

2009 ASL Mathematics & Statistics 數學及統計學

評卷參考 Marking Scheme (此部分只設英文版本)

AS Mathematics and Statistics

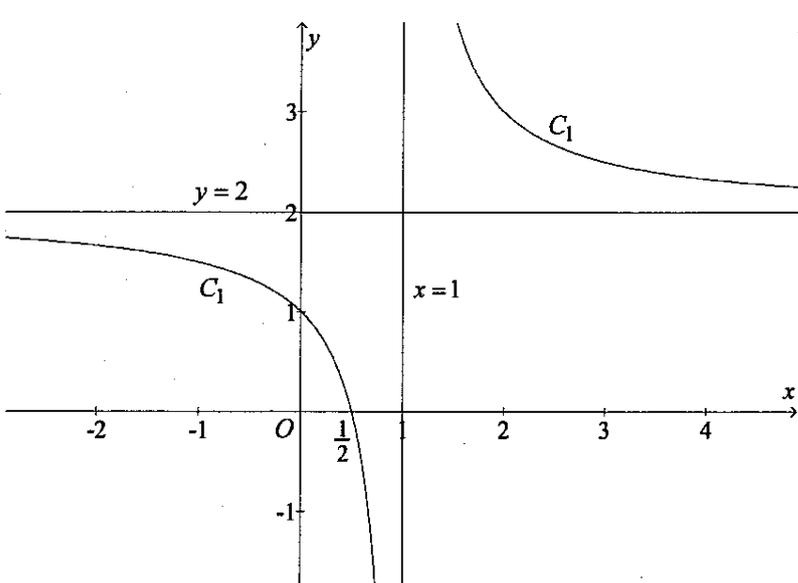
General Instructions To Markers

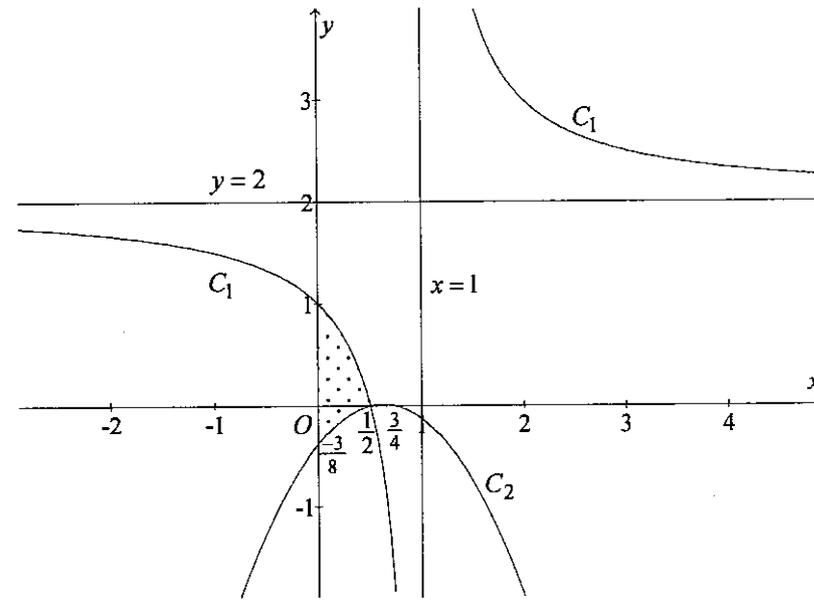
1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.
3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.
5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.
6. In the marking scheme, marks are classified into the following three categories:
 - 'M' marks – awarded for applying correct methods
 - 'A' marks – awarded for the accuracy of the answers
 - Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.
7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.
8. Marks may be deducted for poor presentation (*pp*). The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*.
 - (a) At most deduct 1 mark for *pp* in each section.
 - (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.
9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol $\textcircled{a-1}$ should be used to denote 1 mark deducted for *a*.
 - (a) At most deduct 1 mark for *a* in each section.
 - (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.
10. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
<p>1. (a) $\frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$</p> $= 1 - \left(\frac{-1}{2}\right)x + \frac{1}{2!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)x^2 - \frac{1}{3!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)x^3 + \dots$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \dots$ <p>(b) $\therefore \frac{1}{\sqrt{1-\frac{1}{5}}} = 1 + \frac{1}{2}\left(\frac{1}{5}\right) + \frac{3}{8}\left(\frac{1}{5}\right)^2 + \frac{5}{16}\left(\frac{1}{5}\right)^3 + \dots$</p> $\frac{\sqrt{5}}{2} \approx \frac{447}{400}$ $\sqrt{5} \approx \frac{447}{200}$ <p>(c) The expansion in (a) is valid only when $x < 1$. So Josephine cannot put $x = -4$ into the expansion and her claim is incorrect.</p>	<p>IM</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>I</p> <p>(6)</p>	<p>For any two terms correct</p> <p>(pp-1) if ... was omitted</p> <p>For L.H.S.</p> <p>OR $2 \frac{47}{200}$ OR 2.235</p>
<p>2. (a) $\frac{dP}{dt} = \frac{-0.09}{\sqrt{3t+1}}$</p> $P = \int \frac{-0.09}{\sqrt{3t+1}} dt$ $= -0.06\sqrt{3t+1} + C$ <p>When $t = 0$, $P = 1$.</p> $\therefore 1 = -0.06\sqrt{1} + C$ $C = 1.06$ <p>i.e. $P = -0.06\sqrt{3t+1} + 1.06$</p> <p>(b) When $t = 5$, $P = -0.06\sqrt{3(5)+1} + 1.06 = 0.82$ Thus, 18% of the population has died off.</p> <p>(c) When $P = 0$, $0 = -0.06\sqrt{3T+1} + 1.06$</p> $T = 103 \frac{19}{27}$	<p>IM+1A</p> <p>1A</p> <p>IM</p> <p>1A</p> <p>1A</p> <p>(6)</p>	<p>Withhold 1A if C was omitted</p> <p>OR 103.7037</p>
<p>3. (a) $x = y^4 - y$</p> $\frac{dx}{dy} = 4y^3 - 1$ $\therefore \frac{dy}{dx} = \frac{1}{4y^3 - 1}$ <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><u>Alternative Solution</u></p> $1 = 4y^3 \frac{dy}{dx} - \frac{dy}{dx}$ $\therefore \frac{dy}{dx} = \frac{1}{4y^3 - 1}$ </div>	<p>IM</p> <p>IM+1A</p> <p>IM+IM</p> <p>1A</p>	<p>For finding $\frac{dx}{dy}$</p> <p>IM for $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$</p> <p>IM for finding $\frac{dy}{dx}$</p> <p>IM for chain rule</p>

Solution	Marks	Remarks
(b) $\therefore \frac{1}{4y^3 - 1} = \frac{1}{3}$ $y = 1$ $\therefore x = 1^4 - 1 = 0$ Hence the required equation of the tangent is $y - 1 = \frac{1}{3}(x - 0)$ i.e. $x - 3y + 3 = 0$	1M 1A 1M 1A (7)	
4. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\therefore \frac{5}{12} = a + \frac{1}{4} - P(A \cap B)$ i.e. $P(A \cap B) = a - \frac{1}{6}$ (b) $P(B')P(A B') = P(A \cap B')$ $[1 - P(B)] \cdot P(A B') = P(A) - P(A \cap B)$ $\therefore \left(1 - \frac{1}{4}\right)k = a - \left(a - \frac{1}{6}\right)$ i.e. $k = \frac{2}{9}$ (c) Since A and B are independent, $P(A \cap B) = P(A) \times P(B)$. $\therefore a - \frac{1}{6} = a \times \frac{1}{4}$ $a = \frac{2}{9}$	1A 1A M+1M+1M 1A 1M 1A	OR ... = $P(A \cup B) - P(B)$ OR ... = $\frac{5}{12} - \frac{1}{4}$
<u>Alternative Solution</u> Since A and B are independent, $P(A B') = P(A)$. $\therefore a = k$ $= \frac{2}{9}$	1M 1A	
	(8)	
5. (a) The required probability $= 1 - (0.64)^{15} - C_1^{15}(0.36)(0.64)^{14} - C_2^{15}(0.36)^2(0.64)^{13} - C_3^{15}(0.36)^3(0.64)^{12}$ ≈ 0.8469 (b) The required probability $= \frac{3 \times C_1^5(0.36)(0.64)^4 \times C_1^5(0.36)(0.64)^4 \times C_2^5(0.36)^2(0.64)^3}{C_4^{15}(0.36)^4(0.64)^{11}}$ $= \frac{50}{91}$	1M+1A 1A 1M+1A 1A	OR = $\frac{C_1^3 \cdot \frac{4!}{2!1!1!} \cdot \frac{11!}{3!4!4!}}{\frac{15!}{5!5!5!}}$ OR 0.5495
	(6)	

Solution	Marks	Remarks
<p>6. (a) Let a_i, b_i be the salaries in groups A and B respectively and \bar{x} be the common mean. Let $\sigma_A, \sigma_B, \sigma_{\text{total}}$ be the s.d. of the two groups and the pooled group respectively.</p> $\sigma_A^2 = \frac{\sum_{i=1}^{11} (a_i - \bar{x})^2}{11} \quad \text{and} \quad \sigma_B^2 = \frac{\sum_{i=1}^7 (b_i - \bar{x})^2}{7}$ $\sigma_{\text{total}}^2 = \frac{\sum_{i=1}^{11} (a_i - \bar{x})^2 + \sum_{i=1}^7 (b_i - \bar{x})^2}{11+7}$ $\therefore \sigma_{\text{total}} = \sqrt{\frac{11(2.5)^2 + 7(2.8)^2}{18}}$ $\approx 2.6208 \text{ thousand dollars}$ <p>(b) (i) The salary of the manager with the second highest salary in group A is <u>at least \$40000</u> and that in group B is exactly <u>\$40000</u>. Hence, the salary of the manager with the second highest salary in group A is <u>not lower than</u> that in group B.</p> <p>(ii) The upper quartile will be changed from the 6th observation to the 5th one and so it will be <u>less than or equal to</u> the original one. The lower quartile will be <u>unchanged</u> (the 2nd observation). Hence, the inter-quartile range will be <u>unchanged or decreased</u>.</p>	<p>IM</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>(7)</p>	<p>OR $\sigma_A^2 = \frac{\sum_{i=1}^{11} a_i^2}{11} - \bar{x}^2, \dots$</p> <p>OR $\sigma_{\text{total}}^2 = \frac{\sum_{i=1}^{11} a_i^2 + \sum_{i=1}^7 b_i^2}{11+7} - \bar{x}^2$</p> <p>OR \$2620.7505</p> <p>For either salary</p> <p>Follow through</p> <p>Before: <input type="radio"/> <input checked="" type="radio"/> <input type="radio"/> <input type="radio"/> <input type="radio"/> <input checked="" type="radio"/> <input type="radio"/></p> <p>After: <input type="radio"/> <input checked="" type="radio"/> <input type="radio"/> <input type="radio"/> <input checked="" type="radio"/> <input type="radio"/></p> <p>Follow through</p>
<p>7. (a) (i) The vertical asymptote of $C_1 : y = \frac{2x-1}{hx-1}$ is $x = \frac{1}{h}$.</p> <p>Hence, $\frac{1}{h} = 1$ i.e. $h = 1$</p> <p>The horizontal asymptote of $C_1 : y = \frac{2x-1}{hx-1}$ is $y = \frac{2}{h}$.</p> <p>Hence, $\frac{2}{h} = k$ (1) i.e. $k = 2$</p> <p>(ii)</p> 	<p>1A</p> <p>1A</p> <p>1A</p> <p>(5)</p>	<p>For asymptotes</p> <p>For intercepts</p> <p>For shape</p> <p>(pp-1) for all labels of axes and origin omitted</p>

Solution	Marks	Remarks
<p>(b) (i) Since $C_2: y = -x^2 + px + q$ passes through $\left(\frac{1}{2}, 0\right)$,</p> $\frac{-1}{4} + \frac{p}{2} + q = 0 \quad \text{----- (*)}$ $\frac{d}{dx} \left(\frac{2x-1}{x-1} \right) = \frac{(x-1)(2) - (2x-1)(1)}{(x-1)^2}$ $= \frac{-1}{(x-1)^2}$ $\frac{d}{dx} (-x^2 + px + q) = -2x + p$ <p>Since the tangents of C_1 and C_2 at $\left(\frac{1}{2}, 0\right)$ are perpendicular to each other,</p> $\frac{-1}{\left(\frac{1}{2}-1\right)^2} \cdot \left[-2\left(\frac{1}{2}\right) + p \right] = -1$ $p = \frac{5}{4}$ <p>By (*), $q = \frac{-3}{8}$</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p></p> <p></p> <p></p> <p></p> <p></p> <p>For both</p>
<p>(ii)</p> 	<p>1A</p>	<p>For the shape of C_2</p>
<p>(iii) The required area = $\int_0^1 \left[\frac{2x-1}{x-1} - \left(-x^2 + \frac{5}{4}x - \frac{3}{8} \right) \right] dx$</p> $= \int_0^1 \left(2 + \frac{1}{x-1} + x^2 - \frac{5}{4}x + \frac{3}{8} \right) dx$ $= \left[\ln x-1 + \frac{x^3}{3} - \frac{5x^2}{8} + \frac{19}{8}x \right]_0^1$ $= \frac{103}{96} - \ln 2$	<p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>(10)</p>	<p>For $2 + \frac{1}{x-1}$</p> <p>For $\frac{x^3}{3} - \frac{5x^2}{8} + \frac{19}{8}x$</p> <p>OR 0.3798</p>

	Solution	Marks	Remarks								
8. (a) (i)	$R = kt^{1.2} e^{\frac{\lambda t}{20}}$ $\ln R = \ln k + 1.2 \ln t + \frac{\lambda t}{20}$ $\ln R - 1.2 \ln t = \frac{\lambda}{20} t + \ln k \text{ which is a linear function of } t$	1A									
(ii)	intercept on the vertical axis = $\ln k = 2.89$ $k \approx 18$ (correct to the nearest integer) slope = $\frac{\lambda}{20} = -0.05$ $\lambda = -1$	1A 1A									
(iii)	$\therefore R = 18t^{1.2} e^{-0.05t}$ $\frac{dR}{dt} = 18[1.2t^{0.2} e^{-0.05t} + t^{1.2} e^{-0.05t}(-0.05)]$ $= 0.9t^{0.2} e^{-0.05t} (24 - t)$ <table border="1" style="margin: 10px auto;"> <tr> <td></td> <td>$0 < t < 24$</td> <td>$t = 24$</td> <td>$24 < t \leq 30$</td> </tr> <tr> <td>$\frac{dR}{dt}$</td> <td>> 0</td> <td>0</td> <td>< 0</td> </tr> </table>		$0 < t < 24$	$t = 24$	$24 < t \leq 30$	$\frac{dR}{dt}$	> 0	0	< 0	1M 1M	
	$0 < t < 24$	$t = 24$	$24 < t \leq 30$								
$\frac{dR}{dt}$	> 0	0	< 0								
	Hence, R will attain maximum after 24 months.	1A									
	$R = 18(24)^{1.2} e^{-0.05(24)}$ ≈ 245.6815916										
	Hence, the maximum population size is 246 hundreds.	1A									
		(7)									
(b) (i)	When $t = 0$, $L - 20(6e^0 + 0^3) = Q = 240$ $\therefore L = 360$	1A									
(ii)	$e^{-t} = 1 + (-t) + \frac{(-t)^2}{2!} + \frac{(-t)^3}{3!} + \dots$ $= 1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \dots$ $\therefore Q = 360 - 20 \left[6 \left(1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \dots \right) + t^3 \right]$ $= 360 - 20(6 - 6t + 3t^2 + \dots)$ $\approx 240 + 120t - 60t^2 \text{ which is a quadratic polynomial}$	1A 1A									
(iii)	Let $300 = Q = 240 + 120t - 60t^2$ (by (b)(ii)) i.e. $t^2 - 2t + 1 = 0$ Hence, when $t = 1$, the species of fish will reach a population size of 300.	1M 1A	Follow through								
(iv)	$Q = L - 20(6e^{-t} + t^3)$ $= 360 - 20 \left[6 \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!} + \frac{t^6}{6!} - \frac{t^7}{7!} + \dots \right) + t^3 \right]$ $= 240 + 120t - 60t^2 - 120 \left[\left(\frac{t^4}{4!} - \frac{t^5}{5!} \right) + \left(\frac{t^6}{6!} - \frac{t^7}{7!} \right) + \dots \right]$										

Solution	Marks	Remarks
(c) For $n > 6$, $R_n = R_6 + \int_6^n 0 dt \approx 10.5316$ (by (a)(i))	1M	For $\int_6^n 0 dt = 0$
When $n \rightarrow \infty$, $e^{-n} \rightarrow 0$ and so $Q_n \rightarrow 4.5799 + \frac{15}{3+0} = 9.5799$	1M	For $e^{-n} \rightarrow 0$
Therefore, over a long period of time, plan A produces approximately 10.5316 million dollars and plan B produces 9.5799 million dollars of revenue.	} 1A 1M 1A	Follow through
Moreover, the revenue of plan A is even an under-estimate.		
Hence, plan A will produce more revenue over a long period of time.		
	(5)	
10. (a) (i) The required probability = $\frac{1}{C_3^9} = \frac{1}{84}$	1A	OR 0.0119
(ii) The required probability = $\frac{C_3^4}{C_3^9} = \frac{1}{21}$	1A	OR 0.0476
	(2)	
(b) $(1-p)\left(\frac{1}{84}\right) + p\left(\frac{1}{21}\right) \leq \frac{1}{60}$ $5-5p+20p \leq 7$ $p \leq \frac{2}{15}$	1M+1A	(pp-1) for using “=”
Hence, the largest value of p should be $\frac{2}{15}$.	1A	OR 0.1333
	(3)	
(c) (i) The required probability = $\frac{2}{15} \cdot \frac{1}{C_4^9}$ $= \frac{1}{945}$	1M 1A	For using (b) OR 0.0011
(ii) The required probability = $\left(1 - \frac{2}{15}\right) \cdot \frac{C_3^3 C_1^6}{C_4^9} + \frac{2}{15} \cdot \frac{C_3^4 C_1^5}{C_4^9}$ $= \frac{59}{945}$	1M+1A 1A	OR 0.0624
(iii) The probability of exactly 2 logos are found on 1 card $= \left(1 - \frac{2}{15}\right) \cdot \frac{C_2^3 C_2^6}{C_4^9} + \frac{2}{15} \cdot \frac{C_2^4 C_2^5}{C_4^9}$ $= \frac{47}{126}$	1M 1A	OR 0.3730
Hence, the required probability $= \left(\frac{1}{945} + \frac{59}{945}\right)^2 + 2\left(\frac{1}{945} + \frac{59}{945}\right)\left(1 - \frac{1}{945} - \frac{59}{945}\right) + \left(\frac{47}{126}\right)^2$ $= \frac{1387}{5292}$	1M+1A 1A	OR 0.2621
	(10)	

Solution	Marks	Remarks
11. Let X_N cm and X_D cm be the widths of the tongue of a normal baby and a baby having inherited disease A respectively.		
(a) $P(X_N < 2.22) = 0.242$ $\frac{2.22 - \mu}{0.4} = -0.7$ $\mu = 2.5$	1A	
	(1)	
(b) (i) The required probability $= P(X_N > 2.5 + 0.5)$ $= P\left(Z > \frac{3 - 2.5}{0.4}\right)$ $= 0.5 - 0.3944$ $= 0.1056$	IM 1A	
(ii) The required probability $= 0.05 \times P(X_D < 2.5 + 0.5) + 0.95 \times P(X_N > 2.5 + 0.5)$ $= 0.05 \times P\left(Z < \frac{0.2}{0.2}\right) + 0.95 \times 0.1056$ $= 0.05 \times 0.8413 + 0.95 \times 0.1056$ $= 0.142385$	IM+1A 1A	OR 0.1424
(iii) The required probability $= \frac{0.05(0.8413)}{0.05(0.8413) + 0.95(1 - 0.1056)}$ ≈ 0.0472	IM+1A 1A	
	(8)	
(c) (i) The required probability $= \frac{C_3^8 C_1^{12} (0.142385)^4 (1 - 0.142385)^{16}}{C_4^{20} (0.142385)^4 (1 - 0.142385)^{16}}$ $= \frac{224}{1615}$	IM 1A	OR 0.1387
(ii) The required probability $= \frac{C_2^7 (0.142385)^3 (1 - 0.142385)^{17} + C_2^7 C_1^{12} (0.142385)^4 (1 - 0.142385)^{16}}{(1 - 0.142385)^{20} + C_1^{20} (0.142385)(1 - 0.142385)^{19} + C_2^{20} (0.142385)^2 (1 - 0.142385)^{18} + C_3^{20} (0.142385)^3 (1 - 0.142385)^{17} + C_4^{20} (0.142385)^4 (1 - 0.142385)^{16}}$ ≈ 0.0156	IM+IM+1A 1A	IM for numerator IM for denominator
	(6)	

Solution		Marks	Remarks	
12. (a)	(i)			
		$\frac{e^{-\lambda} \lambda^1}{1!} = \frac{41.04}{200}$ ————— (1)	} 1A	For both
		$\frac{e^{-\lambda} \lambda^4}{4!} = \frac{26.72}{200}$ ————— (2)		
		(2) ÷ (1):		
		$\frac{\lambda^3}{24} = \frac{334}{513}$		
		$\lambda \approx 2.5$ (correct to 1 decimal place)	1A	
		(ii) $a = 7.22$, $b = 16.42$, $c = 51.30$, $d = 42.75$, $e = 13.36$	2A	1A for any 1 correct 2A for all correct
		(iii) For the number of passengers > 5 , the expected frequency by Po(2) is $200 - 27.07 - 54.13 - 54.13 - 36.09 - 18.04 - 7.22 = 3.32$ For the number of passengers > 5 , the expected frequency by Po(2.5) is $200 - 16.42 - 41.04 - 51.30 - 42.75 - 26.72 - 13.36 = 8.41$ The sum of errors for model fitted by Po(2) is $E_1 = 28 - 27.07 + 50 - 54.13 + 52 - 54.13 + 40 - 36.09 + 24 - 18.04 $ $\quad + 6 - 7.22 + 0 - 3.32 $ $\quad = 21.6$ The sum of errors for model fitted by Po(2.5) is $E_2 = 28 - 16.42 + 50 - 41.04 + 52 - 51.30 + 40 - 42.75 + 24 - 26.72 $ $\quad + 6 - 13.36 + 0 - 8.41 $ $\quad = 42.48$ Since $E_1 < E_2$, Po(2) fits the observed data better.	IM	For either one
			1A	For both
		1A	Follow through	
		(7)		
(b)	Let X_i be the number of passengers waiting at the i^{th} stop.			
	(i) The required probability $= P(X_1 \geq 4)$ $= 1 - \frac{e^{-2} 2^0}{0!} - \frac{e^{-2} 2^1}{1!} - \frac{e^{-2} 2^2}{2!} - \frac{e^{-2} 2^3}{3!}$ ≈ 0.1429	IM 1A		
	(ii) The required probability $= P(X_1 \leq 2) \cdot P(X_2 \geq 2)$ $= \left(\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} \right) \left(1 - \frac{e^{-2} 2^0}{0!} - \frac{e^{-2} 2^1}{1!} \right)$ ≈ 0.4019	IM 1A		
	(iii) The required probability $= [P(X_1 = 1) \cdot P(X_2 = 1) + P(X_1 = 1) \cdot P(X_2 = 2) + P(X_1 = 2) \cdot P(X_2 = 1)]$ $\quad \cdot [1 - P(X_3 = 0)]$ $= \left[\left(\frac{e^{-2} 2^1}{1!} \right)^2 + \left(\frac{e^{-2} 2^1}{1!} \right) \left(\frac{e^{-2} 2^2}{2!} \right) + \left(\frac{e^{-2} 2^2}{2!} \right) \left(\frac{e^{-2} 2^1}{1!} \right) \right] \cdot \left(1 - \frac{e^{-2} 2^0}{0!} \right)$ ≈ 0.1900	IM+1M+1M 1A	$\left. \begin{array}{l} \text{1M for } \left(\frac{e^{-2} 2^1}{1!} \right)^2 \\ \text{1M for } \left(\frac{e^{-2} 2^1}{1!} \right) \left(\frac{e^{-2} 2^2}{2!} \right) \times 2 \\ \text{1M for } 1 - \frac{e^{-2} 2^0}{0!} \end{array} \right\}$	
		(8)		