2008-ASL
M & S

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 2008

MATHEMATICS AND STATISTICS AS-LEVEL

8.30 am – 11.30 am (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer **ALL** questions in Section A, **using the AL(E) answer book.**
3. Answer any **FOUR** questions in Section B, **using the AL(C) answer book.**
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Section A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(E) answer book.

1. (a) The binomial expansion of $\frac{1}{\sqrt{1+ax}}$ in ascending powers of x is $1+\frac{3}{2}x+bx^2+\cdots$, where a and b are constants.

   (i) Find the values of a and b .

   (ii) State the range of values of x for which the expansion is valid.

   (b) Using (a) with $x=\frac{1}{30}$, find an approximate value of $\sqrt{10}$.

(7 marks)

2. Suppose $y^3-uy=1$ and $u=2^{x^2}$.

   (a) Find $\frac{dy}{du}$ in terms of u and y .

   (b) Find $\frac{du}{dx}$ in terms of x .

   (c) Find $\frac{dy}{dx}$ in terms of x and y .

(7 marks)

3. The rate of change of concentration of a drug in the blood of a patient can be modelled by

$$\frac{dx}{dt}=5.3\left(\frac{1}{t+2}-\frac{1}{t+5}\right)+1.2e^{-0.1t} ,$$

   where x is the concentration measured in mg/L and t is the time measured in hours after the patient has taken the drug. It is given that $x=0$ when $t=0$.

   (a) Find x in terms of t .

   (b) Find the concentration of the drug after a long time.

(6 marks)

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4. A and B are two events. A' and B' are the complementary events of A and B respectively. Suppose $\mathrm{P}(A)=\frac{1}{5}$, $\mathrm{P}(A\cup B)=\frac{9}{20}$, $\mathrm{P}(A\mid B)=\frac{1}{6}$ and $\mathrm{P}(B)=k$, where $0<k<1$.

(a) Using $\mathrm{P}(A\mid B)$, express $\mathrm{P}(A\cap B)$ in terms of k.

(b) Find the value of k.

(c) Find $\mathrm{P}(A'\cap B)$.

(d) Are the two events A' and B' mutually exclusive? Explain your answer.

(7 marks)

5. The amount of money spent by a randomly selected customer of a jewellery shop is assumed to be normally distributed with a mean of $\$\mu$ and a standard deviation of \$6 000. Suppose 24.2% of the customers spend more than \$30 000 in the shop.

(a) Find the value of μ.

(b) It is given that Mrs. Chan spends less than \$30 000 in the shop. Find the probability that she spends more than \$16 500.

(6 marks)

6. A test is taken by a class of 18 students. The marks are as follows:

55	82	74	70	91	75	79	89	68
79	59	72	79	73	60	71	82	k

where k is Jane's mark.

It is known that the mean mark of the class is the same irrespective of including or excluding Jane's.

(a) Find the value of k.

(b) If 3 student marks are selected randomly from the set of the 18 student marks, find the probability that exactly 1 of them is the mode of the set of the 18 student marks.

(c) A student mark is classified as an *outlier* if it lies outside the interval $(\mu-2\sigma,\ \mu+2\sigma)$, where μ is the mean and σ is the standard deviation of the set of marks.

(i) Find all the *outlier(s)* of the set of the 18 student marks.

(ii) In order to assess the students' performance in the test, all *outliers* are removed from the set. Describe the change in the median and the standard deviation of the student marks due to such removal.

(7 marks)

Section B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C) answer book.

7. Define $\mathrm{f}(x)=\frac{3x-2}{x+2}$ for all $x\neq -2$. Let C be the curve $y=\mathrm{f}(x)$.

(a) Find the equations of the horizontal asymptote(s) and vertical asymptote(s) to C.

(2 marks)

(b) Sketch C and its asymptotes. Indicate the point(s) where the curve cuts the axes.

(3 marks)

(c) $P(h, k)$ is a point on C in the first quadrant. Let L_1 and L_2 be respectively the tangent and normal to C at P.

(i) Show that the equation of L_1 is

$$8x-(h+2)^2y+3h^2-4h-4=0\ .$$

(ii) If L_1 passes through the origin, find

(1) the equation of L_2,

(2) the area of the region bounded by C, L_2 and the x-axis.

(10 marks)

Provided by dse.life

8. A biologist studied the population of fruit fly A under limited food supply. Let t be the number of days since the beginning of the experiment and $N(t)$ be the number of fruit fly A at time t. The biologist modelled the rate of change of the number of fruit fly A by

$$N'(t) = \frac{20}{1+he^{-kt}} \quad (t \geq 0),$$

where h and k are positive constants.

(a) (i) Express $\ln\left[\frac{20}{N'(t)} - 1\right]$ as a linear function of t.

(ii) It is given that the intercepts on the vertical axis and the horizontal axis of the graph of the linear function in (i) are 1.5 and 7.6 respectively. Find the values of h and k.

(4 marks)

(b) Take $h = 4.5$, $k = 0.2$ and assume that $N(0) = 50$.

(i) Let $v = h + e^{kt}$, find $\frac{dv}{dt}$.

Hence, or otherwise, find $N(t)$.

(ii) The population of fruit fly B can be modelled by

$$M(t) = 21\left(t + \frac{h}{k}e^{-kt}\right) + b,$$

where b is a constant. It is known that $M(20) = N(20)$.

(1) Find the value of b.

(2) The biologist claims that the number of fruit fly A will be smaller than that of fruit fly B for $t > 20$. Do you agree? Explain your answer.

[Hint: Consider the difference between the rates of change of the two populations.]

(11 marks)

9. The rate of change of yearly average temperature of a city is predicted to be

$$\frac{dx}{dt} = \frac{1}{40}\sqrt{1+t^2} \quad (t \geq 0),$$

where x is the temperature measured in °C and t is the time measured in years. It is given that $x = 22$ when $t = 0$.

(a) (i) Using the trapezoidal rule with 4 sub-intervals, estimate the increase of temperature from $t = 0$ to $t = 10$.

(ii) Determine whether this estimate is an over-estimate or an under-estimate.

(4 marks)

(b) It is known that the electricity consumption $W(x)$, in appropriate units, depends on the yearly average temperature x and is given by

$$W(x) = 100(\ln x)^2 - 630\ln x + 1960 \quad (x \geq 22).$$

(i) If $W(x_0) = 968$, find all possible value(s) of x_0.

(ii) Find the range of values of x while $W'(x) < 0$.

(iii) Find the rate of change of electricity consumption at $t = 0$.

(iv) Using (a), estimate the electricity consumption at $t = 10$. Determine and explain whether the actual electricity consumption is larger than or smaller than this estimate.

(11 marks)

Provided by dse.life

10. Assume that the number of visitors arriving at each counter in an immigration hall is independent and follows a Poisson distribution with a mean of 3.9 visitors per minute. A counter is classified as *busy* if at least 4 visitors arriving at it in one minute.

(a) Find the probability that a counter is *busy* in a certain minute.

(3 marks)

(b) An officer checks 4 counters in a certain minute. Find the probability that at least one *busy* counter is found.

(2 marks)

(c) If 10 counters are open, find the probability that more than 7 of them are *busy* in a certain minute.

(3 marks)

(d) Suppose 10 counters are open and one of them is randomly selected. Find the probability that more than 7 of them are busy and the randomly selected counter is not busy in a certain minute.

(3 marks)

(e) The immigration hall is called *congested* if more than 90% of the open counters are *busy* in a minute. Suppose 15 counters in the hall are open. A senior officer checks the counters in a certain minute. It is given that more than 7 of the first 10 checked counters are *busy*. Find the probability that the hall is *congested*.

(4 marks)

11. A manager of a maintenance centre launches an appraisal system to assess the performance of technicians in terms of the time spent to complete a task. A technician can get 2 points if he takes less than 2 hours to complete a task, 1 point if he takes between 2 and 4.6 hours, and 0 point if he takes longer than 4.6 hours.

Assume the time for a technician to complete a task is normally distributed with a mean of 3 hours and a standard deviation of 0.8 hour, and the number of tasks assigned to a technician follows a Poisson distribution with a mean of 1.8 tasks per day.

(a) Find the probability that a technician is assigned not more than 4 tasks on a certain day.

(3 marks)

(b) Let p_i be the probability of a technician getting i point(s) upon completing a task, where $i = 0, 1, 2$.

Find the values of p_0, p_1 and p_2 .

(3 marks)

(c) Find the probability that a technician gets exactly 4 points on a certain day under each of the following conditions:

(i) 3 tasks are assigned,

(ii) 4 tasks are assigned.

(5 marks)

(d) It is given that a technician is assigned fewer than 5 tasks on a certain day. Find the probability that the technician gets exactly 4 points.

(4 marks)

Provided by dse.life

12. Officials of the Food Safety Centre of a city inspect the imported "Choy Sum" by selecting 40 samples of "Choy Sum" from each lorry and testing for an unregistered insecticide. A lorry of "Choy Sum" is classified as *risky* if more than 2 samples show positive results in the test.

Farm A supplies "Choy Sum" to the city. Past data indicated that 1% of the Farm A "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A is transported by a number of lorries to the city.

(a) Find the probability that a lorry of "Choy Sum" is *risky*.

(3 marks)

(b) Find the probability that the 5^{th} lorry is the first lorry transporting *risky* "Choy Sum".

(2 marks)

(c) If k lorries of "Choy Sum" are inspected, find the least value of k such that the probability of finding at least one lorry of *risky* "Choy Sum" is greater than 0.05 .

(3 marks)

(d) Farm B also supplies "Choy Sum" to the city. It is known that 1.5% of the Farm B "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A and Farm B is transported by 8 and 12 lorries respectively to the city.

(i) Find the probability that a lorry of "Choy Sum" supplied by Farm B is *risky*.

(ii) Find the probability that exactly 2 of these 20 lorries of "Choy Sum" are *risky*.

(iii) It is given that exactly 2 of these 20 lorries of "Choy Sum" are *risky*. Find the probability that these 2 lorries transport "Choy Sum" from Farm B .

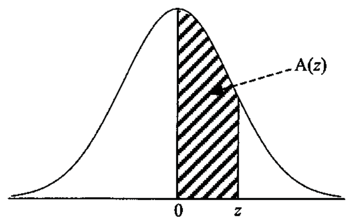
(7 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z . Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx$$

Provided by dse.life