88-CE
A MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1988

附加數學　試卷一

ADDITIONAL MATHEMATICS　PAPER I

8.30 am–10.30 am　(2 hours)

This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (39 marks)

Answer ALL questions in this section.

1. (a) Simplify $(\sqrt{x+1+\Delta x} - \sqrt{x+1})(\sqrt{x+1+\Delta x} + \sqrt{x+1})$.

   (b) Let $y = \sqrt{x+1}$.

   Find $\frac{dy}{dx}$ from first principles.

   (5 marks)

2. Find the equations of the two tangents to the curve $y^2 = x^2y + 2$ at the points where $x = 1$.

   (5 marks)

3.

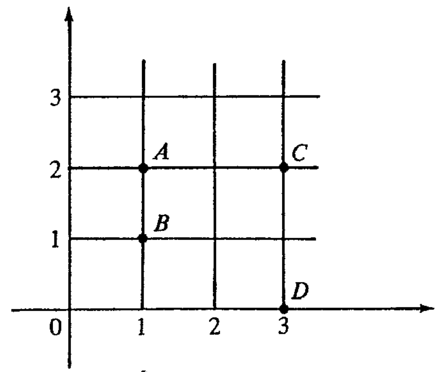

Figure 1

A, B, C and D are four points in the Argand diagram (see Figure 1), and A, B, C represent the three complex numbers z_1, z_2 and z_3 respectively.

(a) Express $\frac{z_1 z_2}{z_3}$ in the form $a + bi$ where a and b are rational.

(b) Using the result of (a), find $\angle AOD + \angle BOD - \angle COD$.

(5 marks)

4. Let $y = \sin x - 2\cos x$ where $0 \leqslant x \leqslant 2\pi$.

Find (a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$,

(b) the minimum value of y .

(6 marks)

5. Let $f(x) = x^2 + 4mx + 4m + 15$, where m is a constant.

Find the discriminant of the equation $f(x) = 0$.

Hence, or otherwise, find the range of values of m so that $f(x) > 0$ for all real values of x .

(5 marks)

6. Let $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 8\mathbf{i} + 6\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} + k\mathbf{j}$ where k is a constant.

Find $\mathbf{a} \cdot \mathbf{c}$ and $\mathbf{b} \cdot \mathbf{c}$ in terms of k .

If $\mathbf{c}$ makes equal angles with $\mathbf{a}$ and $\mathbf{b}$, evaluate k .

(6 marks)

7. Solve the inequality $\frac{|x-3|}{2x} < 1$ by considering each of the following cases:

(i) $x \geqslant 3$,

(ii) $3 > x > 0$,

(iii) $0 > x$.

(7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. The curve $C : y = \frac{x^2 + 4x - 2}{x^2 + 4}$ cuts the line $y = 1$ at P .

(a) Find the coordinates of P .

(1 mark)

(b) For the curve C , find

(i) the x- and y- intercepts;

(ii) the range of values of x for which the slope is negative;

(iii) the turning points; and for each point, state whether it is a maximum point or a minimum point.
(Testing for maximum/minimum is not required.)

(11 marks)

(c) In Figure 2(a), sketch the curve C for $-5 \leqslant x \leqslant 5$.

(4 marks)

(d) (i) Express the equation of the curve C in the form $y = a + \frac{bx + c}{x^2 + 4}$ (a , b , c are constants).

Hence show that if $x > 1\frac{1}{2}$ then $y > 1$.

(ii) In Figure 2(b), sketch the curve C for $x \geqslant 5$.

(4 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

8. **If you attempt Question 8, fill in the details in the first three boxes above and tie this sheet into your answer book.**

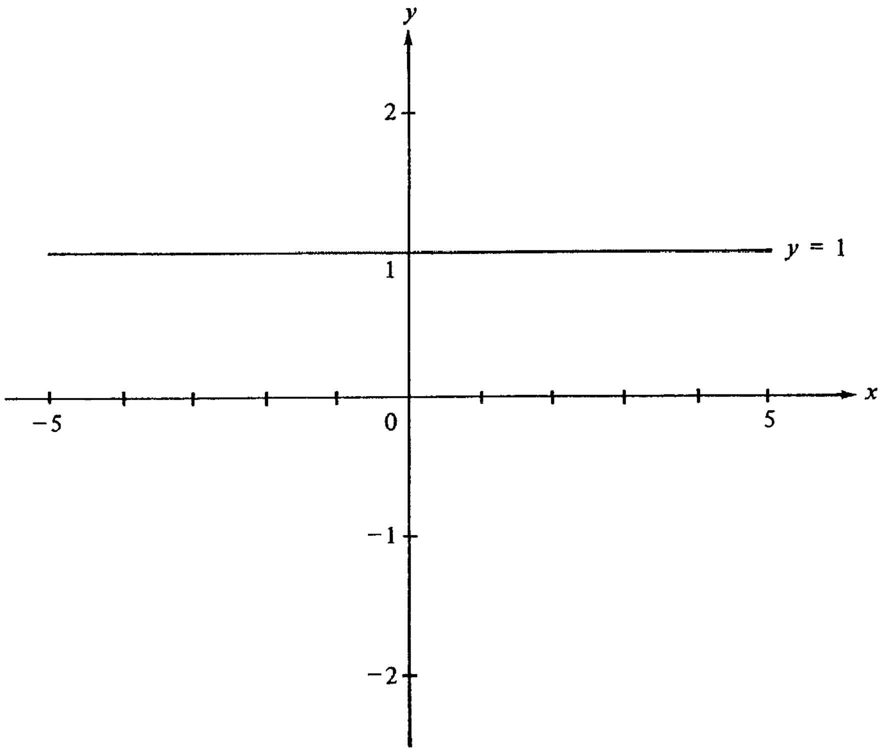

Figure 2(a)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

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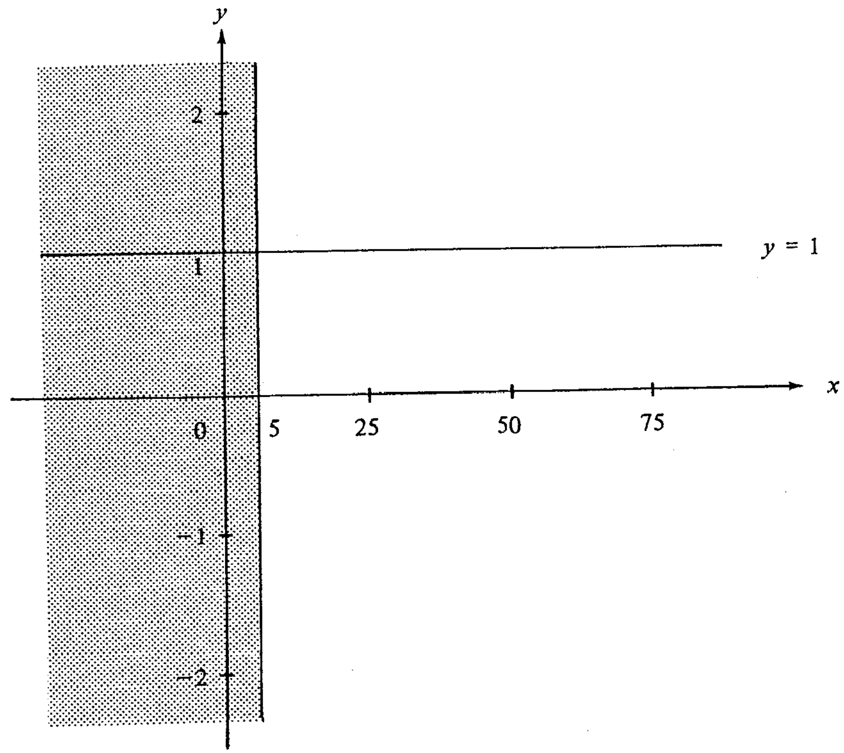

Figure 2(b)

9. A, B and C are three points on a plane such that

$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j}$,

$\overrightarrow{OB} = -6\mathbf{i} - 2\mathbf{j}$

and $\overrightarrow{AB} - \overrightarrow{BC} = -12\mathbf{i} + 6\mathbf{j}$,

where O is the origin.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(5 marks)

(b) X is a point on the plane such that $\overrightarrow{AX} = k\overrightarrow{OX}$.

(i) Express $\overrightarrow{OX}$ in terms of k , $\mathbf{i}$ and $\mathbf{j}$.

(ii) If $OX \perp BX$, find the value of k and hence find $\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX}$.

Furthermore, if M is the mid-point of BC, find $\overrightarrow{AM}$ and hence show that X lies on AM .

(15 marks)

10. Let $f(x) = x^2 + 2x - 1$

and $g(x) = -x^2 + 2kx - k^2 + 6$ (where k is a constant).

(a) Suppose the graph of $y = f(x)$ cuts the x-axis at the points P and Q ; and the graph of $y = g(x)$ cuts the x-axis at the points R and S .

(i) Find the lengths of PQ and RS .

(ii) Find, in terms of k , the x-coordinate of the mid-point of RS .

If the mid-points of PQ and RS coincide with each other, find the value of k .

(9 marks)

(b) If the graphs of $y = f(x)$ and $y = g(x)$ intersect at only one point, find the possible values of k ; and for each value of k , find the point of intersection.

(6 marks)

(c) Find the range of values of k such that $f(x) > g(x)$ for any real value of x .

(5 marks)

11. (a) Let $z = \cos\theta + i\sin\theta$ where $i = \sqrt{-1}$.

(i) Show that, for any positive integer n,

$$z^n + \frac{1}{z^n} = 2\cos n\theta,$$

and $$z^n - \frac{1}{z^n} = 2i\sin n\theta.$$

(ii) Using the results of (i), find all complex numbers z such that

$$\frac{\left(z^2 - \frac{1}{z^2}\right)i}{z^2 + \frac{1}{z^2}} = -\sqrt{3},$$

and represent them in an Argand diagram.

(10 marks)

(b) (i) Solve $x^2 - x + 1 = 0$, giving the roots in polar form.

(ii) Let α and β be the roots of the equation in (b)(i).

Find the quadratic equation whose roots are $\left(\frac{\alpha}{\beta}\right)^k$ and $\left(\frac{\beta}{\alpha}\right)^k$, where k is an integer. Write the answers when

(1) $k = 3n$,

(2) $k = 3n + 1$,

(3) $k = 3n + 2$,

where n is an integer.

(10 marks)

12. (a)

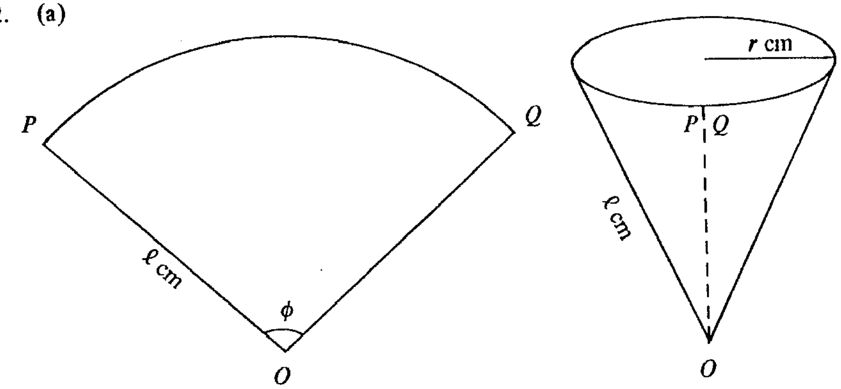

Figure 3(a) Figure 3(b)

Figure 3(a) shows a piece of paper in the shape of a sector of radius ℓ cm and $\angle POQ = \phi$ radians. It is made to form a conical vessel of radius r cm where OP coincides with OQ (see Figure 3(b)).

(i) Express r in terms of ℓ and ϕ.

Let V cm^3 be the capacity of the vessel.

(ii) Show that

$$V^2 = k(4\pi^2\phi^4 - \phi^6),$$

where $k = \dfrac{\ell^6}{576\pi^4}$.

(iii) By finding $\dfrac{d(V^2)}{d\phi}$, determine the value of ϕ for which the capacity of the vessel is a maximum.

[Note: You may use the fact that V is a maximum when V^2 is a maximum.]

(10 marks)

12. (b)

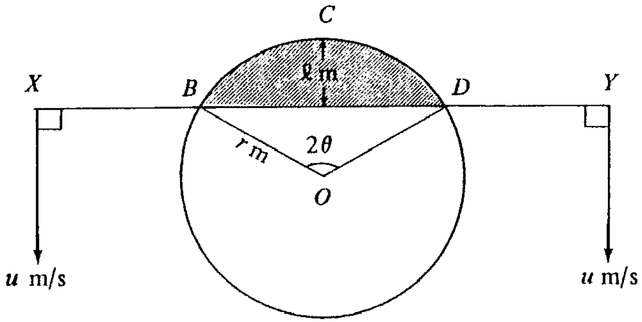

Figure 4

Figure 4 shows a circular pool of radius r metres centred at O. Two men, X and Y, holding the ends of a long rod, are walking in the direction shown at a speed of u metres per second. At a certain instant, the portion BD of the rod subtends an angle of 2θ radians at O and is at a distance ℓ metres from the mid-point C of the rim BD of the pool.

(i) Express ℓ in terms of r and θ.

Let A square metres be the area of the shaded region.

(ii) Express A in terms of r and θ.

(iii) Let $\frac{dA}{dt}$ (in $m^2 s^{-1}$) be the rate of change of the area of the shaded region with respect to time.

Express $\frac{dA}{dt}$ in terms of r, θ and u. (Hint: $\frac{d\ell}{dt} = u$.)

Hence deduce that $\frac{dA}{dt} = ru$ when $\theta = \frac{\pi}{6}$.

(10 marks)

END OF PAPER

88-CE
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PAPER II

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附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER II

11.15 am–1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

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