92-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1992

ADDITIONAL MATHEMATICS PAPER I

8.30 am-10.30 am (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions in Section B.

All working must be clearly shown.

Unless otherwise specified in a question, numerical answers must be given in **exact value**.

Section A (42 marks)

Answer **ALL** questions in this section.

1. Given $\vec{OA} = 5\mathbf{i} - \mathbf{j}$, $\vec{OB} = -3\mathbf{i} + 5\mathbf{j}$ and APB is a straight line.

   (a) Find $\vec{AB}$ and $|\vec{AB}|$.

   (b) If $|\vec{AP}| = 4$, find $\vec{AP}$.

   (5 marks)

2. (a) Express the complex number $\frac{\sqrt{3}+i}{\sqrt{3}-i}$ in polar form.

   (b) Using the result of (a), express $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{92}$ in the form $a + bi$.

   (6 marks)

3. Solve $|x(x+5)| > 6$ for real values of x .

   (6 marks)

4.

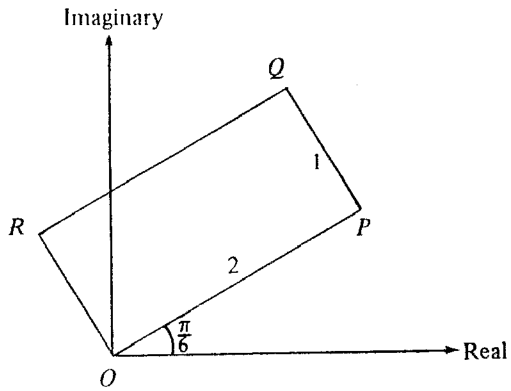

Figure 1

Figure 1 shows an Argand diagram in which $OPQR$ is a rectangle. $OP = 2$, $PQ = 1$ and OP makes an angle $\frac{\pi}{6}$ with the positive real axis. Let z_1, z_2 and z_3 be the complex numbers represented by vertices P, Q and R respectively.

(a) Express z_1 and z_3 in the form $a + bi$.

(b) Using the result of (a), find z_2.

(6 marks)

5. The curve $(x - 2)(y^2 + 3) = -8$ cuts the y-axis at two points. Find

(a) the coordinates of the two points;

(b) the slope of the tangent to the curve at each of the two points.

(6 marks)

6. α, β are the real roots of the equation

$$x^2 + (k - 2)x - (k - 1) = 0.$$

If $|\alpha| = |\beta|$, find k.

(6 marks)

7.

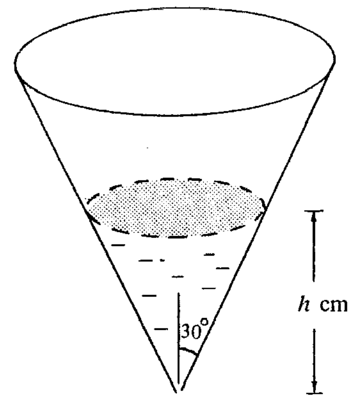

Figure 2

Figure 2 shows a vessel in the shape of a right circular cone with semi-vertical angle 30°. Water is flowing out of the cone through its apex at a constant rate of π cm^{3}s^{-1}.

(a) Let V cm^3 be the volume of water in the vessel when the depth of water is h cm. Express V in terms of h.

(b) How fast is the water level falling when the depth of water is 4 cm ?

(7 marks)

Section B (48 marks)

Answer any **THREE** questions in this section.

Each question carries 16 marks.

8.

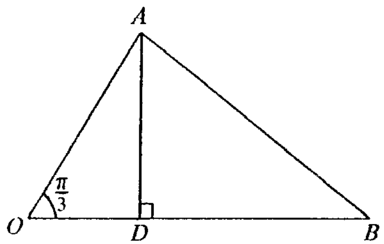

Figure 3

In Figure 3, $OA = 2$, $OB = 3$ and $\angle AOB = \frac{\pi}{3}$. D is a point on OB such that AD is perpendicular to OB. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$. (3 marks)

(b) Find the length of OD.
Hence express $\overrightarrow{OD}$ in terms of $\mathbf{b}$. (2 marks)

(c) Let H be a point on AD such that $AH : HD = 1 : k$ and $\overrightarrow{OH}$ is perpendicular to $\overrightarrow{AB}$.

(i) Express $\overrightarrow{OH}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Hence find the value of k.

(ii) OH produced meets AB at a point C. Let $AC : CB = 1 : m$ and $OH : HC = 1 : n$.

(1) Express $\overrightarrow{OC}$ in terms of m, $\mathbf{a}$, $\mathbf{b}$.

(2) Express $\overrightarrow{OC}$ in terms of n, $\mathbf{a}$, $\mathbf{b}$.

(3) Hence find m and n. (11 marks)

9. α, β are the roots of the quadratic equation

$$x^2 + (p + 1)x + (p - 1) = 0,$$

where p is a real number.

(a) Show that α, β are real and distinct. (3 marks)

(b) Express $(\alpha - 2)(\beta - 2)$ in terms of p. (3 marks)

(c) Given $\beta < 2 < \alpha$.

(i) Using the result of (b), show that $p < -\frac{5}{3}$.

(ii) If $(\alpha - \beta)^2 < 24$, find the range of possible values of p.

Hence write down the possible integral value(s) of p.

(10 marks)

10. (a) Using De Moivre's Theorem, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(3 marks)

(b) Let the complex number $z = \cos\theta + i\sin\theta$.

Using the fact that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \quad \text{and} \quad z^n - \frac{1}{z^n} = 2i\sin n\theta$$

for any positive integer n, show that

$$16\sin^3\theta\cos^2\theta = 2\sin\theta + \sin 3\theta - \sin 5\theta.$$

(5 marks)

(c) Using (a) and (b), express $\sin 5\theta + 9\sin 3\theta$ as a polynomial in $\sin\theta$.

Hence, or otherwise, solve

$$\sin 5\theta + 9\sin 3\theta - 8\sin\theta = 0$$

for $0 \le \theta \le \pi$.

(8 marks)

11.

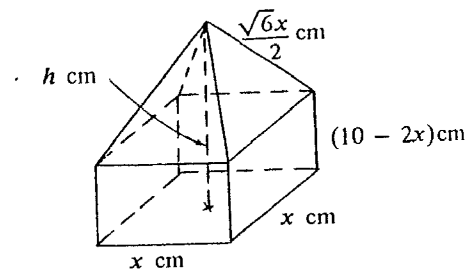

Figure 4 (a)

Figure 4 (a) shows a solid consisting of a right pyramid and a cuboid with a common face which is a square of side x cm. The slant edge of the pyramid is $\frac{\sqrt{6}x}{2}$ cm and the height of the cuboid is $(10 - 2x)$cm, where $0 < x < 5$.

(a) Let h cm be the height of the solid. Show that $h = 10 - x$.

(3 marks)

(b) Let V cm^3 be the volume of the solid.

(i) Show that $V = 10x^2 - \frac{5}{3}x^3$.

(ii) Find the range of values of x for which V is increasing.

Hence write down the range of values of x for which V is decreasing.

(6 marks)

(c)

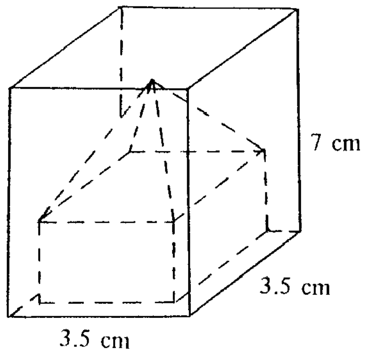

Figure 4 (b)

The solid is placed COMPLETELY inside a rectangular box as shown in Figure 4 (b). The base of the box is a square of side 3.5 cm and the height of the box is 7 cm.

(i) Show that $3 \le x \le 3.5$.

(ii) Hence find, correct to one decimal place, the greatest volume of the solid.

(4 marks)

(d) The side of the square base of the box in (c) is now changed to 4.7 cm and the height 5.5 cm. Find, correct to one decimal place, the greatest volume of the solid that can be placed COMPLETELY inside the box.

(3 marks)

12. (a) C_1 is the curve $y = 2\cos 2x - 4\sin x + 1$, where $0 \le x \le \pi$.

(i) Find the x- and y- intercepts of C_1.

(ii) Find the turning point(s) of C_1.

(iii) Sketch the curve C_1 in Figure 5 (a).

(12 marks)

(b) C_2 is the curve $y = |2\cos 2x - 4\sin x + 1|$, where $0 \le x \le \pi$.

(i) Using the result of (a) (iii), sketch the curve C_2 in Figure 5 (b).

(ii) Hence write down the greatest and least values of $|2\cos 2x - 4\sin x + 1|$ for $0 \le x \le \pi$.

(4 marks)

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12. **(Continued)**

If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet into your answer book.

(a) (iii)

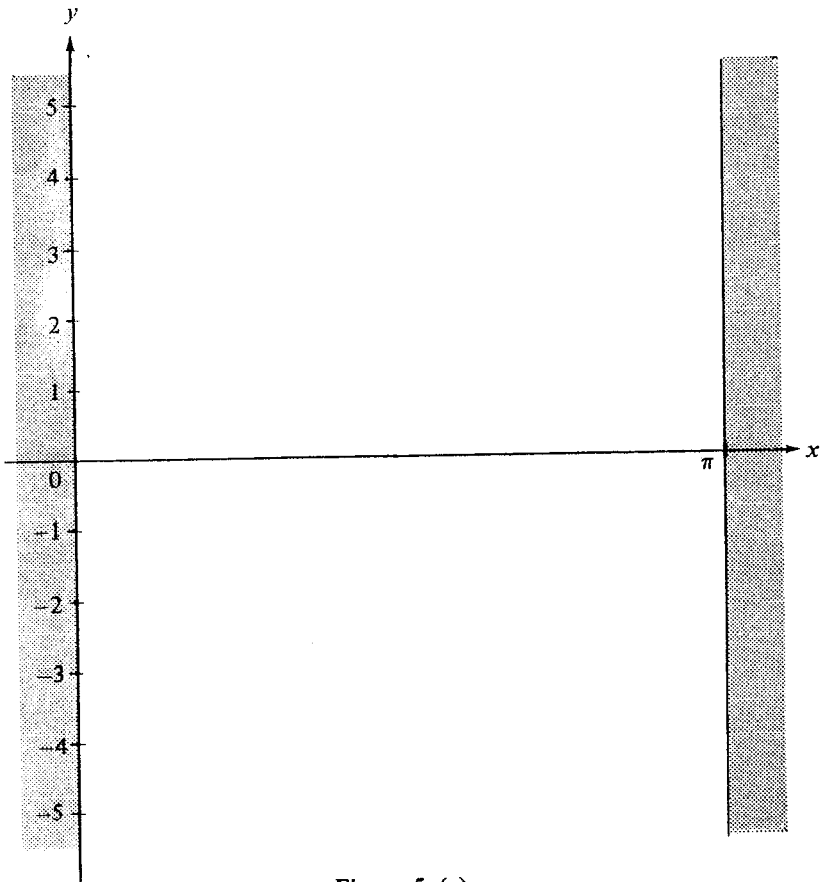

Figure 5 (a)

12. **(Continued)**

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(b) (i)

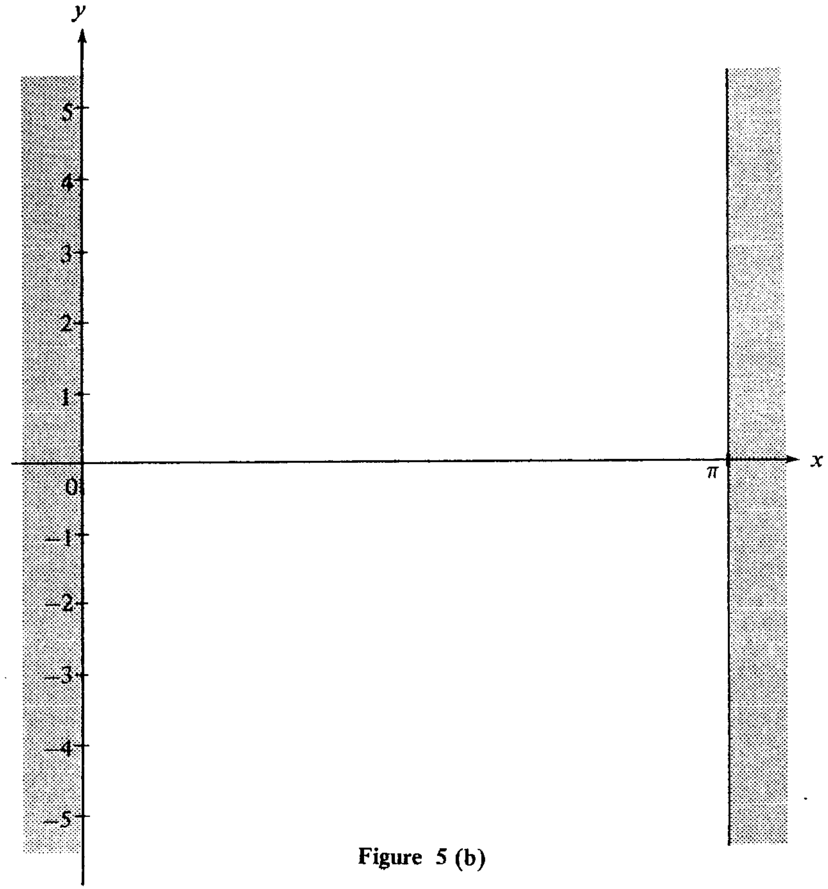

Figure 5 (b)

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