香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2010年香港高級程度會考
HONG KONG ADVANCED LEVEL EXAMINATION 2010

數學及統計學　高級補充程度

MATHEMATICS AND STATISTICS AS-LEVEL

評卷參考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

'M' marks	–	awarded for applying correct methods
'A' marks	–	awarded for the accuracy of the answers
Marks without 'M' or 'A'	–	awarded for correctly completing a proof or arriving at an answer given in the question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.

8. Marks may be deducted for poor presentation (*pp*). The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*.
   (a) At most deduct 1 mark for *pp* in each section.
   (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

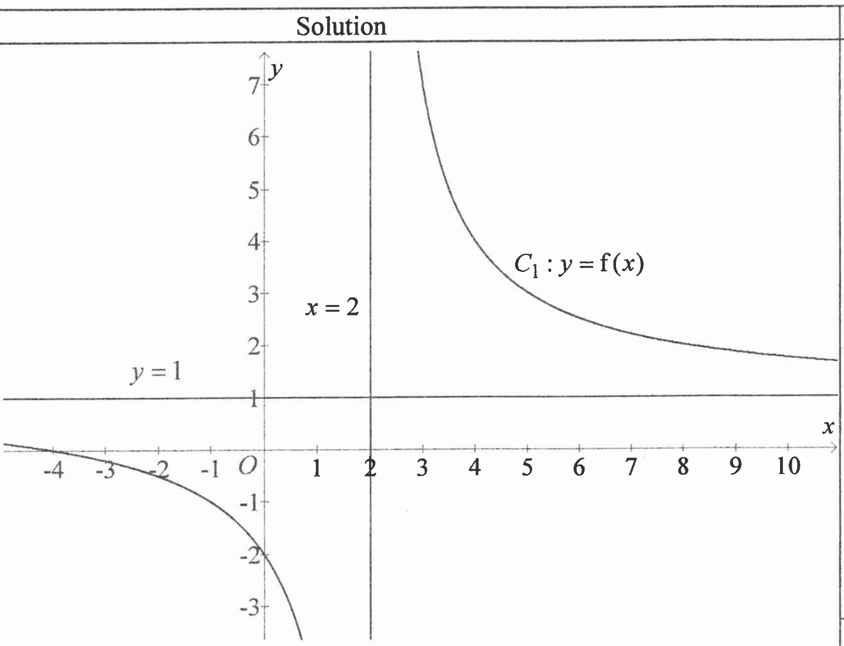
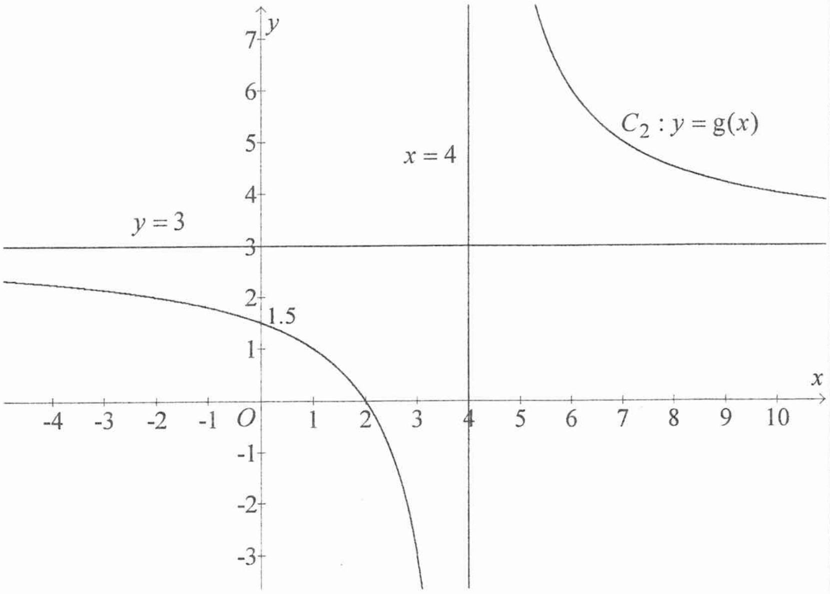
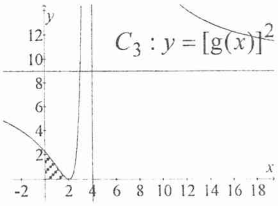
9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a-1*) should be used to denote 1 mark deducted for *a*.
   (a) At most deduct 1 mark for *a* in each section.
   (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

10. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. (a) $\left(1+\frac{x}{a}\right)^r = 1+r\left(\frac{x}{a}\right)+\frac{r(r-1)}{2}\left(\frac{x}{a}\right)^2+\cdots$	1A	pp-1 for omitting "…"
$=1+\frac{rx}{a}+\frac{r(r-1)x^2}{2a^2}+\cdots$		
$\therefore\ \frac{r}{a}=\frac{1}{24}$ and $\frac{r(r-1)}{2a^2}=\frac{-1}{576}$	1M	
Solving, $r=\frac{1}{3}$ and $a=8$.	1A	For both
(b) $(8+x)^{\frac{1}{3}}=8^{\frac{1}{3}}\left(1+\frac{x}{8}\right)^{\frac{1}{3}}$	1M	
$=2\left(1+\frac{x}{24}-\frac{x^2}{576}+\cdots\right)$		
$=2+\frac{x}{12}-\frac{x^2}{288}+\cdots$	1A	
The expansion is valid when $\left\lvert\frac{x}{8}\right\rvert<1$.		
i.e. $-8<x<8$	1A	OR $\lvert x\rvert<8$
	(6)	
2. (a) $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x \approx \frac{0.5}{2}(1+e^2+2e)$	1M	
$=\frac{(e+1)^2}{4}$	1A	OR $\frac{e^2+2e+1}{4}$ OR 3.4564
(b) $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x=\left[\frac{e^{2x}}{2}\right]_0^1$		
$=\frac{e^2-1}{2}$	1A	
(c) (i) $A=\frac{(1+e^{2h})h}{2}+\frac{(e^{2h}+e^2)(1-h)}{2}$		
$=\frac{e^{2h}+(1-e^2)h+e^2}{2}$	1	Follow through
(ii) $\frac{\mathrm{d}A}{\mathrm{d}h}=\frac{2e^{2h}+1-e^2}{2}$	1A	
$\frac{\mathrm{d}A}{\mathrm{d}h}=0$ when $h=\frac{1}{2}\ln\frac{e^2-1}{2}$	1A	OR 0.5807
$\frac{\mathrm{d}^2A}{\mathrm{d}h^2}=2e^{2h}>0$	1M	OR by using sign test
Hence A is minimum when $h=\frac{1}{2}\ln\frac{e^2-1}{2}$.		
The minimum value of A is $\frac{3e^2-1}{4}+\frac{1-e^2}{4}\ln\frac{e^2-1}{2}$.	1A	OR 3.4367
	(8)	

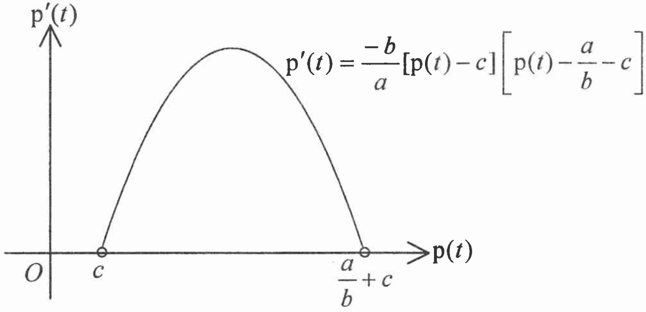
Solution	Marks	Remarks
3. (a) $\frac{dA}{dt} = -kA$		
$\therefore \frac{dt}{dA} = \frac{-1}{kA}$	1A	
$t = \frac{-1}{k}\int\frac{dA}{A}$		
$= \frac{-1}{k}\ln\|A\| + C$	1A	
When $t = 0$, $A = A_0$ and when $t = 5730$, $A = \frac{A_0}{2}$.		
$0 = \frac{-1}{k}\ln A_0 + C$ and $5730 = \frac{-1}{k}\ln\frac{A_0}{2} + C$	1M	
$\therefore 5730 = \frac{-1}{k}\ln\frac{A_0}{2} + \frac{1}{k}\ln A_0$		
$= \frac{1}{k}\ln 2$		
i.e. $k = \frac{\ln 2}{5730}$		
$\approx 1.21 \times 10^{-4}$ (correct to 3 significant figures)	1A	
(b) $A = 0.3A_0$		
$\therefore t = \frac{-5730}{\ln 2}\ln(0.3A_0) + \frac{5730}{\ln 2}\ln A_0$	1M	
$= \frac{5730}{\ln 2}\ln\frac{10}{3}$		
≈ 9950 years (correct to the nearest ten years)	1A	
	(6)	
4. (a) $\because$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
$\therefore 1 = P(A) + b - c$	1A	For $P(A \cup B) = 1$
i.e. $P(A) = 1 + c - b$	1A	
(b) (i) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$		
$\frac{1}{2} = \frac{c}{b}$	1M	For conditional probability
$b = 2c$ -- (1)	1A	
$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$		
$\frac{2}{3} = \frac{c}{1 + c - b}$		For either one
$c = 2 - 2b$ -- (2)		
Solving (1) and (2), we have $b = 0.8$ and $c = 0.4$.	1A+1A	
(ii) $P(A)P(B) = (0.6)(0.8) \neq 0.4 = P(A \cap B)$		
Alternative Solution 1 $P(A \mid B) = 0.5 \neq 0.6 = P(A)$		
Alternative Solution 2 $P(B \mid A) = \frac{2}{3} \neq 0.8 = P(B)$		
Hence the events A and B are not independent.	1	Follow through
	(7)	

Solution	Marks	Remarks
5. (a) $a=2, b=6$ and $\bar{x}=62$	1A+1A+1A	
(b) (i) The required probability		
$=\frac{15}{20}\times\frac{14}{19}\times\frac{13}{18}\times\frac{12}{17}\times\frac{11}{16}\times\frac{10}{15}$		OR $\frac{{}_{15}C_6}{{}_{20}C_6}$
$=\frac{1001}{7752}$	1A	OR 0.1291
(ii) The required probability		
$=1-\frac{{}_5C_0\times{}_{15}C_6+{}_5C_1\times{}_{15}C_5}{{}_{20}C_6}$	1M	
$=\frac{937}{1938}$	1A	OR 0.4835
	(6)	
6. (a) $\mathrm{P}\left(Z>\frac{152-\mu}{5}\right)=0.117$	1A	
$\mathrm{P}\left(0<Z<\frac{152-\mu}{5}\right)=0.383$	1M	
$\frac{152-\mu}{5}\approx 1.19$		
$\mu\approx 146.05$	1A	
(b) (i) The required probability		
$=0.117\times(1-0.2)+(1-0.117)\times 0.1$		
$=0.1819$	1A	
(ii) The required probability		
$=\frac{(1-0.117)\times(1-0.1)}{1-0.1819}$	1M+1M	
$=\frac{883}{909}$	1A	OR 0.9714
	(7)	
7. (a) (i) $a=-2$	1A	
(ii) The equation of the horizontal asymptote is $y=1$.	1A	
(iii) $\mathrm{f}'(x)=\frac{(x-2)-(x+4)}{(x-2)^2}$	1M	
$=\frac{-6}{(x-2)^2}$	1A	
<0 for $x\neq 2$		
Hence $\mathrm{f}(x)$ is decreasing on $(-\infty, 2)$ or $(2,\infty)$.	1A	Accept '... for all $x \boxed{\neq 2}$.'

Solution	Marks	Remarks
(iv) 	1A 1A 1A (8)	For both asymptotes For both intercepts of C_1 For the shape of C_1
(b) (i) 	1A 1A 1A	For both asymptotes For both intercepts of C_2 For the shape of C_2
(ii) $g(x)=\frac{(x-2)+4}{(x-2)-2}+2$ $=\frac{3x-6}{x-4}$	1A	May be awarded in (b)(i)
Hence the required area $=\int_0^2\left(3\cdot\frac{x-2}{x-4}\right)^2\mathrm{d}x$	1A	For lower and upper limits
$=9\int_{-4}^{-2}\left(\frac{u+2}{u}\right)^2\mathrm{d}u$ where $u=x-4$	1M	For substitution 
$=9\int_{-4}^{-2}\left(1+\frac{4}{u}+\frac{4}{u^2}\right)\mathrm{d}u$		
$=9\left[u+4\ln\lvert u\rvert-\frac{4}{u}\right]_{-4}^{-2}$		
$=27-36\ln 2$	1A	OR 2.0467
	(7)	

Solution	Marks	Remarks
8. (a) $f'(t) = -500ae^{2at} + 300ae^{at}$	1A	
Since $f(t)$ attains maximum when $t = 5$, $f'(5) = 0$	1M	
$-500ae^{10a} + 300ae^{5a} = 0$		
$a = 0.2\ln 0.6$	1A	OR -0.1022
	(3)	
(b) (i) $-250e^{0.4T_1 \ln 0.6} + 300e^{0.2T_1 \ln 0.6} - 50 = 0$	1M	
$e^{0.2T_1 \ln 0.6} = 0.2$ or 1 (rejected as $T_1 > 0$)		
$T_1 = \dfrac{5\ln 0.2}{\ln 0.6}$	1A	OR 15.7533
(ii) The total amount of sales increased		
$= \int_0^{T_1} (-250e^{2at} + 300e^{at} - 50)\,dt$	1M	
$= \left[\dfrac{-125e^{0.4t\ln 0.6}}{0.2\ln 0.6} + \dfrac{300e^{0.2t\ln 0.6}}{0.2\ln 0.6} - 50t \right]_0^{\frac{5\ln 0.2}{\ln 0.6}}$	1A	
$= \dfrac{-125}{0.2\ln 0.6}(0.2^2 - 1) + \dfrac{300}{0.2\ln 0.6}(0.2 - 1) - 50\left(\dfrac{5\ln 0.2}{\ln 0.6}\right)$		
$= \dfrac{-600 + 250\ln 5}{\ln 0.6}$ thousand dollars	1A	OR 386.9041 thousand dollars OR \$ 386904.0876
	(5)	
(c) (i) $E = 100 + \int \dfrac{100}{t+9}\,dt$		
$= 100 + 100\ln(t+9) + C$	1A	
When $t = 0, E = 100$.		
$100 = 100 + 100\ln 9 + C$	1M	
$C = -100\ln 9$		
$\therefore \quad E = 100[\ln(t+9) + 1 - \ln 9]$	1A	
(ii) $200 = 100\ln(t+9) + 100 - 100\ln 9$		
$T_2 = 9(e-1)$	1A	OR 15.4645
(iii) Total sales increased		
$= \int_\alpha^{2\alpha} -(t-\alpha)(t-2\alpha)\,dt$	1M	
$= \int_\alpha^{2\alpha} (-t^2 + 3\alpha t - 2\alpha^2)\,dt$		
$= \left[\dfrac{-t^3}{3} + \dfrac{3\alpha t^2}{2} - 2\alpha^2 t \right]_\alpha^{2\alpha}$		
$= \dfrac{\alpha^3}{6}$	1A	
Hence the maximum total increase of sales can be achieved when		
$\alpha = T_2$	1M	
$= 9(e-1)$		
Hence the plan should be started $9(e-1)$ months after the launching of the campaign.		
	(7)	

Solution	Marks	Remarks
9. (a) (i) $p(t)=\dfrac{a}{b+e^{-t}}+c$		
$p'(t)=\dfrac{ae^{-t}}{(b+e^{-t})^2}$	1A	
$p''(t)=\dfrac{(b+e^{-t})^2(-ae^{-t})-(ae^{-t})2(b+e^{-t})(-e^{-t})}{(b+e^{-t})^4}$		
$=\dfrac{ae^{-t}(e^{-t}-b)}{(b+e^{-t})^3}$	1A	
Hence $p''(t)=0$ when $e^{-t}-b=0$. i.e. $t=-\ln b$		
$\begin{array}{cccc} \hline t & t<-\ln b & t=-\ln b & t>-\ln b \\ \hline p''(t) & + & 0 & - \\ \hline \end{array}$		
Hence the growth rate attains the maximum value when $t=-\ln b$	1A	Follow through
(ii) *primordial population* $=\lim\limits_{t\to-\infty}\left(\dfrac{a}{b+e^{-t}}+c\right)=c$	1A	
(iii) *ultimate population* $=\lim\limits_{t\to\infty}\left(\dfrac{a}{b+e^{-t}}+c\right)=\dfrac{a}{b}+c$	1A	
	(5)	
(b) $\ln[p(t)-c]=-\ln(b+e^{-t})+\ln a$		
$\therefore\ \ln a=\ln 8000$ $a=8000$	1A	
$\therefore\ p'(0)=\dfrac{8000}{(b+1)^2}=2000$ $b=1\ \boxed{\text{or } -3\ \text{(rejected)}}$	1A	
$\therefore\ p(0)=\dfrac{8000}{1+1}+c=6000$ $c=2000$	1A	
	(3)	
(c) The population at the time of maximum growth rate is $p(-\ln b)=\dfrac{a}{2b}+c$	1A	
The mean of the *primordial population* and *ultimate population* is $\dfrac{1}{2}\left[c+\left(\dfrac{a}{b}+c\right)\right]=\dfrac{a}{2b}+c$ Hence the scientist's claim is agreed.	1	
	(2)	

Solution	Marks	Remarks
(d) $\mathrm{p}(t)=\dfrac{a}{b+e^{-t}}+c$		
$e^{-t}=\dfrac{a}{\mathrm{p}(t)-c}-b$	1A	
$\therefore\ \mathrm{p}'(t)=\dfrac{a\left[\dfrac{a}{\mathrm{p}(t)-c}-b\right]}{\left[b+\left(\dfrac{a}{\mathrm{p}(t)-c}-b\right)\right]^2}$	1M	
$=\dfrac{a\,[\mathrm{p}(t)-c]\,\{a-b[\mathrm{p}(t)-c]\}}{a^2}$		
$=\dfrac{-b}{a}[\mathrm{p}(t)-c]\left[\mathrm{p}(t)-\dfrac{a}{b}-c\right]$		
Hence $\alpha=c$ and $\beta=\dfrac{a}{b}+c$.	1A	
	1A	
From the graph, we can see that $\mathrm{p}'(t)$ is maximum when $\mathrm{p}(t)$ is the mean of c and $\dfrac{a}{b}+c$, i.e. the mean of the *primordial population* and *ultimate population*.	1	Follow through
	(5)	

Solution	Marks	Remarks
10. (a) (i) The sample mean is 1.75 .	1A	
(ii) $\frac{e^{-\lambda_2}{\lambda_2}^{1}}{1!} \approx \frac{12.68}{52}$ ------------------------------------- (1)	1M	OR by using other pairs of data
$\frac{e^{-\lambda_2}{\lambda_2}^{2}}{2!} \approx \frac{13.94}{52}$ ------------------------------------- (2)		
$(2) \div (1):\ \frac{\lambda_2}{2} \approx \frac{13.94}{12.68}$		
$\lambda_2 \approx 2.20$ (correct to 2 decimal places)	1A	
(iii) $a = 9.04, b = 15.81, c = 13.84, d = 3.53$ and $k = 5.76$	1A+1A	1A for any one correct 1A for all others correct
(iv) For the number of wedding banquets per week > 4, the expected frequency by Po(1.75) is $52 - 9.04 - 15.81 - 13.84 - 8.07 - 3.53 = 1.71$	1M	For either one
For the number of wedding banquets per week > 4, the expected frequency by Po(2.20) is $52 - 5.76 - 12.68 - 13.94 - 10.23 - 5.62 = 3.77$		
The sum of errors for model fitted by Po(1.75) is $E_1 = \lvert 7-9.04\rvert + \lvert 18-15.81\rvert + \lvert 12-13.84\rvert + \lvert 11-8.07\rvert + \lvert 4-3.53\rvert + \lvert 0-1.71\rvert$ $= 11.18$		For both
The sum of errors for model fitted by Po(2.20) is $E_2 = \lvert 7-5.76\rvert + \lvert 18-12.68\rvert + \lvert 12-13.94\rvert + \lvert 11-10.23\rvert + \lvert 4-5.62\rvert + \lvert 0-3.77\rvert$ $= 14.66$	1A	
Since $E_1 < E_2$, Po(1.75) fits the observed data better.	1A	Follow through
	(8)	
(b) (i) P(expense between \$6188 and \$8888) $= \mathrm{P}\left(\frac{6188-7388}{1200} < Z < \frac{8888-7388}{1200}\right)$	1A	
$= \mathrm{P}(-1 < Z < 1.25)$ $\approx 0.3413 + 0.3944$ $= 0.7357$	1A	
(ii) P(at most 3 banquets in a certain week) $= \frac{e^{-1.75}(1.75)^0}{0!} + \frac{e^{-1.75}(1.75)^1}{1!} + \frac{e^{-1.75}(1.75)^2}{2!} + \frac{e^{-1.75}(1.75)^3}{3!}$	1M	
≈ 0.89918965	1A	
P(1 banquet between \$6188 and \$8888 | at most 3 banquets) $\approx \frac{0 + \frac{e^{-1.75}(1.75)^1}{1!}(0.7357) + \frac{e^{-1.75}(1.75)^2}{2!} \cdot C_1^2(1-0.7357)(0.7357) + \frac{e^{-1.75}(1.75)^3}{3!} \cdot C_1^3(1-0.7357)^2(0.7357)}{0.89918965}$	1M+1M	1M for joint prob in numerator 1M for denominator using above
≈ 0.3905	1A	
	(7)	

Solution	Marks	Remarks
11. (a) (i) P(getting 3 points $\mid$ Gold VIP)		
$=2(0.4)(0.3)$		
$=0.24$	1A	
(ii) P(getting 3 points)		
$=(0.25)(0.2)+(0.6)(0.24)+(0.15)(0.4)^3$	1M	
$=0.2036$	1A	
(iii) P(Gold VIP $\mid$ 3 points are obtained)		
$=\dfrac{(0.6)(0.24)}{0.2036}$	1M	
≈ 0.7073	1A	
	(5)	
(b) P($ 20 cash rebate)		
$=(0.25)(0.4)+[(0.25)(0.3)+(0.6)(0.4)^2]+0.2036$	1M	
$=0.4746$	1A	
	(2)	
(c) (i) P(getting 10 points)		
$=(0.15)[3(0.3)(0.1)^2+3(0.2)^2(0.1)]$		
$=0.00315$	1A	
(ii) P($ 200 cash rebate)		
$=0.00315+(0.15)[3(0.2)(0.1)^2+(0.1)^3]$	1M	
$=0.0042$	1A	
	(3)	
(d) (i) Expected cash rebate using the online game		
$=\$\{0.7[20(0.4746)+50(1-0.4746-0.0042)+200(0.0042)]+(1-0.7)(0)\}$	1M	
$=\$25.4744$		
The minimum cash rebate under the 4% direct cash rebate plan		
$>\$[(0.25)(400)+(0.6)(800)+(0.15)(1000)](0.04)$	1M	
$=\$29.2$		
Since $29.2>25.4744$, Winnie is agreed with.	1	Follow through
(ii) The maximum cash rebate under the 2% direct cash rebate plan		
$=\$[(0.25)(800)+(0.6)(1000)+(0.15)(3000)](0.02)$	1M	
$=\$25$		
Since $25<25.4744$, John is agreed with.	1	Follow through
	(5)	

Solution	Marks	Remarks
12. (a) P(a tablet is contaminated)		
$=1-(1-0.6\%)(1-0.6\%)(1-0.1\%)$	1M+1M	
≈ 0.012952036		
≈ 0.0130	1A	
	(3)	
(b) P(a bag is *unsafe*)		
$=1-(1-0.012952036)^{20}-20(1-0.012952036)^{19}(0.012952036)$	1M	
≈ 0.027306899		
≈ 0.0273	1A	
	(2)	
(c) (i) P(the 10th bag is the first *unsafe* bag)		
$\approx (1-0.027306899)^{10-1}(0.027306899)$	1M	
≈ 0.0213	1A	
(ii) P(the supply will be suspended in a certain week)		
$\approx 1-(1-0.027306899)^{100}-C_1^{100}(1-0.027306899)^{99}(0.027306899)$		
$-C_2^{100}(1-0.027306899)^{98}(0.027306899)^2$		
$-C_3^{100}(1-0.027306899)^{97}(0.027306899)^3-C_4^{100}(1-0.027306899)^{96}(0.027306899)^4$	1M+1A	
≈ 0.1390	1A	
	(5)	
(d) (i) P(the ingredient A is contaminated)		
$=\dfrac{0.006+0.004n}{n+1}$	1M	
(ii) P(the ingredient B is contaminated) $=\dfrac{0.006+0.004n}{n+1}$		
$\therefore\ 1-\left(1-\dfrac{0.006+0.004n}{n+1}\right)\left(1-\dfrac{0.006+0.004n}{n+1}\right)(1-0.001)<0.01$	1A	
$\left(1-\dfrac{0.006+0.004n}{n+1}\right)^2>\dfrac{110}{111}$		
$1-\dfrac{0.006+0.004n}{n+1}>\sqrt{\dfrac{110}{111}}$ or $1-\dfrac{0.006+0.004n}{n+1}<-\sqrt{\dfrac{110}{111}}$ (rejected)	1M	
$n>2.885790831$	1A	OR $n>2.8858$
Hence the least number of n is 3 .	1A	
	(5)	