=\frac{\sqrt{3}}{2}x^2+\frac{64\sqrt{3}}{x}$	1	
	4	
(b) $\frac{dS}{dx}=\sqrt{3}x-\frac{64\sqrt{3}}{x^2}$	1A	
$\frac{dS}{dx}=0$		
$\sqrt{3}x=\frac{64\sqrt{3}}{x^2}$	1M	
$x=4$	1A	
$\frac{d^2S}{dx^2}=\sqrt{3}+\frac{128\sqrt{3}}{x^3}>0$ at $x=4$		
$\therefore$ S is minimum at $x=4$.	1M	For checking
When $x=4$, $\ell=2\sqrt{3}$.	1A	Withhold this mark if checking is omitted
	5	
(c) (i) $V=\frac{1}{2}h(2h\tan 30°)(2\sqrt{3})$		
$=2h^2$	1	
$A=(2h\tan 30°)(2\sqrt{3})$		
$=4h$	1	
$\frac{dV}{dt}=-\frac{1}{10}A$		
$\frac{dV}{dh}\frac{dh}{dt}=-\frac{1}{10}A$	1M	For chain rule
$(4h)\frac{dh}{dt}=-\frac{1}{10}(4h)$	1A	For $\frac{dV}{dh}=4h$ only.
$\frac{dh}{dt}=-\frac{1}{10}$	1A	

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Solution	Marks	Remarks
<u>Alternative Solution (1)</u>		
$V = 2h^2$		
$\frac{dV}{dt} = 4h\frac{dh}{dt}$	1A	
$-\frac{1}{10}(4h) = 4h\frac{dh}{dt}$	1M	For substitution in LHS
$\frac{dh}{dt} = -\frac{1}{10}$	1A	
<u>Alternative Solution (2)</u>		
$\frac{dV}{dt} = -\frac{1}{10}A$		
$\frac{dV}{dh}\frac{dh}{dt} = -\frac{1}{10}A$	1M	For chain rule
$A\frac{dh}{dt} = -\frac{1}{10}A$ $\quad \because \frac{dV}{dh} = A$	1A	For $\frac{dV}{dh} = A$
$\frac{dh}{dt} = -\frac{1}{10}$	1A	
(ii) At $t = 0$, $h = 4\cos 30° = 2\sqrt{3}$		
$\therefore$ Time required $= \frac{2\sqrt{3}}{\frac{1}{10}}$	1M	
$= 20\sqrt{3}$	1A	
	7	

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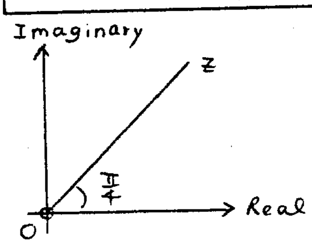
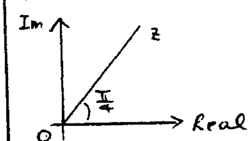
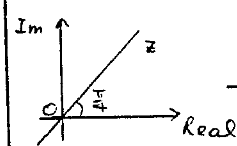
Solution	Marks	Remarks
12. (a) $\arg(1-i) = -\frac{\pi}{4}$ OR $-45°$	1A	Accept $-\frac{\pi}{4}+2k\pi$
$\arg(\frac{z}{1-i}) = \frac{\pi}{2}$		
$\arg z - \arg(1-i) = \frac{\pi}{2}$	2M	
$\arg z - (-\frac{\pi}{4}) = \frac{\pi}{2}$		
$\arg z = \frac{\pi}{4}$ OR $45°$	1A	Accept $\frac{\pi}{4}+2k\pi$
<u>Alternative solution for evaluating arg (z)</u>		
Let $z=x+yi$		
$\frac{z}{1-i} = \frac{x+yi}{1-i}(\frac{1+i}{1+i})$	1M	
$= \frac{(x-y)}{2} + \frac{x+y}{2}i$		
Since $\arg(\frac{z}{1-i}) = \frac{\pi}{2}$, $\therefore \text{Re}(\frac{z}{1-i}) = 0$ and $\text{Im}(\frac{z}{1-i})>0$	1M	
i.e. $x-y=0$; $x+y>0$		
$x=y$; $x+y>0$		
$\therefore \arg z = \tan^{-1}(\frac{y}{x}) = \tan^{-1}(1)$	1A	
$= \frac{\pi}{4}$		
	1A+1A	1A for a straight line through O 1A if argument is also correct (pp−1) if axes are not labelled  —— 2A  —— 1A only
	6	
(b) Let $w=r(\cos\theta+i\sin\theta)$		
$w^3=(\overline{w})^3$		
$r^3(\cos3\theta+i\sin3\theta) = [r(\cos(-\theta)+i\sin(-\theta))]^3$	1A+1M	1A for LHS
$= r^3[\cos(-3\theta)+i\sin(-3\theta)]$		1M for $\overline{w}=r[\cos(-\theta)+i\sin(-\theta)]$ (can be om

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Solution	Marks	Remarks
$= r^3(\cos 3\theta - i\sin 3\theta)$	1A	
$\therefore r^3 \sin 3\theta = 0$		
$\sin 3\theta = 0$ $\quad \because r > 0$	1A	
$3\theta = n\pi$		(can be omitted)
$\theta = \frac{2\pi}{3}$ $\quad \because \frac{\pi}{2} < \theta < \pi$	1A	Accept degrees
$\therefore \arg w = \frac{2\pi}{3}$		
Alternative Solution (1)		
Let $w = r(\cos\theta + i\sin\theta)$		
$w^3 = (\overline{w})^3$		
$w^3 = \overline{w^3}$	1M	
$\therefore$ Imaginary part of $w^3 = 0$	1A	
Since $w^3 = r^3(\cos 3\theta + i\sin 3\theta)$	1A	(can be omitted)
$r^3 \sin 3\theta = 0$		
$\sin 3\theta = 0$	1A	
$\theta = \frac{2\pi}{3}$	1A	
Alternative Solution (2)		
Let $w = x + yi$		
$w^3 = (\overline{w})^3$		
$(x+yi)^3 = (x-yi)^3$	1M	
$x^3 + 3x^2(yi) + 3x(yi)^2 + (yi)^3 = x^3 - 3x^2(yi) + 3x(yi)^2 - (yi)^3$	1A	
$6x^2yi + 2(yi)^3 = 0$		
$3x^2y - y^3 = 0$	1A	
$y(3x^2 - y^2) = 0$		
$y = 0$ or $\frac{y}{x} = \pm\sqrt{3}$		
Since $\frac{\pi}{2} < \arg w < \pi$, $\frac{y}{x} = -\sqrt{3}$	1A	
$y=0$ and $\frac{y}{x} = \sqrt{3}$ are rejected		
$\therefore \arg w = \tan^{-1}(\frac{y}{x})$		
$= \tan^{-1}(-\sqrt{3})$		
$= \frac{2\pi}{3}$	1A	
	5	

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Solution	Marks	Remarks
(c) $z=2\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})$ $w=2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$	1M	Awarded if either one is correct
The complex number represented by Q $=z+w$ $=2\sqrt{2}(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})+2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$	2M	
$=2\sqrt{2}(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i)+2(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)$	1A	(can be omitted)
$=1+(2+\sqrt{3})i$	1A	
	5	

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Solution	Marks	Remarks
13. (a) $\alpha+\beta=\lambda$ and $\alpha\beta=1$	1A	(can be omitted)
$S_2=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	1M	
$=\lambda^2-2$	1A	
$S_3=\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2$	1M	OR $=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$
$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$		$=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$
$=\lambda^3-3\lambda$	1A	
	5	
(b) $\alpha^5-\lambda\alpha^4+\alpha^3$		
$=\alpha^3(\alpha^2-\lambda\alpha+1)$	1A	
$=0$ $\because \alpha$ is a root of $x^2-\lambda x+1=0$	1A	
Alternative Solution		
Since α is a root of $x^2-\lambda x+1=0$		
$\alpha^2-\lambda\alpha+1=0$	1A	
$\alpha^3(\alpha^2-\lambda\alpha+1)=0$		
$\alpha^5-\lambda\alpha^4+\alpha^3=0$ —— (1)	1A	
$S_5-\lambda S_4+S_3=(\alpha^5+\beta^5)-\lambda(\alpha^4+\beta^4)+(\alpha^3+\beta^3)$		
$=(\alpha^5-\lambda\alpha^4+\alpha^3)+(\beta^5-\lambda\beta^4+\beta^3)$		
$=0+\beta^3(\beta^2-\lambda\beta+1)$		
$=0+\beta^3(0)$	1A	For $\beta^2-\lambda\beta+1=0$ or $\beta^5-\lambda\beta^4+\beta^3=0$
$=0$	1	
	4	
(c) Let $S_3=10k, S_4=7\lambda k, S_5=25k$, where $k\neq 0$.	1M	
$S_5-\lambda S_4+S_3=0$		
$25k-\lambda(7\lambda k)+10k=0$		
$7\lambda^2=35$		
$\lambda=\sqrt{5}$ $\because \lambda\geq 2$	1A	

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Solution	Marks	Remarks
Alternative Solution		
Put $S_5=\frac{25}{10}S_3$, $S_4=\frac{7\lambda}{10}S_3$.	1M	OR Putting $S_3=\frac{10}{25}S_5$, $S_4=\frac{7\lambda}{25}S_5$; $S_3=\frac{10}{7\lambda}S_4$, $S_5=\frac{25}{7\lambda}S_4$ etc.
$S_5-\lambda S_4+S_3=0$		
$\frac{25}{10}S_3-\lambda(\frac{7\lambda}{10})S_3+S_3=0$		
$\frac{25}{10}-\frac{7\lambda^2}{10}+1=0$		
$7\lambda^2=35$		
$\lambda=\sqrt{5}$ $\because \lambda\ge 2$	1A	
(i) $S_3=\lambda^3-3\lambda$		
$=(\sqrt{5})^3-3\sqrt{5}$	1M	
$=2\sqrt{5}$	1A	
(ii) Put $\lambda=\sqrt{5}$, $\alpha,\beta=\frac{\sqrt{5}\pm 1}{2}$		
$\therefore (\frac{\sqrt{5}+1}{2})^5+(\frac{\sqrt{5}-1}{2})^5$		
$=S_5$ or $=\alpha^5+\beta^5$	1M	(pp–1) if explanation was not given
$=\frac{25}{10}S_3$	1M	
$=\frac{25}{10}(2\sqrt{5})$		
$=5\sqrt{5}$	1A	
	7	

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