香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八三年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1983

附加數學 試卷一

ADDITIONAL MATHEMATICS PAPER I

二小時完卷

Two hours

上午八時三十分至十時三十分

8.30 a.m.—10.30 a.m.

本試卷必須用英文作答

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)
Answer ALL questions in this section.

1. Determine the range of values of λ for which the equation

$$x^2 + 4x + 2 + \lambda(2x + 1) = 0$$

has no real roots.

(5 marks)

2. Given that a, b, c are in arithmetic progression and the positive numbers x, y, z are in geometric progression, prove that

$$(b - c)\log x + (c - a)\log y + (a - b)\log z = 0 .$$

(6 marks)

3. Figure 1 shows an isosceles triangle ABC with $BC = 2x$ and $AB = AC$. The perimeter of the triangle is 2 metres. The triangle is revolved about BC so as to form a solid consisting of two cones with a common base of radius AD. Express the volume of this solid in terms of x. Hence find the value of x for which this volume is a maximum.

(6 marks)

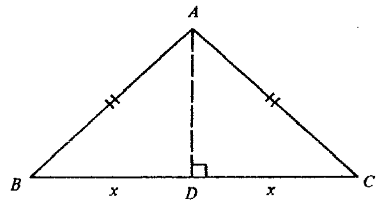

Figure 1

4. Expand $(1 + ax)^4(1 - 4x)^3$ in ascending powers of x up to and including the term containing x^2.

Given that the coefficient of x is zero, evaluate the coefficient of x^2.

(7 marks)

5. Solve the inequality $|x(x - 2)| < 1$. (8 marks)

6. The complex number z satisfies the condition

$$|z - (3 + i)| = |z - (5 + 5i)|.$$

If $z = x + iy$, where x and y are real, find and simplify the relation between x and y.

Find also the values of x and y for which $|z|$ is a minimum.

(8 marks)

SECTION B (60 marks)
Answer any THREE questions from this section.
Each question carries 20 marks.

7. (a) Using De Moivre's theorem, or otherwise, show that

(i) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

(ii) $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$.

(5 marks)

(b) Using (a), or otherwise, show that

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}.$$

(3 marks)

(c) By putting $x = \tan\theta$ and using the result of (b), show that the equation

$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ .... (*)

can be transformed to

$\tan 4\theta = 1$. .... (**)

Find the general solution of equation (**) in terms of π.

Hence deduce the four roots of (*) leaving your answers in terms of π.

(12 marks)

8. Let $f(x) = x^3 + ax^2 + bx - 72$.

(a) Given that $x = 4$ is a double root of $\frac{df}{dx} = 0$, find the values of a and b. (5 marks)

(b) Show that $f(x)$ can be expressed in the form $(x + p)^3 + q$, and find p and q. Hence find the three roots of $f(x) = 0$.

(9 marks)

(c) Represent the three roots x_1, x_2, x_3 of $f(x) = 0$ on an Argand diagram by the points A, B, C, respectively, x_1 and x_2 being complex conjugates and $0 < \arg(x_1) < \pi$.

By considering triangle ABC, or otherwise, determine $\arg\left(\frac{x_2 - 4}{x_1 - 4}\right)$.

(6 marks)

9. (a) Prove, by mathematical induction, that for all positive integers n,

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2).$$

(6 marks)

(b) On a battle field, cannon-balls are stacked as shown in Figure 2. For a stack with n layers, the balls in the bottom layer are arranged as shown in Figure 3 with n balls on each side. For the second bottom layer, the arrangement is similar but each side consists of $(n - 1)$ balls; for the third bottom layer, each side has $(n - 2)$ balls, and so on. The top layer consists of only one ball.

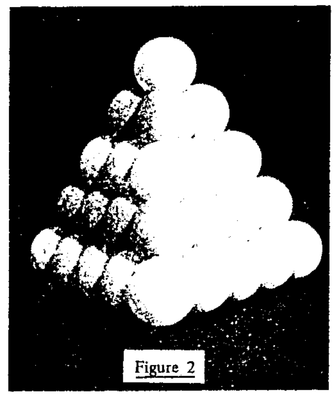

Figure 2

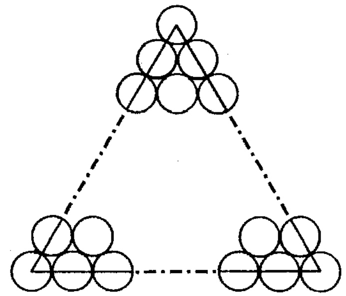

Figure 3

(i) Find the number of balls in the r-th layer counting from the top.

(ii) Using the result of (a), or otherwise, find the total number of cannon-balls in a stack consisting of n layers.

(iii) If the time required to deliver and fire a cannon-ball taken from the r-th layer is $\frac{2}{r}$ minutes, find the time required to deliver and fire all the cannon-balls in the r-th layer.

Hence find the total time needed to use up all the cannon-balls in a stack of 10 layers.

(14 marks)

10. In Figure 4, PQR is an isosceles triangle with base $QR = 2r$. N is the mid-point of QR. L and M are variable points on PQ and PR, respectively, such that $LM \parallel QR$. Let $LM = x$.

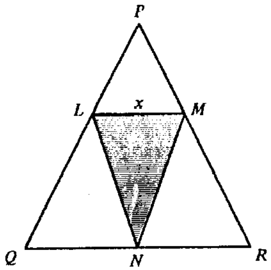

Figure 4

(a) Find x such that the area of ΔLMN is a maximum.

(8 marks)

(b) If the figure is revolved about PN, find x so that the volume of the cone generated by ΔLMN is a maximum.

(6 marks)

(c) Show that the volume of the cone generated by revolving the ΔLMN specified in (a) about PN is only $\frac{27}{32}$ of the volume generated in (b).

(6 marks)

11. Figure 5 shows a rail POQ with $\angle POQ = 120°$. A rod AB of length $\sqrt{7}$ m is free to slide on the rail with its end A on OP and end B on OQ. Let $OA = x$ metres and $OB = y$ metres.

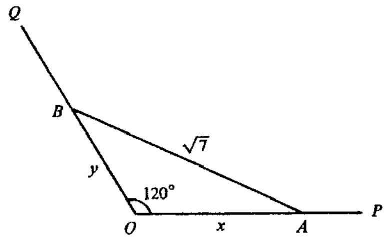

Figure 5

(a) (i) Find a relation between x and y and hence find the value of y when $x = 2$.

(ii) Find $\frac{dy}{dx}$.

Given that x and y are functions of time t (in seconds), show that

$$\frac{dy}{dt} = -\left(\frac{2x + y}{x + 2y}\right)\frac{dx}{dt} .$$

(10 marks)

(b) The end A is pushed towards O with a uniform speed of $\frac{1}{2}$ m/s. When A is at a distance of 2 metres from O, find the speed of the end B.

(4 marks)

(c) Suppose the perpendicular distance from O to the rod is p metres. Show that

$$p = \frac{xy}{2}\sqrt{\frac{3}{7}} .$$

Hence find $\frac{dp}{dt}$ when $x = 2$.

(6 marks)

END OF PAPER

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附加數學 試卷二

ADDITIONAL MATHEMATICS PAPER II

二小時完卷

Two hours

上午十一時十五分至下午一時十五分

11.15 a.m.—1.15 p.m.

本試卷必須用英文作答

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)

Answer ALL questions in this section.

1. A triangle has vertices $P(k, -1)$, $Q(7, 11)$ and $R(1, 3)$. Given that the area of the triangle is 20 units, find the two values of k.

(5 marks)

2. Use the substitution $u = x^2$ to find the indefinite integral

$$\int x \sin^2(x^2)\,dx .$$

(5 marks)

3. Use the substitution $u = 1 + 3x^2$ to evaluate

$$\int_0^1 x^3\sqrt{1 + 3x^2}\,dx .$$

(5 marks)