7. Further Probability

Learning Unit	Learning Objective
Statistics Area	
Further Probability	
10. Conditional probability and independence	10.1 Understand the concepts of conditional probability and independent events 10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events C and D to solve problems
11. Bayes' theorem	11.1 use Bayes' theorem to solve simple problems

Set notation

1. Let A and B be two events. Suppose that $P(A)=0.8$, $P(B \mid A)=0.45$ and $P(B \mid A')=0.6$, where A' is the complementary event of A. Find
   (a) $P(B)$,
   (b) $P(A \mid B)$,
   (c) $P(A \cup B)$.

(5 marks) (2018 DSE-MATH-M1 Q1)

2. Let A and B be two events. Suppose that $P(A)=0.2$, $P(B')=0.7$ and $P(A \mid B)=0.6$, where B' is the complementary event of B.
   (a) $P(B \mid A)$.
   (b) Are A and B mutually exclusive? Explain your answer.
   (c) Are A and B independent? Explain your answer.

(6 marks) (2017 DSE-MATH-M1 Q2)

3. Let X and Y be two events such that $P(X)=0.4$, $P(Y)=0.7$ and $P(Y \mid X)=0.5$.
   (a) Are X and Y independent? Explain your answer.
   (b) Find $P(X \cup Y)$.

(5 marks) (2016 DSE-MATH-M1 Q1)

4. A and B are two events. Suppose that $P(A)=0.3$, $P(B)=0.28$ and $P(B' \mid A')=0.6$, where A' and B' are the complementary events of A and B respectively.
   (a) Find $P(A' \cap B')$ and $P(A' \cap B)$.
   (b) Are A and B mutually exclusive? Explain your answer.

(6 marks) (2015 DSE-MATH-M1 Q2)

5. Let A and B be two events such that $P(A \mid B)=0.4$, $P(A \cup B)=0.45$ and $P(B')=0.75$, where B' is the complementary event of B .
   (a) Find $P(A \cap B)$ and $P(A)$.
   (b) Are events A and B independent? Justify your answer.

(6 marks) (2014 DSE-MATH-M1 Q7)

6. Suppose A and B are two events. Let A' and B' be the complementary events of A and B respectively. It is given that $P(A \mid B')=0.6$, $P(A \cap B)=0.12$ and $P(A \cap B')=k$, where $k>0$.
   (a) Find $P(A)$, $P(B)$ and $P(A \cup B)$ in terms of k.
   (b) If A and B are independent, find the value of k.

(6 marks) (PP DSE-MATH-M1 Q9)

7. Let A and B be two events. It is given that $P(A)=a$, $P(B' \mid A)=\frac{27}{32}$ and $P(A \mid B')=\frac{27}{31}$.
   (a) Find $P(A \cap B')$ in terms of a .
   (b) Find $P(B)$ in terms of a .
   (c) It is given that $P(A \cap B)=0.1$.
      (i) Find the value of a .
      (ii) Determine whether A and B are independent or not.

(7 marks) (2013 ASL-M&S Q5)

8. Let A and B be two events. It is given that $P(A \mid B)=\frac{3}{4}$, $P(B \mid A)=\frac{3}{8}$ and $P(A)=a$.
   (a) Find $P(A \cap B)$ in terms of a .
   (b) Find $P(B)$ in terms of a .
   (c) It is given that $P(A' \cap B')=\frac{7}{16}$.
      (i) Find the value of a .

(ii) Find the value of $P(A|B')$.

(7 marks) (2012 ASL-M&S Q5)

9. Let A and B be two events of a certain sample space such that $P(A \cup B) = 1$. Denote $P(B) = b$ and $P(A \cap B) = c$, where $0 < b < 1$ and $0 < c < 1$.

(a) Express $P(A)$ in terms of b and c .

(b) Suppose that $P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$.

(i) Find the values of b and c .

(ii) Are the events A and B independent? Explain your answer.

(7 marks) (*modified from* 2010 ASL-M&S Q4)

10. Let A and B be two events. Suppose $P(A \cup B) = \frac{5}{12}$, $P(A) = a$, $P(B) = \frac{1}{4}$ and $P(A \mid B') = k$.

(a) Find $P(A \cap B)$ in terms of a .

(b) Find the value of k .

(c) If A and B are independent, find the value of a .

(8 marks) (2009 ASL-M&S Q4)

11. A and B are two events. A' and B' are the complementary events of A and B respectively. Suppose $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{9}{20}$, $P(A \mid B) = \frac{1}{6}$ and $P(B) = k$, where $0 < k < 1$.

(a) Using $P(A \mid B)$, express $P(A \cap B)$ in terms of k .

(b) Find the value of k .

(c) Find $P(A' \cap B)$.

(d) Are the two events A' and B' mutually exclusive? Explain your answer.

(7 marks) (2008 ASL-M&S Q4)

12. Let A and B are two events with $P(A) = a$ and $P(B) = b$, where $0 < a < 1$ and $0 < b < 1$. Suppose that $P(A' \mid B) = 0.6$, $P(B \mid A') = 0.3$ and $P(B' \mid A) = 0.7$, where A' and B' are complementary events of A and B respectively.

(a) By considering $P(A' \cap B)$, prove that $a + 2b = 1$.

(b) Using the fact that $A \cup B'$ is the complementary event of $A' \cap B$, or otherwise, find the value of a and b.

(c) Are A and B independent events? Explain your answer.

(7 marks) (2007 ASL-M&S Q5)

13. A and B are two events. Suppose that $P(A \cap B) = 0.2$ and $P(A \mid B') = 0.5$, where B' is the complementary event of B. Let $P(B) = b$, where $b < 1$.

(a) Express $P(A \cap B')$ and $P(A)$ in terms of b.

(b) If A and B are independent events, find the value(s) of b.

(7 marks) (2006 ASL-M&S Q5)

14. A and B are two events. Suppose that $P(A \mid B') = \frac{5}{12}$, $P(B \mid A') = \frac{8}{15}$ and $P(B) = \frac{2}{5}$, where A' and B' are complementary events of A and B respectively. Let $P(A) = a$, where $0 < a < 1$.

(a) Find $P(A \cap B')$.

(b) Express $P(A' \cap B)$ in terms of a .

(c) Using the fact that $A' \cup B$ is the complementary event of $A \cap B'$, or otherwise, find the value of a .

(d) Are A and B mutually exclusive? Explain your answer.

(7 marks) (2005 ASL-M&S Q5)

15. A and B are two events. Suppose that $P(A) = 0.75$ and $P(B) = 0.8$. Let $P(A \cap B) = k$.

(a) Express $P(A \cup B)$ in terms of k .

(b) (i) Prove that $0.55 \le k \le 1$.

(ii) Let A' and B' are complementary events of A and B respectively. Using (b)(i) and $A' \cup B'$ is the complementary event of $A \cap B$, or otherwise, prove that $P(A' \cup B') \le 0.45$

(7 marks) (2004 ASL-M&S Q1)

16. A and B are two events. Suppose that $P(A \mid B) = 0.5$, $P(B \mid A) = 0.4$ and $P(A \cup B) = 0.84$. Let $P(A) = a$, where $a > 0$.

(a) Express $P(A \cap B)$ and $P(B)$ in terms of a .

(b) Using the result of (a), or otherwise, find the value of a .

(c) Are A and B independent events? Explain your answer briefly.

(7 marks) (2003 ASL-M&S Q4)

17. A and B are two independent events. If $P(A) = 0.4$ and $P(A \cup B) = 0.7$, find $P(B)$.

(4 marks) (2001 ASL-M&S Q1)

Tree Diagram, Conditional Probability and Bayes' Theorem

18. A box contains six cards numbered 1 , 2 , 3 , 4 , 5 and 6 respectively.
    (a) Three cards are drawn randomly from the box one by one with replacement. Given that the sum of the numbers drawn is 7, find the probability that the number 1 is drawn exactly two times.
    (b) If the card numbered 6 is taken away before three cards are drawn, will the probability described in (a) change? Explain your answer.

(6 marks) (2015 DSE-MATH-M1 Q2)

19. A bag contains 2 white balls and 5 yellow balls. In a survey, each interviewee draws a ball randomly from the bag. If a white ball is drawn, then the interviewee considers the question 'Are you a smoker?'. If a yellow ball is drawn, then the interviewee considers the question 'Are you a non-smoker?'.
    Finally, the interviewee answers either 'Yes' or 'No'. Let p be the probability that a randomly selected interviewee is a smoker.
    (a) Express, in terms of p, the probability that a randomly selected interviewee answers 'Yes'.
    (b) In this survey, 50 out of 91 interviewees answer 'Yes'.
        (i) Find p .
        (ii) Given that an interviewee answers 'No', find the probability that the interviewee is a non-smoker.

(6 marks) (2015 DSE-MATH-M1 Q3)

20. A company produces microwave ovens by production lines A and B. It is known that 4% of all microwave ovens fail to function properly and that 2% of microwave ovens produced by line A fail to function properly. Among the microwave ovens which function properly, $\frac{2}{3}$ of them are produced by line B. Suppose a microwave oven is randomly selected.
    (a) What is the probability that the microwave oven is produced by line B and functions properly?
    (b) What is the probability that the microwave oven is produced by line A?
    (c) If the microwave oven is produced by line B, what is the probability that it functions properly?

(5 marks) (2014 DSE-MATH-M1 Q8)

21. In a shooting game, one member from each team will be selected to shoot a target three times. The team will get a prize if the target is hit at least once. Team A consists of Mabel and Owen, with the probability that Mabel is selected to shoot being 0.7. Suppose that the probabilities of Mabel and Owen to hit the target in each shot are 0.6 and 0.5 respectively.
    (a) Find the probability that Team A will get a prize if Mabel is selected.
    (b) Find the probability that Team A will get a prize.
    (c) Given that Team A does not get a prize, find the probability that Owen is selected.

(6 marks) (2013 DSE-MATH-M1 Q8)

22. In a game, there are two bags, A and B, each containing 5 balls. Bag A contains 3 red and 2 blue balls, while bag B contains 4 red and 1 blue balls. A player first chooses a bag at random and then draws a ball randomly from the bag. The player will be rewarded if the ball drawn is blue. The ball is then replaced for the next player's turn.
(a) Find the probability that a player is rewarded in a particular game.
(b) Two players participate in the game. Given that at least one of them is rewarded, find the probability that both of them are rewarded.
(c) If 60 players are rewarded, find the expected number of players among them having drawn a blue ball from bag A.

(5 marks) (PP DSE-MATH-M1 Q7)

23. The percentage of local Year One students in a certain university is 90%, among whom 5% are enrolled with a scholarship. For non-local Year One students, 35% of them are enrolled **without** a scholarship.
(a) If a Year One student is selected at random, find the probability that the student is enrolled with a scholarship.
(b) Given that a selected Year One student is enrolled with a scholarship, find the probability that this student is a non-local student.

(4 marks) (SAMPLE DSE-MATH-M1 Q4)

24. Twelve boys and ten girls in a class are divided into 3 groups as shown in the table below:

	Group A	Group B	Group C
Number of boys	6	4	2
Number of girls	2	3	5

To choose a student as the class representative, a group is selected at random, then a student is chosen at random from the selected group.
(a) Find the probability that a boy is chosen as the class representative.
(b) Suppose that a boy is chosen as the class representative. Find the probability that the boy is from Group A.

(5 marks) (2002 ASL-M&S Q5)

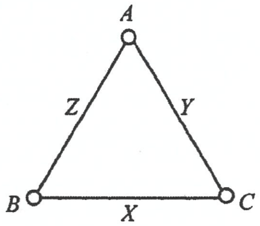
25. In the election of the Legislative Council, 48% of the votes supports Party A, 39% Party B and 13% Party C. Suppose on the polling day, 65%, 58% and 50% of the supporting voters of Parties A, B and C respectively cast their votes.
(a) A voter votes on the polling day. Find the probability that the voter support Party B.
(b) Find the probability that exactly 2 out of 5 voting voters support Party B.

(7 marks) (2001 ASL-M&S Q7)

26. A department store uses a machine to offer prizes for customers by playing games A or B. The probability of a customer winning a prize in game A is $\frac{5}{9}$ and that in game B is $\frac{5}{6}$. Suppose each time the machine randomly generates either game A or game B with probabilities 0.3 and 0.7 respectively.
(a) Find the probability of a customer winning a prize in 1 trial.
(b) The department store wants to adjust the probabilities of generating game A and game B so that the probability of a customer winning a prize in 1 trial is $\frac{2}{3}$. Find the probabilities of generating game A and game B respectively.

(6 marks) (2000 ASL-M&S Q8)

27. Three control towers A, B and C are in telecommunication contact by means of three cables X, Y and Z as shown in the figure. A and B remain in contact only if Z is operative or if both cables X and Y are operative. Cables X, Y and Z are subject to failure in anyone day with probabilities 0.015, 0.025 and 0.030 respectively. Such failures occur independently.



(a) Find, to 4 significant figures, the probability that, on a particular day,
(a) both cables X and Z fail to operate,
(b) all cables X, Y and Z fail to operate,
(c) A and B will not be able to make contact.
(b) Given that cable X fails to operate on a particular day, what is the probability that A and B are not able to make contact?
(c) Given that A and B are not able to make contact on a particular day, what is the probability that cable X has failed?

(7 marks)
(7 marks) (1999 ASL-M&S Q7)

28. A factory produced 3 kinds of ice-cream bars A, B and C in the ratio 1 : 2 : 5. It was reported that some ice-cream bars produced on 1 May, 1998 were contaminated. All ice-cream bars produced on that day were withdrawn from sale and a test was carried out. The test results showed hat 0.8% of kind A, 0.2% of kind B and 0% of kind C were contaminated.
(a) An ice-cream bar produced on that day is selected randomly. Find the probability that
(i) the bar is of kind A and is NOT contaminated,
(ii) the bar is NOT contaminated.
(b) If an ice-cream bar produced on that day is contaminated, find the probability that is of kind A.

(6 marks) (1998 ASL-M&S Q6)

29. A company buys equal quantities of fuses, in 100-unit lots, from two suppliers A and B. The company tests two fuses randomly drawn from each lot, and accepts the lot if both fuses are non-defective.
It is known that 4% of the fuses from supplier A and 1% of the fuses from supplier B are defective. Assume that the qualities of the fuses are independent of each other.
(a) What is the probability that a lot will be accepted?
(b) What is the probability that an accepted lot came from supplier A?
(6 marks) (1996 ASL-M&S Q6)

30. An insurance company classifies the aeroplanes it insures into class L (low risk) and class H (high risk), and estimates the corresponding proportions of the aeroplanes as 70% and 30% respectively. The company has also found that 99% of class L and 88% of class H aeroplanes have no accident within a year. If an aeroplane insured by the company has no accident within a year, what is the probability that it belongs to
(a) class H?
(b) class L?
(7 marks) (1995 ASL-M&S Q5)

31. In asking some sensitive question such as "Are you homosexual?", a *randomized response technique* can be applied: The interviewee will be asked to draw a card at random from a box with one red card and two black cards and then consider the statement 'I am homosexual' if the card is red and the statement 'I am not homosexual' otherwise. He will give the response either 'True' or 'False'. The colour of the card drawn is only known to the interviewee so that nobody knows which statement he has responded to. Suppose in a survey, 790 out of 1200 interviewees give the response 'True'.
(a) Estimate the percentage of persons who are homosexual.
(b) For an interviewee who answered 'True', what is the probability that he is really homosexual?
(7 marks) (1994 ASL-M&S Q7)

Section B

32.

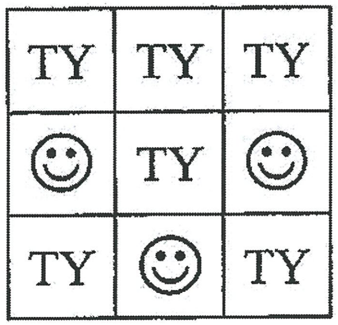


Figure (a)

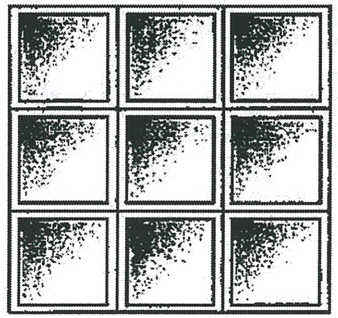


Figure (b)

A soft-drink company proposes a promotion programme by attaching a scratch card to each can of soft drink. Every card has nine squares, with 3 or 4 randomly selected squares each containing a smiley face and in each of the rest a 'TY' denoting 'Thank You'. An example is shown in Figure (a). All squares are covered by metallic films (see Figure (b)).

(a) A customer is asked to rub off the metallic films on 3 squares of a scratch card. If 3 smiley faces are found, the customer will win a prize. Find the probability that the customer can win a prize if the card has
(i) 3 smiley faces,
(ii) 4 smiley faces.
(2 marks)

(b) If the company wants to set the probability of winning a prize to be at most $\frac{1}{60}$, what should be the largest value of the proportion (p) of the cards with 4 smiley faces?
(3 marks)

(c) The company then produces the scratch card according to the proportion p found in (b). The company changes the rule of the game that customers will be asked to rub off the metallic films on 4 squares now and the prizes will be given as follows:
Gold Prize — exactly 4 smiley faces are found on 1 card
Silver Prize — exactly 3 smiley faces are found on 1 card
Bronze Prize — exactly 2 smiley faces are found on each of 2 cards
Find the probability of winning
(i) a Gold Prize with 1 card,
(ii) a Silver Prize with 1 card,
(iii) one or two prizes with 2 cards.
(10 marks)
(2009 ASL-M&S Q10)

33. In a promotion period of an electronic shopping card with spending limit of $3 000 , cardholders who spend over $400 in the maximum amount transaction are classified as VIPs and are eligible for entering an online "click-and-get-point" game once. The rules of the game are detailed in the following table.

Spending ($ x)	VIP Category	Number of clicks allowed
$400 < x \le 800$	Silver	1
$800 < x \le 1000$	Gold	2
$1000 < x \le 3000$	Platinum	3

The probabilities to get 1 , 2 , 3 and 4 points on a single click are 0.4, 0.3, 0.2 and 0.1 respectively. The total number of points got in a game can be exchanged for a cash rebate according to the following table.

Total number of points	Cash rebate
1 to 3	$20
4 to 9	$50
10 to 12	$200

It is known that among the VIPs, 25% belong to Silver, 60% belong to Gold and 15% belong to Platinum.

(a) In a certain completed game, find the probability

(i) of getting exactly 3 points if the player is a Gold VIP;

(ii) of getting exactly 3 points;

(iii) that the player is a Gold VIP given that the player gets exactly 3 points.

(5 marks)

(b) Find the probability that the player gets a cash rebate of exactly $ 20 in a certain completed game.

(2 marks)

(c) In a certain completed game, find the probability that the player gets

(i) exactly 10 points;

(ii) a cash rebate of exactly $ 200 .

(3 marks)

(d) Research data reveal that 70% of each category of the VIPs will complete the game. A manager of the card company proposes offering a 4% direct cash rebate of the transaction to all VIPs instead of the online game. However, a senior manager, Winnie, thinks that the cost of that proposal will certainly be higher than the expected cash rebate of the online game.

(i) Do you agree with Winnie? Explain your answer.

(ii) Another senior manager, John, thinks that the cost of offering a 2% direct cash rebate to all VIPs will certainly be lower than the expected cash rebate of the online game. Do you agree with John? Explain your answer.

(5 marks)

(2010 ASL-M&S Q11)

34. Boys B_1 , B_2 and girls G_1 , G_2 are students who have qualified to represent their school in a singing contest. One boy and one girl will form one team. The team formed by B_i and G_j is denoted by $B_i G_j$, where $i = 1$, 2 and $j = 1$, 2 . A team can enter the second round of the contest if both team members do not make any mistakes during their performance. Suppose that a student making mistakes in a performance is an independent event, and the probabilities that B_1 , B_2 , G_1 and G_2 do not make any mistakes in a performance are 0.9 , 0.7 , 0.8 and 0.6 respectively.

(a) List all the possible teams that can be formed.

(1 mark)

(b) Find the probability that $B_1 G_1$ can enter the second round of the contest.

(1 mark)

(c) If a team is selected randomly to represent the school, find the probability that the team can enter the second round of the contest.

(2 marks)

(d) If two teams $B_1 G_1$ and $B_2 G_2$ are formed to represent the school, find the probability that

(i) exactly one team can enter the second round of the contest,

(ii) at least one team can enter the second round of the contest.

(5 marks)

(e) Suppose that two teams are allowed to represent the school and each student can only join one team.

(i) If the two teams are formed randomly, find the probability that exactly one team can enter the second round of the contest.

(ii) How should the teams be formed so that the school has a better chance of having at least one team that can enter the second round of the contest?

(6 marks)

(2000 ASL-M&S Q13)

2021 DSE Q1

The table below shows the probability distribution of a discrete random variable X, where a and b are constants.

x	-1	0	1	2	3	4
$P(X=x)$	a	0.15	0.15	b	0.05	0.25

It is given that $E(5X+1)=10$.

(a) Find a and b.

(b) Let C be the event that $X>0$ and D be the event that $X \le 2$. Find $P(C \mid D)$.

(6 marks)

2021 DSE Q2

The probability that a person has disease D is 0.12. Test T is used to show whether a person has disease D or not. For a person who has disease D, the probability that test T shows that the person has disease D is 0.97. For a person who does not have disease D, the probability that test T shows that the person does not have disease D is 0.89.

(a) Find the probability that test T shows a correct result.

(b) Find the probability that test T shows that a person has disease D.

(c) Given that a person is shown to have disease D by test T, is the probability that the person actually has disease D less than 0.6? Explain your answer.

(6 marks)

2021 DSE Q3

In an examination, there are 10 questions. For each question, the probability that Peter knows how to do the question is 0.8. For each question that Peter knows how to do, the probability that he carelessly answers the question wrongly is 0.1; otherwise, Peter will answer the question correctly for the question that he knows how to do. For questions that Peter does not know how to do, he will answer them wrongly. Peter gets grade A if he answers 8 or more questions correctly.

(a) Find the probability that Peter gets grade A.

(b) Find the probability that Peter knows how to do all the questions and gets grade A.

(c) Given that Peter gets grade A, find the probability that he knows how to do all the questions.

(7 marks)

7. Further Probability

Set notation

1. (2018 DSE-MATH-M1 Q1)

2. (2017 DSE-MATH-M1 Q2)

(a) $P(B \mid A)$		
$=\dfrac{P(A\mid B)P(B)}{P(A)}$		
$=\dfrac{P(A\mid B)(1-P(B'))}{P(A)}$		
$=\dfrac{0.6(1-0.7)}{0.2}$	1M	
$=0.9$	1A	
(b) $P(A\cap B)$		
$=P(A\mid B)P(B)$		
$=P(A\mid B)(1-P(B'))$		
$=0.6(1-0.7)$		
$=0.18$		
$\neq 0$	1M	
Thus, A and B are not mutually exclusive.	1A	f.t.
(c) Note that $P(A\mid B)=0.6\neq 0.2=P(A)$.	1M	
Thus, A and B are not independent.	1A	f.t.
Note that $P(A\cap B)=0.18\neq 0.06=P(A)P(B)$.	1M	
Thus, A and B are not independent.	1A	f.t.
	----------(6)	

(a)	Very good. Over 90% of the candidates were able to find the value of $P(B\mid A)$ by using Bayes' Theorem.
(b)	Very good. Most candidates were able to conclude that A and B are not mutually exclusive events.
(c)	Very good. About 80% of the candidates were able to conclude that A and B are not independent events.

3. (2016 DSE-MATH-M1 Q1)

(a) $P(Y \mid X)$
$= 0.5$
$\neq 0.7$
$= P(Y)$ — 1M
Thus, X and Y are not independent. — 1A f.t.

> $P(X)P(Y)$
> $= (0.4)(0.7)$
> $= 0.28$
>
> $P(X \cap Y)$
> $= P(Y \mid X)P(X)$
> $= (0.5)(0.4)$
> $= 0.2$
>
> $P(X \cap Y) \neq P(X)P(Y)$ — 1M
> Thus, X and Y are not independent. — 1A f.t.

(b) $P(X \cap Y)$
$= P(Y \mid X)P(X)$
$= (0.5)(0.4)$ — 1M
$= 0.2$

$P(X \cup Y)$
$= P(X) + P(Y) - P(X \cap Y)$
$= 0.4 + 0.7 - 0.2$ — 1M
$= 0.9$ — 1A
----------(5)

(a)	Very good. More than 70% of the candidates were able to mention $P(Y \mid X) \neq P(Y)$ or $P(X \cap Y) \neq P(X)P(Y)$ to conclude that A and B were not independent events. Only some candidates were unable to show their numerical values in comparison.
(b)	Very good. A very high proportion of the candidates were able to use the identity $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ to find the value of $P(X \cup Y)$ while a few candidates were unable to find the value of $P(X \cap Y)$.

4. (2015 DSE-MATH-M1 Q2)

(a) $P(A' \cap B')$
$= P(B' \mid A')P(A')$
$= 0.6(1 - 0.3)$ — 1M
$= 0.42$ — 1A

$P(A' \cap B)$
$= P(A') - P(A' \cap B')$
$= 1 - 0.3 - 0.42$ — 1M
$= 0.28$ — 1A

(b) Note that $P(B) = P(A \cap B) + P(A' \cap B)$.
Since $P(B) = P(A' \cap B) = 0.28$, we have $P(A \cap B) = 0$. — 1M
Thus, A and B are mutually exclusive. — 1A f.t.
----------(6)

(a)	Very good. Most candidates were able to find the value of $P(A' \cap B')$ while a few candidates failed to find the value of $P(A' \cap B)$ properly.
(b)	Fair. Many candidates mixed up mutually exclusive events with independent events. Only some candidates were able to mention $P(A \cap B) = 0$ to conclude that A and B are mutually exclusive events.

5. (2014 DSE-MATH-M1 Q7)

(a) $P(A \cap B) = P(B)P(A \mid B)$
$= (1 - 0.75) \times 0.4$ — 1M
$= 0.1$ — 1A

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.45 = P(A) + (1 - 0.75) - 0.1$ — 1M
$P(A) = 0.3$ — 1A

(b) $P(A \mid B) = 0.4$
$\neq P(A)$ — 1M

> Alternative Solution
> $P(A)P(B) = 0.3 \times 0.25$
> $= 0.075$
> $\neq P(A \cap B)$ — 1M

Hence A and B are not independent events. — 1
(6)

(a) (b)	Excellent. Good. Few candidates wrote $P(A \cap B) = 0.1$ for (a) and then $P(A) \cdot P(B) = \cdots \neq 0.1 \neq P(A \cap B)$ for (b); while others tried to make conclusion by comparing $P(A \cap B)$ with 0, or $P(A \mid B)$ with $P(A) \cdot P(B)$.

6. (PP DSE-MATH-M1 Q9)

(a) $P(A) = P(A \cap B) + P(A \cap B')$	
$= 0.12 + k$	1A
$P(A \mid B') = \dfrac{P(A \cap B')}{P(B')}$	
$0.6 = \dfrac{k}{1 - P(B)}$	
$P(B) = 1 - \dfrac{5k}{3}$	1A
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
$= (0.12 + k) + \left(1 - \dfrac{5k}{3}\right) - 0.12$	1M
$= 1 - \dfrac{2k}{3}$	1A
(b) If A and B are independent, $P(A)P(B) = P(A \cap B)$.	
$(0.12 + k)\left(1 - \dfrac{5k}{3}\right) = 0.12$	1M
$0.8k - \dfrac{5k^2}{3} = 0$	
$k = 0.48$ or 0 (rejected)	1A
Alternative solution 1	
If A and B are independent, $P(A) = P(A \mid B')$.	
$0.12 + k = 0.6$	1M
$k = 0.48$	1A
Alternative solution 2	
If A and B are independent, $P(A)P(B') = P(A \cap B')$.	
$(0.12 + k)\left(\dfrac{5k}{3}\right) = k$	1M
$\dfrac{5k^2}{3} - 0.8k = 0$	
$k = 0.48$ or 0 (rejected)	1A
Alternative solution 3	
If A and B are independent, $P(A \mid B) = P(A \mid B')$.	
$\therefore \dfrac{P(A \cap B)}{P(B)} = P(A \mid B')$	
$\dfrac{0.12}{1 - \dfrac{5k}{3}} = 0.6$	1M
$k = 0.48$	1A
	(6)

(a)	平平。部分學生忽略了 $P(A) = P(A \cap B) + P(A \cap B')$ 及誤以爲 $P(A \cup B) = P(A) + P(B)$ 。
(b)	平平。少數學生不懂如何利用 A 與 B 爲獨立事件這個條件。

7. (2013 ASL-M&S Q5)

(a) $P(A \cap B') = P(B' \mid A) \cdot P(A)$	
$= \dfrac{27}{32} a$	1A
(b) $P(A \cap B') = P(A \mid B') \cdot P(B')$	
$\dfrac{27}{32} a = \dfrac{27}{31} \cdot [1 - P(B)]$	1M
$P(B) = 1 - \dfrac{31}{32} a$	1A
(c) (i) $P(A) = P(A \cap B) + P(A \cap B')$	
$a = 0.1 + \dfrac{27}{32} a$	1M
$a = 0.64$	1A
(ii) $P(A) \cdot P(B) = (0.64)\left[1 - \dfrac{31}{32}(0.64)\right]$	
$= 0.2432$	1A
$\neq P(A \cap B)$	
Hence A and B are not independent.	1
	(7)

Good. Some candidates were not able to apply the definition of independent events and some mixed up the events B and its complement B' .

8. (2012 ASL-M&S Q5)

Solution	Marks	Remarks
(a) $P(A\cap B)=P(A)P(B\mid A)$		
$=\dfrac{3a}{8}$	1A	
(b) $P(A\cap B)=P(B)P(A\mid B)$		
$\dfrac{3a}{8}=\dfrac{3}{4}P(B)$	1M	
$P(B)=\dfrac{a}{2}$	1A	
(c) (i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$1-\dfrac{7}{16}=a+\dfrac{a}{2}-\dfrac{3a}{8}$	1M	
$a=\dfrac{1}{2}$	1A	
(ii) $P(A\mid B')=\dfrac{P(A\cap B')}{P(B')}$		
$=\dfrac{P(A)-P(A\cap B)}{1-P(B)}$		
$=\dfrac{\frac{1}{2}-\frac{3}{8}\times\frac{1}{2}}{1-\frac{1}{2}\times\frac{1}{2}}$	1M	For numerator
$=\dfrac{5}{12}$	1A	
	(7)	

Very good.
Nevertheless, some candidates were not familiar with the operations of complement and intersection of events.

9. (2010 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore 1=P(A)+b-c$	1A	For $P(A\cup B)=1$
i.e. $P(A)=1+c-b$	1A	
(b) (i) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$		
$\dfrac{1}{2}=\dfrac{c}{b}$	1M	For conditional probability
$b=2c$ -------------------- (1)	1A	For either one
$P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$		
$\dfrac{2}{3}=\dfrac{c}{1+c-b}$		
$c=2-2b$ -------------------- (2)		For either one
Solving (1) and (2), we have $b=0.8$ and $c=0.4$.	1A+1A	
(ii) $P(A)P(B)=(0.6)(0.8)\neq 0.4=P(A\cap B)$		
Alternative Solution 1: $P(A\mid B)=0.5\neq 0.6=P(A)$		
Alternative Solution 2: $P(B\mid A)=\dfrac{2}{3}\neq 0.8=P(B)$		
Hence the events A and B are not independent.	1	Follow through
	(7)	

Poor. Many candidates were not familiar with the basic concept of exhaustive events. The definition and concept of independent events were also not well mastered.

10. (2009 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore \dfrac{5}{12}=a+\dfrac{1}{4}-P(A\cap B)$	1A	
i.e. $P(A\cap B)=a-\dfrac{1}{6}$	1A	
(b) $P(B')P(A\mid B')=P(A\cap B')$		
$[1-P(B)]\cdot P(A\mid B')=P(A)-P(A\cap B)$		OR $\cdots=P(A\cup B)-P(B)$
$\therefore \left(1-\dfrac{1}{4}\right)k=a-\left(a-\dfrac{1}{6}\right)$	1M+1M+1M	OR $\cdots=\dfrac{5}{12}-\dfrac{1}{4}$
i.e. $k=\dfrac{2}{9}$	1A	
(c) Since A and B are independent, $P(A\cap B)=P(A)\times P(B)$.		
$\therefore a-\dfrac{1}{6}=a\times\dfrac{1}{4}$	1M	
$a=\dfrac{2}{9}$	1A	
Alternative Solution: Since A and B are independent, $P(A\mid B')=P(A)$.		
$\therefore a=k$	1M	
$=\dfrac{2}{9}$	1A	
	(8)	

Very good. Most candidates were able to master the basic rules of probability.

11. (2008 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $P(A\cap B)=P(A\mid B)\cdot P(B)$		
$=\dfrac{k}{6}$	1A	
(b) $\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore \dfrac{9}{20}=\dfrac{1}{5}+k-\dfrac{k}{6}$	1M	
i.e. $k=\dfrac{3}{10}$	1A	
(c) $P(A'\cap B)=P(B)-P(A\cap B)$		
$=\left(\dfrac{3}{10}\right)-\dfrac{1}{6}\left(\dfrac{3}{10}\right)$	1M	
Alternative Solution $P(A'\cap B)=P(A\cup B)-P(A)$ $=\dfrac{9}{20}-\dfrac{1}{5}$	1M	
$=\dfrac{1}{4}$	1A	
(d) $P(A'\cap B')=1-P(A\cup B)$		
$=1-\dfrac{9}{20}$		
Alternative Solution 1 $P(A'\cap B')=P(A')-P(A'\cap B)$ $=\left(1-\dfrac{1}{5}\right)-\left(\dfrac{1}{4}\right)$		
Alternative Solution 2 $P(A'\cap B')=P(A')+P(B')-P(A'\cup B')$ $=P(A')+P(B')-P((A\cap B)')$ $=\left(1-\dfrac{1}{5}\right)+\left(1-\dfrac{3}{10}\right)-\left(1-\dfrac{1}{6}\cdot\dfrac{3}{10}\right)$		
$=\dfrac{11}{20}$		
$\neq 0$	1M	For $P(A'\cap B')\neq 0$
Hence A' and B' are not mutually exclusive.	1	Follow through
Alternative Solution $P(A')+P(B')=\left(1-\dfrac{1}{5}\right)+\left(1-\dfrac{3}{10}\right)$ $=\dfrac{3}{2}$		
$\neq P(A'\cup B')$ since $P(A'\cup B')\leq 1$	1M	For $P(A')+P(B')\neq P(A'\cup B')$
Hence A' and B' are not mutually exclusive.	1	Follow through
	(7)	

Very good, except part (d) where many candidates were not sure of the definition of "mutually exclusive" events and confused with that of "independent" events.

12. (2007 ASL-M&S Q5)

Solution	Marks / Remarks
(a) $P(A'\cap B)$ $=P(B\mid A')P(A')$ $=0.3(1-a)$	1M (either one)
$P(A'\cap B)$ $=P(A'\mid B)P(B)$ $=0.6b$	(either one)
Hence, we have $0.6b=0.3(1-a)$.	
Thus, we have $a+2b=1$.	1
(b) $P(A\cap B')$ $=P(B'\mid A)P(A)$ $=0.7a$	(either one)
$P(A\cup B')$ $=1-P(A'\cap B)$ $=1-0.6b$	1M for complementary events
Note that $P(A\cup B')=P(A)+P(B')-P(A\cap B')$.	
Hence, we have $1-0.6b=a+(1-b)-0.7a$	1M
So, we have $3a=4b$.	
Solving $a+2b=1$ and $3a=4b$, we have $a=0.4$ and $b=0.3$.	1A for both correct
$P(A\cap B')$ $=P(B'\mid A)P(A)$ $=0.7a$	(either one)
$P(A\cup B')$ $=1-P(A'\cap B)$ $=1-0.3(1-a)$ $=0.7+0.3a$	1M for complementary events
Note that $P(A\cup B')=P(A)+P(B')-P(A\cap B')$.	
Hence, we have $0.7+0.3a=a+1-b-0.7a$	1M
So, we have $b=0.3$.	
By (a), we have $a+2(0.3)=1$.	
Thus, we have $a=0.4$.	1A (both correct)
(c) Since $P(A\cap B)=P(A)-P(A\cap B')$, $P(A)=0.4$ and $P(A\cap B')=0.28$, we have $P(A\cap B)=0.12=(0.4)(0.3)=P(A)P(B)$.	1M for relating $P(A\cap B)$ and $P(A)P(B)$
Thus, A and B are independent events.	1A f.t.
Since $P(A)=a$, we have $P(A\cap B)=P(A)-P(A\cap B')=a-0.7a=0.3a$. With the help of $P(B)=0.3$, we have $P(A\cap B)=P(A)P(B)$.	1M for relating $P(A\cap B)$ and $P(A)P(B)$
Thus, A and B are independent events.	1A f.t.
Since $P(A'\mid B)=0.6$, we have $P(A\mid B)=1-P(A'\mid B)=1-0.6=0.4$. With the help of $P(A)=0.4$, we have $P(A\mid B)=P(A)$.	1M for relating $P(A\mid B)$ and $P(A)$
Thus, A and B are independent events.	1A f.t.
	(7)

Fair. Some candidates could not apply the laws of probability especially when complementary events are also involved.

13. (2006 ASL-M&S Q5)

Solution	Marks
(a) $P(A\mid B')=\frac{P(A\cap B')}{P(B')}$	
$0.5=\frac{P(A\cap B')}{1-b}$	1M
$P(A\cap B')=0.5(1-b)$	1A accept $0.5-0.5b$
$P(A)$ $=P(A\cap B)+P(A\cap B')$	
$=0.2+0.5(1-b)$	1M
$=0.7-0.5b$	1A
(b) $P(A\cap B)=P(A)P(B)$	
$0.2=(0.7-0.5b)b$	1M for using (a) + 1M for using independence
$5b^2-7b+2=0$	
$b=0.4$ or $b=1$ (rejected)	1A
Thus, we have $b=0.4$.	
Since A and B are independent events, we have $P(A\cap B)=P(A)P(B)$. So, we have $P(A\mid B')P(B')=P(A\cap B')=P(A)-P(A)P(B)=P(A)P(B')$. Since $P(B')\neq 0$, we have $P(A\mid B')=P(A)$.	
By (a), we have $0.5=0.7-0.5b$. Therefore, we have $0.5b=0.2$.	1M for using (a) + 1M for using independence
Thus, we have $b=0.4$.	1A
	----------(7)

Good. Some candidates confused the definition of 'mutually exclusive events' with the definition of 'independent events'.

14. (2005 ASL-M&S Q5)

Solution	Marks
(a) $P(A\cap B')$ $=P(A\mid B')P(B')$ $=(\frac{5}{12})(1-\frac{2}{5})$	1M can be absorbed (either one)
$=\frac{1}{4}$	1A (either one)
(b) $P(A'\cap B)$ $=P(B\mid A')P(A')$	
$=(\frac{8}{15})(1-a)$	1A or equivalent
(c) $P(A'\cup B)$ $=1-P(A\cap B')$	
$=1-\frac{1}{4}$ (by (a))	1M accept $P(A)+P(A'\cap B)=P(B)+P(A\cap B')$
$=\frac{3}{4}$	
Note that $P(A'\cup B)=P(A')+P(B)-P(A'\cap B)$.	
Hence, we have $\frac{3}{4}=(1-a)+\frac{2}{5}-(\frac{8}{15})(1-a)$ (by (b)) .	1M for using (b)
Thus, we have $a=\frac{1}{4}$.	1A
(d) $\because$ $P(A)=P(A\cap B)+P(A\cap B')$, $P(A)=\frac{1}{4}$ (by (c)) and $P(A\cap B')=\frac{1}{4}$ (by (a)) $\therefore$ $P(A\cap B)=0$	
Thus, A and B are mutually exclusive.	1A must show reasons
	----------(7)

Good. Some candidates confused the definition of 'mutually exclusive events' with the definition of 'independent events'.

15. (2004 ASL-M&S Q1)

(a) $P(A\cup B)$	
$=P(A)+P(B)-P(A\cap B)$	
$=0.75+0.8-k$	1M can be absorbed
$=1.55-k$	1A
(b) (i) $\because P(A\cup B)\le 1$	1M can be absorbed (either one)
$\therefore 1.55-k\le 1$ (by (a))	
Therefore, we have $k\ge 0.55$.	1 (either one)
$\because P(A\cap B)\le 1$	
$\therefore k\le 1$	
Thus, we have $0.55\le k\le 1$.	1
(ii) $P(A'\cup B')$	
$=1-P(A\cap B)$ (since $A'\cup B'$ is the complementary event of $A\cap B$)	
$=1-k$	1M accept $k=1-P(A'\cup B')$
$\le 1-0.55$ (by (b)(i))	
$=0.45$	1
Thus, we have $P(A'\cup B')\le 0.45$.	

$P(A')$	
$=1-P(A)$	
$=1-0.75$	1M (either one)
$=0.25$	
$P(B')$	
$=1-P(B)$	
$=1-0.8$	
$=0.2$	
$P(A'\cup B')$	
$=P(A')+P(B')-P(A'\cap B')$	
$=0.25+0.2-P(A'\cap B')$	
$=0.45-P(A'\cap B')$	
≤ 0.45 (since $P(A'\cap B')\ge 0$)	1

----------(7)

Good. Most candidates knew that any probability is between 0 and 1 but some could not state the fact that $k\le 1$.

16. (2003 ASL-M&S Q4)

(a) $P(A\cap B)=P(A)\,P(B\mid A)$	1M can be absorbed (either one)
$P(A\cap B)=0.4P(A)$	
$P(A\cap B)=0.4a$	1A
$P(A\cap B)=P(B)\,P(A\mid B)=0.5P(B)$	
$0.4a=0.5P(B)$	
$P(B)=\dfrac{0.4}{0.5}a$	
$P(B)=0.8a$	1A

$P(A\cap B)=P(A)\,P(B\mid A)$	
$P(A\cap B)=0.4P(A)$	1M can be absorbed
$P(A\cap B)=0.4a$	1A
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+P(B)-0.4a$	1M can be absorbed
$P(B)=0.84-0.6a$	1A

$P(A\cap B)=P(B)\,P(A\mid B)$	
$P(B)=2\,P(A\cap B)$	1M can be absorbed
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+2\,P(A\cap B)-P(A\cap B)$	1M can be absorbed
$P(A\cap B)=0.84-a$	1A accept $P(A\cap B)=1.8a-0.84$
$P(B)=1.68-2a$	1A accept $P(B)=0.8a$

(b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+0.8a-0.4a$ (by (a))	1M can be absorbed
$0.84=1.4a$	
$a=0.6$	1A

$P(A\cap B)=P(B)\,P(A\mid B)=0.5P(B)$	
$0.4a=0.5P(B)$	
$P(B)=\dfrac{0.4}{0.5}a$	
$P(B)=0.8a$	
$0.8a=0.84-0.6a$ (by (a))	
$1.4a=0.84$	
$a=0.6$	1A

$P(A\cap B)=P(A)\,P(B\mid A)$	
$P(A\cap B)=0.4\,P(A)$	
$0.84-a=0.4a$	
$a=0.6$	1A

(c) $\because$ $P(A)\,P(B)=(0.6)(0.8)(0.6)=0.288\ne 0.24=(0.4)(0.6)=P(A\cap B)$	1M
$\therefore$ A and B are not independent.	1A

$\because$ $P(A\mid B)=0.5\ne 0.6=P(A)$	1M
$\therefore$ A and B are not independent.	1A

----------(7)

Fair. Concepts of independent events and mutually exclusive events have to be strengthened. Some candidates were unable to distinguish the formulas for intersection and union under general and specific conditions.

17. (2001 ASL-M&S Q1)

Since $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	1M
and $P(A\cap B)=P(A)P(B)$	1M
$\therefore$ $0.7=0.4+P(B)-0.4\,P(B)$	1A
$P(B)=0.5$	1A

----------(4)

Tree Diagram, Conditional Probability and Bayes' Theorem

18. (2015 DSE-MATH-M1 Q2)

(a) The required probability

$$=\frac{\left(\frac{1}{6}\right)^3(3)}{\left(\frac{1}{6}\right)^3(3+3!+3+3)}$$ 1M+1M+1M (1M for $\left(\frac{1}{6}\right)^3$ + 1M for numerator + 1M for denominator)

$$=\frac{1}{5}$$ 1A

(b) The required probability

$$=\frac{\left(\frac{1}{5}\right)^3(3)}{\left(\frac{1}{5}\right)^3(3+3!+3+3)}$$ 1M

$$=\frac{1}{5}$$

Thus, the required probability will not change. 1A f.t.

----------(6)

(a)	Good. Some candidates were unable to count the correct number of relevant outcomes for the sum of 7, hence unable to work out the denominator of the required probability properly.
(b)	Fair. Although many candidates guessed correctly that the required conditional probability remains unchanged, they were unable to provide a mathematical argument to justify the guess.

19. (2015 DSE-MATH-M1 Q3)

(a) The required probability

$$=\frac{2}{7}p+\frac{5}{7}(1-p)$$ 1M for $rs+(1-r)(1-s)$

$$=\frac{5-3p}{7}$$ 1A

(b) (i) $\frac{5-3p}{7}=\frac{50}{91}$ 1M for using (a)

$p=\frac{5}{13}$ 1A

$p\approx 0.384615384$ r.t. 0.3846

$p\approx 0.3846$

(ii) The required probability

$$=\frac{\frac{2}{7}\left(1-\frac{5}{13}\right)}{1-\frac{50}{91}}$$ 1M for numerator using (b)(i)

$$=\frac{16}{41}$$ 1A

≈ 0.3902439024 r.t. 0.3902

≈ 0.3902

----------(6)

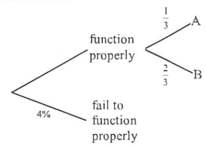
(a)	Good. Some candidates did not simplify the answer and some candidates failed to give an answer as an expression in terms of p.
(b) (i)	Very good. Most candidates were able to set up an equation by using the result of (a).
(ii)	Good. Some candidates failed to use the result of (b)(i) to find the required probability.

20. (2014 DSE-MATH-M1 Q8)

(a) P(the selected microwave oven is produced by line B and functions properly)

$=(1-0.04)\times\frac{2}{3}$

$=0.64$ 1A



(b) $P(A)$ P(functions properly | A) = P(functions properly) P(A | functions properly)

$P(A)\,(1-0.02)=(1-0.04)\left(1-\frac{2}{3}\right)$ 1M

$P(A)=\frac{16}{49}$ 1A OR 0.3265

(c) P(functions properly | B) $=\frac{0.64}{1-\frac{16}{49}}$ 1M

$=\frac{784}{825}$ 1A OR 0.9503

(5)

(a)	Very good.
(b)	Poor. Most candidates did not understand the meanings of the conditional probabilities given.
(c)	Fair. Many candidates used correct methods but obtained wrong answers, because their answers to (a) or (b) were wrong.

21. (2013 DSE-MATH-M1 Q8)

(a) P(get a prize | Mabel) $= 0.6 + 0.4 \times 0.6 + 0.4^2 \times 0.6$	1M	OR $1-(1-0.6)^3$
$= 0.936$	1A	OR $(0.6)^3 + C_1^3(0.6)^2(0.4) + C_1^3(0.6)(0.4)^2$
(b) P(get a prize | Owen) $= 0.5 + 0.5 \times 0.5 + 0.5^2 \times 0.5$ $= 0.875$		OR $1-(1-0.5)^3$
$\therefore$ P(win) $= 0.7 \times 0.936 + 0.3 \times 0.875$	1M	
$= 0.9177$	1A	
(c) P(Owen | does not get a prize) $= \frac{0.3 \times (1-0.875)}{1-0.9177}$	1M	OR $\frac{0.3 \times (1-0.5)^3}{1-0.9177}$
$= \frac{375}{823}$	1A	OR 0.4557
	(6)	

(a)	Fair. Many candidates actually found P(Mabel is selected and Team A gets a prize) instead of the required probability.
(b)	Good.
(c)	Satisfactory. Quite a number of candidates were weak in applying Bayes' theorem.

22. (PP DSE-MATH-M1 Q7)

(a) P(a player is rewarded) $= \frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{1}{5}$ $= 0.3$	1A	
(b) P(both players are rewarded | one player is rewarded) $= \frac{0.3 \times 0.3}{0.3 \times 0.3 + 0.3 \times 0.7 \times 2}$	1M	OR $\frac{0.3 \times 0.3}{1 - 0.7 \times 0.7}$
$= \frac{3}{17}$	1A	OR 0.1765
(c) E(no. of players having drawn a blue ball from A) $= 60 \times \frac{\frac{1}{2} \times \frac{2}{5}}{0.3}$	1M	
$= 40$	1A	
	(5)	

(a)	甚佳。
(b)	尚可。部分學生忘記條件性概率的公式。
(c)	平平。大部分學生不懂如何求期望值。

23. (SAMPLE DSE-MATH-M1 Q4)

(a) The required probability $= 0.9 \times 0.05 + 0.1 \times (1-0.35)$	1M	For $p_1 p_2 + (1-p_1) p_3$
$= 0.11$	1A	
(b) The required probability $= \frac{0.1 \times (1-0.35)}{0.11}$	1M	For denominator using (a)
$= \frac{13}{22}$	1A	OR 0.5909
	(4)	

24. (2002 ASL-M&S Q5)

(a) The required probability $= \frac{6}{8} \times \frac{1}{3} + \frac{4}{7} \times \frac{1}{3} + \frac{2}{7} \times \frac{1}{3}$	1A
$= \frac{15}{28}$ (≈ 0.5357)	1A $a-1$ for r.t. 0.536
(b) P(the boy is selected from group A | a boy is selected) $= \frac{\frac{6}{8} \times \frac{1}{3}}{\frac{15}{28}}$	1M + 1A (1A for numerator)
$= \frac{7}{15}$ (≈ 0.4667)	1A $a-1$ for r.t. 0.467
	----------(5)

25. (2001 ASL-M&S Q7)

(a) The required probability $= \frac{0.39 \times 0.58}{0.48 \times 0.65 + 0.39 \times 0.58 + 0.13 \times 0.5}$	1A numerator 1A denominator
$= 0.375$ (p)	1A
(b) The required probability $= C_2^5 (0.375)^2 (1-0.375)^3$	1M binomial, for any p 1M $C_2^5 p^2 (1-p)^3$, for p in (a)
$= 0.3433$	1A $a-1$ for r.t. 0.343
	---------(6)

26. (2000 ASL-M&S Q8)

(a) The probability of a customer winning a prize in 1 trial $= 0.3\left(\frac{5}{9}\right) + 0.7\left(\frac{5}{6}\right)$	1A
$= 0.75$	1A
(b) Let x and y be the probabilities of generating games A and B respectively. Then $\frac{5}{9}x + \frac{5}{6}y = \frac{2}{3}$	1M
and $x + y = 1$	1M
$\therefore$ $\frac{5}{9}x + \frac{5}{6}(1-x) = \frac{2}{3}$ $\frac{5}{6} - \frac{5}{18}x = \frac{2}{3}$	
$x = \frac{3}{5}$ (or 0.6)	1A or $y = \frac{2}{5}$, $y = 0.4$
$\therefore$ The probabilities of generating game A and game B are $\frac{3}{5}$ (or 0.6) and $\frac{2}{5}$ (or 0.4) respectively.	1A
	-----(6)

27. (1999 ASL-M&S Q7)

Let E_X be the event that cable X is operative,
E_Y be the event that cable Y is operative,
E_Z be the event that cable Z is operative, and
F be the event that A and B are not able to make contact.

Solution	Marks	Remarks
(a) (i) $P(E_X' \cap E_Z') = (0.015)(0.030)$ $= 0.00045$ (p_1)	1A	$a-1$ for r.t. 0.0005 (method must be shown)
(ii) $P(E_X' \cap E_Y' \cap E_Z') = (0.015)(0.025)(0.030)$ $= 0.00001125$ (p_2)	1A	$a-1$ for r.t. 0.0000 (method must be shown)
(iii) $P(F) = P(E_X' \cap E_Z') + P(E_Y' \cap E_Z') - P(E_X' \cap E_Y' \cap E_Z')$ $= 0.00045 + (0.025)(0.030) - 0.00001125$	1M	$p_1 + (0.025)(0.030) - p_2$
$= 0.00118875$ ≈ 0.001189 (p_3)	1A	r.t. 0.001189, $a-1$ for r.t. 0.0012 (method must be shown)
(b) $P(F \mid E_X') = P(E_Z')$ $= 0.030$ (p_4)	1A	or 0.03
(c) $P(E_X' \mid F) = \dfrac{P(E_X')P(F \mid E_X')}{P(F)}$ $= \dfrac{(0.015)(0.030)}{0.00118875}$	1M	$\dfrac{(0.015)p_4}{p_3}$
≈ 0.3785489 ≈ 0.3785	1A (7)	r.t. 0.3785

Remark on (a)(iii):

Note that A and B are not able to make contact when **Z is not operative** and **either X or Y is not cooperative**.

$$F = E_Z' \cap (E_X' \cup E_Y') = (E_Z' \cap E_X') \cup (E_Z' \cap E_Y')$$

So we have

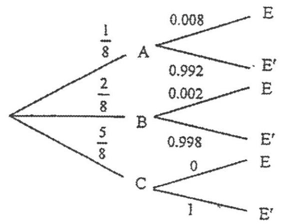
$$P(F) = P(E_Z' \cap E_X') + P(E_Z' \cap E_Y') - P(E_Z' \cap E_X' \cap E_Y')$$

Alternatively, we have

$$\begin{aligned} P(F) &= P(E_Z' \cap (E_X' \cup E_Y')) \\ &= P(E_Z') \times P(E_X' \cup E_Y') \\ &= 0.030 \times (P(E_X') + P(E_Y') - P(E_X' \cap E')) \\ &= 0.030 \times (0.015 + 0.025 - 0.015 \times 0.025) \\ &= 0.00118875 \end{aligned}$$

28. (1998 ASL-M&S Q6)

Let E be the event that an ice-cream bar is contaminated.



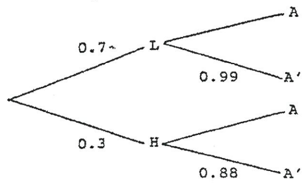
Solution	Marks	Remarks
(tree diagram)	1A	For the tree diagram or all parts in (a) being correct
(a) (i) $P(A)P(E' \mid A) = \frac{1}{8} \times 0.992$ $= 0.124$	1A	(p_1)
(ii) $P(E') = P(A)P(E' \mid A) + P(B)P(E' \mid B) + P(C)P(E' \mid C)$ $= 0.124 + \frac{2}{8} \times 0.998 + \frac{5}{8} \times 1$	1M	for $p_1 + \frac{2}{8}p_2 + \frac{5}{8}p_3$
$= 0.9985$	1A	$a-1$ for r.t. 0.999
(b) $P(A \mid E) = \dfrac{P(A)P(E \mid A)}{1 - P(E')}$ $= \dfrac{\frac{1}{8} \times 0.008}{1 - 0.9985}$ $\left(\text{or } \dfrac{\frac{1}{8} \times 0.8\%}{\frac{1}{8} \times 0.8\% + \frac{2}{8} \times 0.2\%}\right)$	1M	
≈ 0.6667	1A (6)	$a-1$ for r.t. 0.667

29. (1996 ASL-M&S Q6)

Solution	Marks	Remarks
(a) The probability that a lot will be accepted $= (0.5)[(0.99)^2 + (0.96)^2]$	1M+1M	1M for $(0.99)^2 + (0.96)^2$; 1M for $0.5p$
≈ 0.9509 (or 0.95085)	1A	
(b) The probability that a lot came from supplier A $= \dfrac{(0.5)(0.96)^2}{0.95085}$	1A+1M	1A for the numerator; 1M for the denominator
≈ 0.4846	1A (6)	

30. (1995 ASL-M&S Q5)

(a) Let A' be the event of having no accident within a year.



[1A]

> **Alternatively,**
> $P(L) = 0.7$, $P(H) = 0.3$
> $P(A'|L) = 0.99$, $P(A'|H) = 0.88$ [1A]

$P(H|A') = \dfrac{P(A' \cap H)}{P(A')}$

$= \dfrac{P(A'|H)P(H)}{P(A'|H)P(H) + P(A'|L)P(L)}$

$= \dfrac{0.88 \times 0.3}{0.88 \times 0.3 + 0.99 \times 0.7}$ [1M+1M+1A] 1M for the numerator; 1M for the denominator

≈ 0.2759 (or $\dfrac{8}{29}$) [1A]

(b) $P(L|A') = 1 - P(H|A')$

$= 1 - 0.2759$ [1M]

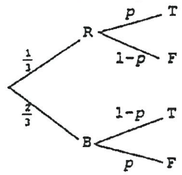
$= 0.7241$ (or $\dfrac{21}{29}$) [1A]

> **Alternatively,**
> $P(L|A') = \dfrac{P(A'|L)P(L)}{P(A'|H)P(H) + P(A'|L)P(L)}$
>
> $= \dfrac{0.99 \times 0.7}{0.88 \times 0.3 + 0.99 \times 0.7}$ [1M]
>
> $= 0.7241$ (or $\dfrac{21}{29}$) [1A]

(7)

31. (1994 ASL-M&S Q7)

(a) R: red card is drawn T: response 'True'
B: black card is drawn F: response 'False'
p: percentage of persons who are homosexual



[1A]

$\dfrac{790}{1200} = \dfrac{1}{3}p + \dfrac{2}{3}(1-p)$ [1M + 1A]

$\dfrac{790}{1200} = \dfrac{2}{3} - \dfrac{1}{3}p$

$p = 2.5\%$ [1A]

> **Alternatively**
> Let x, y be the no. of interviewees who are homosexual and not homosexual respectively, then
>
> $\frac{1}{3}x + \frac{2}{3}y = 790$ [1A]
>
> $\frac{2}{3}x + \frac{1}{3}y = 410$ [1A]
>
> Solving the equations, we have
> $x = 30$, $y = 1170$ [1M] For reducing into one unknown
>
> $\therefore$ The percentage required $= \dfrac{30}{1200}$
> $= 2.5\%$ [1A]

(b) $P(R|T) = \dfrac{P(T|R)P(R)}{P(T)}$

$= \dfrac{(0.025)(\frac{1}{3})}{\frac{79}{120}}$ [1M + 1A]

$= 0.0127$ (or $\dfrac{1}{79}$) [1A]

7

Section B

32. (2009 ASL-M&S Q10)

Solution	Marks	Remarks
(a) (i) The required probability $=\dfrac{1}{C_3^9}=\dfrac{1}{84}$	1A	OR 0.0119
(ii) The required probability $=\dfrac{C_3^4}{C_3^9}=\dfrac{1}{21}$	1A	OR 0.0476
	(2)	
(b) $(1-p)\left(\dfrac{1}{84}\right)+p\left(\dfrac{1}{21}\right)\le\dfrac{1}{60}$	1M+1A	(pp-1) for using "="
$5-5p+20p\le 7$		
$p\le\dfrac{2}{15}$		
Hence, the largest value of p should be $\dfrac{2}{15}$.	1A	OR 0.1333
	(3)	
(c) (i) The required probability $=\dfrac{2}{15}\cdot\dfrac{1}{C_4^9}$	1M	For using (b)
$=\dfrac{1}{945}$	1A	OR 0.0011
(ii) The required probability $=\left(1-\dfrac{2}{15}\right)\cdot\dfrac{C_3^3C_1^6}{C_4^9}+\dfrac{2}{15}\cdot\dfrac{C_3^4C_1^5}{C_4^9}$	1M+1A	
$=\dfrac{59}{945}$	1A	OR 0.0624
(iii) The probability of exactly 2 logos are found on 1 card		
$=\left(1-\dfrac{2}{15}\right)\cdot\dfrac{C_2^3C_2^6}{C_4^9}+\dfrac{2}{15}\cdot\dfrac{C_2^4C_2^5}{C_4^9}$	1M	
$=\dfrac{47}{126}$	1A	OR 0.3730
Hence, the required probability		
$=\left(\dfrac{1}{945}+\dfrac{59}{945}\right)^2+2\left(\dfrac{1}{945}+\dfrac{59}{945}\right)\left(1-\dfrac{1}{945}-\dfrac{59}{945}\right)+\left(\dfrac{47}{126}\right)^2$	1M+1A	
$=\dfrac{1387}{5292}$	1A	OR 0.2621
	(10)	

(a)	(i)		Very good.
	(ii)		Very good.
(b)			Good. Some candidates were unfamiliar with handling inequalities.
(c)	(i)		Fair. Many candidates could not analyse the situation and in fact a simple tree diagram would be helpful.
	(ii)		Poor. Many candidates had difficulties in counting and exhausting the relevant cases.
	(iii)		Very poor. Again many candidates encountered difficulties in counting the relevant cases when the situation was complicated in having two cards.

33. (2010 ASL-M&S Q11)

Solution	Marks	Remarks
(a) (i) P(getting 3 points | Gold VIP)		
$=2(0.4)(0.3)$		
$=0.24$	1A	
(ii) P(getting 3 points)		
$=(0.25)(0.2)+(0.6)(0.24)+(0.15)(0.4)^3$	1M	
$=0.2036$	1A	
(iii) P(Gold VIP | 3 points are obtained)		
$=\dfrac{(0.6)(0.24)}{0.2036}$	1M	
≈ 0.7073	1A	
	(5)	
(b) P(\$ 20 cash rebate)		
$=(0.25)(0.4)+[(0.25)(0.3)+(0.6)(0.4)^2]+0.2036$	1M	
$=0.4746$	1A	
	(2)	
(c) (i) P(getting 10 points)		
$=(0.15)[3(0.3)(0.1)^2+3(0.2)^2(0.1)]$		
$=0.00315$	1A	
(ii) P(\$ 200 cash rebate)		
$=0.00315+(0.15)[3(0.2)(0.1)^2+(0.1)^3]$	1M	
$=0.0042$	1A	
	(3)	
(d) (i) Expected cash rebate using the online game		
$=\$\{0.7[20(0.4746)+50(1-0.4746-0.0042)+200(0.0042)]+(1-0.7)(0)\}$	1M	
$=\$25.4744$		
The <u>minimum</u> cash rebate under the 4% direct cash rebate plan		
$>\$[(0.25)(400)+(0.6)(800)+(0.15)(1000)](0.04)$	1M	
$=\$29.2$		
Since $29.2>25.4744$, Winnie is agreed with.	1	Follow through
(ii) The <u>maximum</u> cash rebate under the 2% direct cash rebate plan		
$=\$[(0.25)(800)+(0.6)(1000)+(0.15)(3000)](0.02)$	1M	
$=\$25$		
Since $25<25.4744$, John is agreed with.	1	Follow through
	(5)	

(a)		Very good.
(b)		Good. Some candidates did not exhaust all possible cases in their counting.
(c)		Fair. Many candidates made some errors in counting the relevant events.
(d)		Poor. Many candidates were not familiar with expected values.

34. (2000 ASL-M&S Q13)

(a) Possible teams: B_1G_1, B_1G_2, B_2G_1 and B_2G_2. — 1A

(b) The probability that B_1G_1 can enter the second round of the contest
$= 0.9 \times 0.8$
$= 0.72$ — 1A

(c) Probability required $= \frac{1}{4}(0.9\times0.8+0.9\times0.6+0.7\times0.8+0.7\times0.6)$ — 1M 4 cases
$= 0.56$ — 1A

(d) Suppose B_1G_1 and B_2G_2 are formed to represent the school.

(i) The probability that exactly one team can enter the second round
$= (0.9\times0.8)(1-0.7\times0.6)+(0.7\times0.6)(1-0.9\times0.8)$ — {1A probability of 1 case; 1M summation of 2 cases}
or $1-(0.9\times0.8)(0.7\times0.6)-(1-0.9\times0.8)(1-0.7\times0.6)$
$= 0.5352$ — 1A

(ii) The probability that at least one team can enter the second round
$= 0.5352 + 0.9\times0.8\times0.7\times0.6$ — 1M
or $1-(1-0.9\times0.8)(1-0.7\times0.6)$
or $0.9\times0.8+0.7\times0.6-0.9\times0.8\times0.7\times0.6$
$= 0.8376$ — 1A

(e) (i) If the two teams are formed randomly, the probability that exactly one team can enter the second round
$= \frac{1}{2}\times0.5352+\frac{1}{2}[(0.9\times0.6)(1-0.7\times0.8)+(0.7\times0.8)(1-0.9\times0.6)]$ — {1M the combination B_1G_2, B_2G_1; 1M multiplying by $\frac{1}{2}$}
$= \frac{1}{2}(0.5352+0.4952)$
$= 0.5152$ — 1A

(ii) If B_1G_2 and B_2G_1 are formed to represent the school, the probability that at least one team can enter the second round
$= 0.4952 + 0.9\times0.8\times0.7\times0.6$ — 1M
or $1-(1-0.9\times0.6)(1-0.7\times0.8)$
or $0.9\times0.6+0.7\times0.8-0.9\times0.6\times0.7\times0.8$
$= 0.7976$ — 1A

From (d)(ii), the combination B_1G_1 and B_2G_2 will have a better chance of having at least one team that can enter the second round of the contest. — 1M

8. Discrete Random Variables

Learning Unit	Learning Objective
Statistics Area	
Binomial, Geometric and Poisson Distributions	
12. Discrete random variables	12.1 recognise the concept of a discrete random variable
13. Probability distribution, expectation and variance	13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae 13.2 recognise the concepts of expectation $E(X)$ and variance $\text{Var}(X)$ and use them to solve simple problems 13.3 use the formulae $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2\ \text{Var}(X)$ to solve simple problems

Section A

1. The table below shows the probability distribution of a discrete random variable Y, where m and p are constants:

y	-2	2	m
$P(Y=y)$	p	0.25	0.5

(a) Prove that $\text{Var}(Y)=0.25m^2+2$.

(b) If $\text{Var}(2Y-1)=8E(2Y-1)$, find m.

(7 marks) (2018 DSE-MATH-M1 Q4)

2. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	0	2	4	5	8	9
$P(X=x)$	k^2	0.16	0.18	0.3	k	0.12

Find

(a) k,

(b) $E(X)$,

(c) $\text{Var}(2-3X)$.

(6 marks) (2017 DSE-MATH-M1 Q1)

3. The table below shows the probability distribution of a discrete random variable X, where a and b are constants:

x	2	3	5	7	9
$P(X=x)$	0.08	0.15	a	0.45	b

It is given that $E(X)=5.64$. Find

(a) a and b,

(b) $E((6-5X)^2)$ and $\text{Var}(6-5X)$.

(6 marks) (2015 DSE-MATH-M1 Q1)

4. Let X be a discrete random variable with probability function as shown in the following table.

x	k	0	4	6
$P(X=x)$	0.1	0.2	0.3	0.4

It is given that $E(X)=3.4$.

(a) Find the value of k.

(b) Find $\text{Var}(3-4X)$.

(c) Let G be the event that $X<4$ and H be the event that $X\geq -1$. Find $P(G\cap H)$.

(5 marks) (2014 DSE-MATH-M1 Q6)

5. Let X and Y be two independent discrete random variables with their respective probability distributions shown as follows:

x	0	1	3	5	7
$P(X=x)$	0.2	0.3	0.3	0.1	0.1

y	1	2	4	m
$P(Y=y)$	0.4	0.3	0.2	0.1

Suppose that $E(Y)=2.4$.

(a) Find the value of m.

(b) Let A be the event that $X+Y\leq 2$ and B be the event that $X=0$.

(i) Find $P(A)$.

(ii) Are events A and B independent? Justify your answer.

(5 marks) (2013 DSE-MATH-M1 Q7)

6. Let X be a discrete random variable with probability function shown below:

x	1	3	4	6	9	13
$P(X=x)$	0.1	a	0.25	0.15	b	0.05

where a and b are constants. It is known that $E(X)=5.5$.

(a) Find the values of a and b.

(b) Let F be the event that $X\geq 4$ and G be the event that $X<8$.

(i) Find $P(F\cap G)$.

(ii) Are F and G independent events? Justify your answer.

(6 marks) (2012 DSE-MATH-M1 Q8)

7. The random variable X has probability distribution $P(X=x)$ for $x=1$, 2 and 3 as shown in the following table.

x	1	2	3
$P(X=x)$	0.1	0.6	0.3

Calculate

(a) $E(X)$,

(b) $\text{Var}(3-2X)$.

(5 marks) (SAMPLE DSE-MATH-M1 Q7)

NEW

Out of syllabus

8. Discrete Random Variable

1. (2018 DSE-MATH-M1 Q4)

2. (2017 DSE-MATH-M1 Q1)

(a) $k^2+0.16+0.18+0.3+k+0.12=1$	1M	
$k^2+k-0.24=0$		
$k=0.2$ or $k=-1.2$ (rejected)	1A	
Thus, we have $k=0.2$.		
(b) $\mathrm{E}(X)$		
$=0(0.04)+2(0.16)+4(0.18)+5(0.3)+8(0.2)+9(0.12)$	1M	
$=5.22$	1A	
(c) $\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9((0-5.22)^2(0.04)+(2-5.22)^2(0.16)+(4-5.22)^2(0.18)$		
$+(5-5.22)^2(0.3)+(8-5.22)^2(0.2)+(9-5.22)^2(0.12))$	1M	
$=56.6244$	1A	
$\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9(\mathrm{E}(X^2)-(\mathrm{E}(X))^2)$		
$=9(33.54-(5.22)^2)$	1M	
$=56.6244$	1A	
	----------(6)	

(a)	Very good. About 98% of the candidates were able to find the value of k by setting up a quadratic equation.
(b)	Very good. Over 90% of the candidates were able to find the value of $\mathrm{E}(X)$.
(c)	Very good. Most candidates were able to find the value of $\mathrm{Var}(2-3X)$.

3. (2015 DSE-MATH-M1 Q1)

(a) $0.08+0.15+a+0.45+b=1$ — 1M — either one
$2(0.08)+3(0.15)+5a+7(0.45)+9b=5.64$
Solving, we have $a=0.25$ and $b=0.07$. — 1A — for both

(b) $\mathrm{E}((6-5X)^2)$
$=\mathrm{E}(36-60X+25X^2)$
$=36-60\mathrm{E}(X)+25\mathrm{E}(X^2)$ — 1M
$=36-60(5.64)+25(35.64)$
$=588.6$ — 1A

$\mathrm{Var}(6-5X)$
$=\mathrm{E}((6-5X)^2)-(\mathrm{E}(6-5X))^2$
$=\mathrm{E}((6-5X)^2)-(6-5\mathrm{E}(X))^2$ — 1M — accept $(-5)^2\mathrm{Var}(X)$
$=588.6-(6-5(5.64))^2$
$=95.76$ — 1A

----------(6)

(a)	Very good. Most candidates were able to find the values of a and b by setting up two equations involving them.
(b)	Good. Many candidates were able to find the value of $\mathrm{Var}(6-5X)$ while some candidates wrongly found the value of $(\mathrm{E}(6-5X))^2$ instead of $\mathrm{E}((6-5X)^2)$.

4. (2014 DSE-MATH-M1 Q6)

(a) $0.1k+0.2(0)+0.3(4)+0.4(6)=3.4$ — 1M
$k=-2$ — 1A

(b) $\mathrm{Var}(3-4X)=16\mathrm{Var}(X)$ — 1M
$=16[\mathrm{E}(X^2)-\mathrm{E}(X)^2]$
$=16[0.1(-2)^2+0.2(0)^2+0.3(4)^2+0.4(6)^2-3.4^2]$

Alternative Solution

x	-2	0	4	6
$3-4x$	11	3	-13	-21
$\mathrm{P}(X=x)$	0.1	0.2	0.3	0.4

$\mathrm{E}(3-4X)=0.1(11)+0.2(3)+0.3(-13)+0.4(-21)$ — 1M — OR $3-4(3.4)$
$=-10.6$
$\mathrm{Var}(3-4X)=0.1(11+10.6)^2+0.2(3+10.6)^2+0.3(-13+10.6)^2+0.4(-21+10.6)^2$

$=128.64$ — 1A

(c) $\mathrm{P}(G\cap H)=\mathrm{P}(-1\le X<4)$
$=\mathrm{P}(X=0)$
$=0.2$ — 1A

(5)

(a)	Excellent.
(b)	Very good. Some candidates equated $\mathrm{Var}(3-4X)$ to $3^2\mathrm{Var}(X)$ or $3-4\mathrm{Var}(X)$.
(c)	Good.

5. (2013 DSE-MATH-M1 Q7)

Solution	Marks	Remarks
(a) $E(Y)=1\times0.4+2\times0.3+4\times0.2+m\times0.1=2.4$ $\therefore\ m=6$	1A	
(b) (i) $P(A)=P(X=0,Y=1)+P(X=0,Y=2)+P(X=1,Y=1)$ $=0.2\times0.4+0.2\times0.3+0.3\times0.4$ $=0.26$	1M 1A	
(ii) $P(A\cap B)=P(X=0,Y=1)+P(X=0,Y=2)$ $=0.2\times0.4+0.2\times0.3$ $=0.14$ $P(A)P(B)=0.26\times0.2$ $=0.052$ $\neq P(A\cap B)$	1A	
Alternative Solution $P(A\mid B)=P(Y=1)+P(Y=2)$ $=0.4+0.3$ $=0.7$ $\neq P(A)$ by (i)	1A	
Thus, A and B are not independent.	1A	Follow through
	(5)	

(a)		Excellent.
(b)	(i)	Good. Mistakes were occasionally found in computations.
	(ii)	Fair. A lot of candidates thought that the independence of two events A and B could be verified by checking $P(A\cap B)=0$. Among those who found correct values of related probabilities, some did not mention $P(A\cap B)\neq P(A)\cdot P(B)$ as the reason to make conclusion, while some made a wrong conclusion that 'A and B are independent'.

6. (2012 DSE-MATH-M1 Q8)

Solution	Marks	Remarks
(a) $P(X=1)+P(X=3)+\cdots+P(X=13)=1$ $0.1+a+0.25+0.15+b+0.05=1$ $a+b=0.45$ ---------- (1)	1M	
$E(X)=5.5$ $1\times0.1+3a+4\times0.25+6\times0.15+9b+13\times0.05=5.5$ $a+3b=0.95$ ---------- (2)	1M	
Solving (1) and (2), we get $a=0.2$ and $b=0.25$.	1A	For both
(b) (i) $P(F\cap G)=0.25+0.15$ $=0.4$	1A	
(ii) $P(F)\times P(G)=(0.25+0.15+0.25+0.05)(0.1+0.2+0.25+0.15)$ $=0.49$ $\neq P(F\cap G)$	1A	
Alternative Solution 1 $P(F\mid G)=\dfrac{P(F\cap G)}{P(G)}$ $=\dfrac{0.4}{0.1+0.2+0.25+0.15}$ ≈0.571428571 $P(F)=0.25+0.15+0.25+0.05$ $=0.7$ $\neq P(F\mid G)$	1A	
Alternative Solution 2 $P(G\mid F)=\dfrac{P(F\cap G)}{P(F)}$ $=\dfrac{0.4}{0.25+0.15+0.25+0.05}$ ≈0.571428571 $P(G)=0.1+0.2+0.25+0.15$ $=0.7$ $\neq P(G\mid F)$	1A	
Hence, F and G are not independent.	1	
	(6)	

(a)	Excellent.
(b)	Satisfactory. Quite a number of candidates did not understand the concept of independence — some calculated $P(F\cap G)$ using $P(F)\times P(G)$ and some mixed up independent events with mutually exclusive events.

7. (SAMPLE DSE-MATH-M1 Q7)

(a) $E(X) = 1(0.1) + 2(0.6) + 3(0.3)$	
$= 2.2$	1A
(b) $Var(X) = [1^2(0.1) + 2^2(0.6) + 3^2(0.3)] - 2.2^2$	1M
$= 0.36$	1A
$\therefore$ $Var(3-2X) = 2^2 Var(X)$	1M
$= 1.44$	1A
<u>Alternative Solution</u>	
$E(3-2X) = 3 - 2E(X)$	1M
$= -1.4$	1A
$Var(3-2X) = (3-2\cdot1+1.4)^2(0.1) + (3-2\cdot2+1.4)^2(0.6) + (3-2\cdot3+1.4)^2(0.3)$	1M
$= 1.44$	1A
	(5)

9. Binomial, Geometric and Poisson Distributions

Learning Unit	Learning Objective
Statistics Area	
Binomial, Geometric and Poisson Distributions	
14. Binomial distribution	14.1 recognise the concept and properties of the binomial distribution 14.2 calculate probabilities involving the binomial distribution
15. Geometric distribution	15.1 recognise the concept and properties of the geometric distribution 15.2 calculate probabilities involving the geometric distribution
16. Poisson distribution	16.1 recognise the concept and properties of the Poisson distribution 16.2 calculate probabilities involving the Poisson distribution
17. Applications of binomial, geometric and Poisson distributions	17.1 use binomial, geometric and Poisson distributions to solve problems

Section A

1. Susan plays a game. In each trial of the game, her probability of winning a doll is 0.6 . Susan plays the game until she wins a doll.
   (a) Find the probability that Susan wins a doll at the 4th trial in the game.
   (b) If Susan cannot win a doll in k trials, then the probability that she wins a doll within 10 trials in the game is greater than 0.95 . Find the greatest value of k .
   (c) In each trial of the game, Susan has to pay \$15. Find the expected amount of money she has to pay to win a doll in the game.

(7 marks) (2017 DSE-MATH-M1 Q4)

2. A museum opens at 10:00 . The number of visitors entering the museum in a minute follows a Poisson distribution with a mean of 1.8 .
   (a) Write down the variance of the number of visitors entering the museum in a minute.
   (b) Find the probability that 3 visitors entered the museum in the first two minutes after the museum opens.
   (c) At 10:00 , only one gate at the entrance of the museum is opened. If in any two consecutive minutes, there are at least 4 visitors entering the museum in each minute, then a second gate will be opened. Find the probability that the second gate is just opened three minutes after the museum opens.

(7 marks) (2016 DSE-MATH-M1 Q3)

3. A manufacturer of brand B biscuits starts a promotion plan by giving one reward points card in each packet of biscuits. It is found that 75% of the packets of brand B biscuits contain 3-point cards and the rest contain 7-point cards. A total of 20 points or more can be exchanged for a gift coupon. John buys 4 packets of brand B biscuits and he opens them one by one.
   (a) Find the probability that John gets the first 7-point card when the 4th packet of brand B biscuits has been opened.
   (b) Find the probability that John can exchange for a gift coupon.
   (c) Given that John can exchange for a gift coupon, find the probability that he gets a 7-point card when the 4th packet of brand B biscuits has been opened.

(7 marks) (2015 DSE-MATH-M1 Q4)

4. The number of goals scored in a randomly selected match by a football team follows a Poisson distribution with mean λ. The probability that the team scores no goals in a match is 0.1653.
   (a) Find the value of λ correct to 1 decimal place.
   (b) Find the probability that the team scores less than 3 goals in a match.
   (c) It is known that the numbers of goals scored by the team in any two matches are independent. Find the probability that the team totally scores less than 3 goals in two randomly selected matches.

(5 marks) (2012 DSE-MATH-M1 Q7)

5. Eggs from a farm are packed in boxes of 30. The probability that a randomly selected egg is rotten is 0.04 .
   (a) Find the probability that a box contains more than 1 rotten egg.
   (b) Boxes of eggs are inspected one by one.
      (i) Find the probability that the 1st box containing more than 1 rotten egg is the 6th box inspected.
      (ii) What is the expected number of boxes inspected until a box containing more than 1 rotten egg is found?

(7 marks) (PP DSE-MATH-M1 Q8)

6. The monthly number of traffic accidents occurred in a certain highway follows a Poisson distribution with mean 1.7. Assume that the monthly numbers of traffic accidents occurred in this highway are independent.
   (a) Find the probability that at least four traffic accidents will occur in this highway in the first quarter of a certain year.
   (b) Find the probability that there is exactly one quarter with at least four traffic accidents in a certain year.

(6 marks) (SAMPLE DSE-MATH-M1 Q8)

7. Let \$$X$ be the amount of money won in playing a certain game. It is known that $X \sim B(10, p)$. Two plans are proposed for calculating the game fee (\$$F$) .

Plan 1: $F = (1+\theta)\mathrm{E}(X)$,
Plan 2: $F = \mathrm{E}(X) + 0.1\mathrm{Var}(X)$,

where θ is a constant, E(X) is the expected value of X and Var(X) is the variance of X.

It is known that the game fees are same for both plans if $p = \frac{1}{4}$.

   (a) Find θ .
   (b) Show that the variance of X is the greatest when $p = \frac{1}{2}$.
   (c) Determine which plan will give a lower game fee when $p = \frac{1}{2}$.

(8 marks) (2013 ASL-M&S Q6)

8. Soft drinks are produced in packs by a production line in a company. Assume that the number of defective packs in a day follows a Poisson distribution with mean λ . The company has decided to inspect the production line whenever 4 or more defective packs are found in a day. It is known that the probability that at least 1 defective pack found in a day is $1-e^{-2}$.
(a) Find the value of λ .
(b) Find the probability that the company will have to inspect the production line in a given day.
(c) It is given that the probability that the production line will not be inspected for n consecutive days is greater than 0.5 . Find the greatest integral value of n.
(6 marks) (2012 ASL-M&S Q4)

9. It is known that 36% of the customers of a certain supermarket will bring their own shopping bags. There are 3 cashiers and each cashier has 5 customers in queue.
(a) Find the probability that among all the customers in queue, at least 4 of them have brought their own shopping bags.
(b) If exactly 4 customers in queue have brought their own shopping bags, what is the probability that each cashier will have at least 1 customer who has brought his/her own shopping bag?
(6 marks) (2009 ASL-M&S Q5)

10. Assume that the number of passengers arriving at a bus stop per hour follows a Poisson distribution with mean 5. The probability that a passenger arriving at the bus strop is male is 0.65.
(a) Find the probability that 4 passengers arrive at the bus stop in an hour.
(b) Find the probability that 4 passengers arrive at the bus stop in an hour and exactly 2 of them are male.
(5 marks) (2002 ASL-M&S Q6)

11. The number of people killed in a traffic accident follows a Poisson distribution with mean 0.1. There are 5 traffic accidents on a given day, find the probability that there is at most 1 accident in which some people are killed.
(5 marks) (2000 ASL-M&S Q7)

12. 60% of passengers who travel by train use Octopus. A certain train has 12 compartments and there are 10 passengers in each compartment.
(a) What is the probability that exactly 5 of the passengers in a compartment use Octopus?
(b) What is the mean number of passengers using Octopus in a compartment?
(c) What is the probability that the third compartment is the first one to have exactly 5 passengers using Octopus?
(6 marks) (1999 ASL-M&S Q5)

13. On the average, 5 cars pass through an auto-toll every minute. Assuming that the cars pass through the auto-toll independently, find the probability that more than 5 cars will pass through the auto-toll
(a) in 1 minute,
(b) in any 3 of the next 4 minutes.
(6 marks) (1997 ASL-M&S Q6)

14. A brewery has a backup motor for its bottling machine. The backup motor will be automatically turned on if the original motor breaks down during operating hours. The probability that the original motor breaks down during operating hours is 0.15 and when the backup motor is turned on, it has a probability of 0.24 of breaking down. Only when both the original and backup motors break down is the machine not able to work.
(a) What is probability that the machine is not working during operating hours?
(b) If the machine is working, what is the probability that it is operated by the original motor?
(c) The machine is working today. Find the probability that the first break down of the machine occurs on the 10th day after today.
(7 marks) (1997 ASL-M&S Q7)

15. 5000 children are divided into 100 groups, each consisting of 50 children. The number of "over-weight" children are counted in each group and the numbers of groups having 0 , 1 , 2 , ... "over-weight" children are recorded. The distributions, Poisson(λ) and Binomial(n , p), are respectively used to approximate the number of "over-weight" children in each group and some of expected frequencies are shown in the table below.

Expected frequencies of the number of groups by number of "over-weight" children

Number of "over-weight" children	Expected frequency *	
	Poisson(λ)	Binomial(n, p)
3	19.5	19.9
4	19.5	20.4
5	15.6	16.3

* Correct to 1 decimal place

It is known that λ is an integer.
(a) Find λ .
(b) If the mean of the two distributions are equal, find p .
(5 marks) (*modified from* 1998 ASL-M&S Q7)

Section B – Binomial and Geometric distribution

16. Tom arrives at the bus stop at 7:10 . A bus arrives at 7:20 and another bus arrives at 7:30 . The probability that Tom can take the bus is 0.9 each time. If Tom takes the bus at 7:20 , the probability for him to be late is 0.1 . If Tom takes the bus at 7:30 , the probability for him to be late is 0.4 . Tom will be late if he cannot take these two buses.

(a) Find the probability that Tom takes a bus on or before 7:30 on a certain day.

(2 marks)

(b) Find the probability that Tom is late on a certain day.

(2 marks)

(c) Find the probability that Tom is late 2 times in 6 days.

(2 marks)

(d) There are 7 persons, including Tom, waiting for a lift at the lobby. If Tom is late, he will go to the second floor; otherwise he will go to the third floor. The probabilities for each of the other 6 persons to go to the second and third floor are 0.7 and 0.3 respectively. When an empty lift arrives, the 7 persons enter the lift. No person enters the lift afterwards.

(i) Find the probability that the 7 persons are going to the same floor.

(ii) Find the probability that exactly 3 persons are going to the third floor.

(iii) Given that exactly 3 persons are going to the third floor, find the probability that Tom is late.

(7 marks)

(2016 DSE-MATH-M1 Q10)

17. According to the school regulation, air-conditioners can only be switched on if the temperature at 8 am exceeds 26 °C . From past experience, the probability that the temperature at 8 am does NOT exceed 26 °C is q ($q > 0$). Assume that there are five school days in a week. For two consecutive school days, the probability that the air-conditioners are switched on for not more than one day is $\frac{7}{16}$.

(a) (i) Show that the probability that the air-conditioners are switched on for not more than one day on two consecutive school days is $2q - q^2$.

(ii) Find the value of q .

(2 marks)

(b) The air-conditioners are said to be *fully engaged* in a week if the air-conditioners are switched on for all five school days in a week.

(i) Find the probability that the fifth week is the second week that the air-conditioners are *fully engaged*.

(ii) What is the expected number of consecutive weeks when the air-conditioners are not *fully engaged*?

(5 marks)

(c) On a certain day, the temperature at 8 am exceeds 26 °C and all the 5 classrooms on the first floor are reserved for class activities after school. There are 2 air-conditioners in each classroom. The number of air-conditioners being switched off in the classroom after school depends on the number of students staying in the classroom. Assume that the number of students in each classroom is independent.

Case	I	II	III
Number of air-conditioners being switched off	2	1	0
Probability	0.25	0.3	0.45

(i) What is the probability that all air-conditioners are switched off on the first floor after school?

(ii) Find the probability that there are exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air-conditioner being switched off on the first floor after school.

(iii) Given that there are 6 air-conditioners being switched off on the first floor after school, find the probability that at least 1 classroom has no air-conditioners being switched off.

(8 marks)

(2013 ASL-M&S Q11)

18. A fitness centre has 8 certified personal trainers providing personal training programmes to its customers in evenings. A trainer can only train one customer each evening.

The customers have to book the service in advance. Assume all bookings are made independently. Past data revealed that 'no show' bookings account for one-third of the bookings and therefore the fitness centre accepts over-bookings every evening. Trainers are assigned based on a first-come-first-serve basis. If a customer has made a booking but cannot get training due to over-booking, the customer will be given a coupon for compensation.

(a) Suppose there are 12 bookings in a particular evening.

(i) Find the probability that the fitness centre needs to give out 2 or 3 coupons.

(ii) Find the probability that every customer with booking who shows up can be assigned a trainer.

(4 marks)

(b) Find the largest number of bookings the fitness centre can accept for an evening so that at least 80% of customers who have made a booking can be assigned a trainer.

(3 marks)

(c) The centre provides three kinds of personal training programmes for its customers in each evening as follows:

Personal training programmes	Fee per programme
Diamond	\$ 3800
Platinum	\$ 2800
Jade	\$ 1800

It is known that 50%, 30% and 20% of the customers select Diamond, Platinum and Jade programmes respectively. In a particular evening, all trainers are assigned customers.

(i) Find the expected income of the centre in that evening.

(ii) Find the probability that the 8th customer is the first one to select Jade programme.

(iii) Find the probability that all programmes are selected and exactly 3 are Diamond programmes.

(iv) It is given that all programmes are selected and exactly 3 are Diamond programmes. Find the probability that more than 2 customers select Platinum programmes.

(8 marks)

(2012 ASL-M&S Q12)

19. A manufacturer produces a specific kind of tablets. He uses one machine to produce ingredient A and ingredient B, and then one mixer to mix the ingredients to produce the tablets and pack them in bags. The bags of tablets are then delivered to a hospital.
Past records indicate that 0.6% of ingredients A and B respectively are contaminated during the ingredient production process, while 0.1% of the tablets are contaminated during the mixing and packing process. A tablet is regarded as a *contaminated tablet* if

- the ingredient A in the tablet is contaminated, or
- the ingredient B in the tablet is contaminated, or
- the tablet is contaminated during the mixing and packing process.

The pharmacist of the hospital draws a random sample of 20 tablets from each bag to test for contamination. A bag is considered *unsafe* if it contains more than 1 tablet tested positive as a *contaminated tablet*.

(a) Find the probability that a randomly selected tablet from a certain bag is a *contaminated tablet*.

(3 marks)

(b) Find the probability that a bag of tablets is regarded *unsafe*.

(2 marks)

(c) In a certain week, 100 bags of such tablets are delivered to the hospital. The hospital will suspend the supply of the tablets from the manufacturer if more than 4 bags are found *unsafe* within a week.

(i) Find the probability that the 10th bag will be the first one which is regarded *unsafe*.

(ii) Find the probability that the supply from the manufacturer will be suspended in a certain week.

(5 marks)

(d) The manufacturer wants to increase the production and requires the probability of a tablet being contaminated to be less than 1% . To achieve this, he plans to add new machines for producing the ingredients A and B which has contamination probability of 0.4% respectively. Suppose equal amount of ingredients A and B are produced by the original machine and each of the n new machines.

(i) Express the probability that the ingredient A is contaminated in terms of n .

(ii) What is the least value of n?

(5 marks)

(2010 ASL-M&S Q12)

20. Officials of the Food Safety Centre of a city inspect the imported "Choy Sum" by selecting 40 samples of "Choy Sum" from each lorry and testing for an unregistered insecticide. A lorry of "Choy Sum" is classified as *risky* if more than 2 samples show positive results in the test.
Farm A supplies "Choy Sum" to the city. Past data indicated that 1% of the Farm A "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A is transported by a number of lorries to the city.

(a) Find the probability that a lorry of "Choy Sum" is *risky*.

(3 marks)

(b) Find the probability that the 5th lorry is the first lorry transporting *risky* "Choy Sum".

(2 marks)

(c) If k lorries of "Choy Sum" are inspected, find the least value of k such that the probability of finding at least one lorry of *risky* "Choy Sum" is greater than 0.05 .

(3 marks)

(d) Farm B also supplies "Choy Sum" to the city. It is known that 1.5% of the Farm B "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A and Farm B is transported by 8 and 12 lorries respectively to the city.

(i) Find the probability that a lorry of "Choy Sum" supplied by Farm B is *risky*.

(ii) Find the probability that exactly 2 of these 20 lorries of "Choy Sum" are *risky*.

(iii) It is given that exactly 2 of these 20 lorries of "Choy Sum" are *risky*. Find the probability that these 2 lorries transport "Choy Sum" from Farm B .

(7 marks)

(2008 ASL-M&S Q12)

21. In game A, two players take turns to draw a ball randomly, with replacement, from a bag containing 4 green balls and 1 red ball. The first player who draws the red ball wins the game. Christine and Donald play the game until one of them wins. Christine draws a ball first.

(a) Find the probability that Donald wins game A before his 4th draw.

(2 marks)

(b) Find the probability that Donald wins game A.

(3 marks)

(c) Given that Donald wins game A, find the probability that Donald does not win game A before his 4th draw.

(3 marks)

(d) After game A, Christine and Donald play game B. In game B, there are box X and box Y. Box X contains 2 cards which are numbered 4 and 8 respectively while box Y contains 7 cards which are numbered 1, 2, … , 7 respectively. A player randomly draws one card from each box without replacement. If the number drawn from box X is greater than that from box Y, then the player wins game B. Christine and Donald take turns to draw cards until one of them wins game B. Donald draws cards first.

(i) Find the probability that Donald wins game B in his 1st draw.

(ii) Find the probability that Christine wins game B.

(iii) Given that Christine and Donald win one game each, find the probability that Donald wins game A.

(7 marks)

(2007 ASL-M&S Q12)

22. A manufacturer of brand E grape juice starts a marketing campaign by issuing points which can be exchanged for gifts. The number of points is shown on the back of the lid of each can of brand E grape juice. The probabilities for a customer to get a can of brand E grape juice with a 2-point lid and 5-point lid are 0.8 and 0.2 respectively. A total of 15 points or more can be exchanged for a packet of potato chips while a total of 20 points or more can be exchanged for a radio.

(a) Find the probability that a customer can exchange for a packet of potato chip in buying 5 cans of brand E grape juice.

(3 marks)

(b) A customer, Peter, buys 7 cans of brand E grape juice.

(i) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter gets a 5-point lid.

(ii) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter can exchange for a radio.

(iii) Given that Peter can exchange for a radio only when the 7th can of brand E grape juice has been opened, find the probability that the 7th can of brand E grape juice has a 5-point lid.

(iv) Given that Peter cannot get a packet of potato chip after opening 5 cans of brand E grape juice, find the probability that he can exchange for a radio only when the 7th can of brand E grape juice has been opened.

(12 marks)

(2006 ASL-M&S Q11)

23. A certain test gives a positive result in 94% of the people who have disease S . The test gives a positive result in 14% of the people who do not have disease S . In a city, 7.5% of the citizens have disease S .

(a) Find the probability that the test gives a positive result for a randomly selected citizen.

(3 marks)

(b) Given that the test gives a positive result for a randomly selected citizen, find the probability that the citizen does not have disease S .

(3 marks)

(c) The test is applied to a group of citizens one by one. Let M be the number of tests carried out when the first positive result is obtained. Denote the mean and the standard deviation of M by μ and σ respectively.

(i) Find $P(M=3)$.

(ii) Find the exact values of μ and σ .

(iii) Using the fact that $P(-k\sigma \le M-\mu \le k\sigma) \ge 1-\frac{1}{k^2}$ for any positive constant k , prove that $P(1 \le M \le 25) \ge 0.95$.

(9 marks)

(2004 ASL-M&S Q10)

24. A manufacturer of brand C potato chips runs a promotion plan. Each packet of brand C potato chips contains either a red coupon or a blue coupon. Four red coupons can be exchanged for a toy. Five blue coupons can be exchanged for a lottery ticket. It is known that 30% of the packets contain red coupons and the rest contain blue coupons.

(a) Find the probability that a lottery ticket can be exchanged only when the 6th packet of brand C potato chips has been opened.

(3 marks)

(b) A person buys 10 packets of brand C potato chips.

(i) Find the probability that at least 1 toy can be exchanged.

(ii) Find the probability that exactly 1 toy and exactly 1 lottery ticket can be exchanged.

(iii) Given that at least 1 toy can be exchanged, find the probability that exactly 1 lottery ticket can also be exchanged.

(8 marks)

(c) Two persons buy 10 packets of brand C potato chips each. Assume that they do not share coupons or exchange coupons with each other.

(i) Find the probability that they can each get at least 1 toy.

(ii) Find the probability that one of them can get at least 1 toy and the other can get 2 lottery tickets.

(4 marks)

(2004 ASL-M&S Q11)

25. In a game, two boxes A and B each contains n balls which are numbered $1, 2, \ldots, n$. A player is asked to draw a ball randomly from each box. If the number drawn from box A is greater than that from box B, the player wins a prize.

(a) Find the probability that the two numbers drawn are the same.

(1 mark)

(b) Let p be the probability that a player wins the prize.

(i) Find, in terms of p only, the probability that the number drawn from box B is greater than that from box A .

(ii) Using the result of (i), express p in terms of n .

(iii) If the above game is designed so that at least 46% of the players win the prize, find the least value of n .

(6 marks)

(c) Two winners, John and Mary, are selected to play another gamer. They take turns to throw a fair six-sided die. The first player who gets a number '6' wins the game. John will throw the die first.

(i) Find the probability that John will win the game on his third throw.

(ii) Find the probability that John will win the game.

(iii) Given that Mary has won the game, find the probability that Mary did not win the game before her third throw.

(8 marks)

(2003 ASL-M&S Q11)

26. You may use the probabilities list in the table to answer this question.

A salesman is promoting a new fertilizer which will improve the growth of potatoes. He claims that using the fertilizer, farmers will produce 65% of Grade A and 35% of Grade B potatoes (referred as *the claim* below). A farmer uses the fertilizer on his potatoes. In order to test the effectiveness of the fertilizer, he randomly selects 8 potatoes as a sample for testing.

(a) If *the claim* is valid, find the probability that there is at most 1 Grade A potato in the sample.

(2 marks)

(b) The farmer will reject the claim if there are not more than 3 Grade A potatoes in the sample.

(i) If *the claim* is valid, find the probability that the farmer will reject *the claim*.

(ii) If the fertilizer can only produce 20% Grade A and 80% Grade B potatoes, find the probability that the farmer will not *reject the claim*.

(5 marks)

(c) The farmer's wife takes 3 independent samples of 8 potatoes each to check *the claim*. She will reject *the claim* if not more than 3 Grade A potatoes are found in 2 or more of the 3 samples. If *the claim* is valid, find the probability that the farmer's wife will reject *the claim*.

(4 marks)

(d) Suppose *the claim* is valid. By comparing the methods described in (b) and (c), determine who, the farmer or his wife, will have a bigger chance of rejecting the claim wrongly.

(1 mark)

(e) The farmer's son will reject *the claim* if there are not more than k Grade A potatoes in a sample of 8 potatoes. Find the greatest value of k such that the probability of rejecting *the claim* is less than 0.05 given that *the claim* is valid.

(3 marks)

Table: Probabilities of two binomial distributions

Number of success	Probability *	
	B(8, 0.65)	B(8, 0.2)
0	0.0002	0.1678
1	0.0033	0.3355
2	0.0217	0.2936
3	0.0808	0.1468
4	0.1875	0.0459
5	0.2786	0.0092
6	0.2587	0.0011
7	0.1373	0.0001
8	0.0319	0.0000

* Correct to 4 decimal places.

(2001 ASL-M&S Q13)

27. Madam Wong purchases cartons of oranges from a supplier every day. Her buying policy is to randomly select five oranges from a carton and accept the carton if all five are not rotten. Under usual circumstance, 2% of the oranges are rotten.

(a) Find the probability that a carton of oranges will be rejected by Madam Wong.

(3 marks)

(b) Every day, Madam Wong keeps on buying all the accepted cartons of oranges and stops the buying exercise when she has to reject a carton. What is the mean, correct to 1 decimal place, of the number of cartons inspected by Madam Wong in a day?

(3 marks)

(c) Today, Madam Wong has a target of buying 20 acceptable cartons of oranges from the supplier. Instead of applying the stopping rule in (b), she will keep on inspecting the cartons until her target is achieved. Unfortunately, the supplier has a stock of 22 cartons only.

(i) Find the probability that she can achieve her target.

(ii) Assuming she can achieve her target, find the probability that she needs to inspect 20 cartons only.

(7 marks)

(d) The supplier would like to import oranges of better quality so that each carton will have at least a 95% probability of being accepted by Madam Wong. If r% of these oranges are rotten, find the greatest acceptable value of r .

(2 marks)

(1995 ASL-M&S Q11)

28. A day is regarded as humid if the relative humidity is over 80 % and is regarded as dry otherwise. In city K, the probability of having a humid day is 0.7.

(a) Assume that whether a day is dry or humid is independent from day to day.

(i) Find the probability of having exactly three dry days in a week (7 days).

(ii) What is the mean number of dry days before the next humid day? Give your answer correct to 3 decimal places.

(iii) Today is dry. What is the probability of having two or more humid days before the next dry day?

(8 marks)

(b) After some research, it is known that the relative humidity in city K depends solely on that of the previous day. Given a dry day, the probability that the following day is dry is 0.9 and given a humid day, the probability that the following day is humid is 0.8.

(i) If it is dry on March 19, what is the probability that it will be humid on March 20 and dry on March 21?

(ii) If it is dry on March 19, what is the probability that it will be dry on March 21?

(iii) Suppose it is dry on both March 19 and March 21. What is the probability that it is humid on March 20?

(7 marks)

(1994 ASL-M&S Q11)

Section B - Poisson distribution

29. A company records the numbers of lateness of its staff monthly. The performance of a staff member in a month is regarded as *good* if the staff member is late for fewer than 2 times in that month. Albert is a staff member of the company. The number of lateness of Albert in a month follows a Poisson distribution with a mean of 1.8 .

(a) Find the probability that Albert's performance in a certain month is *good*.

(2 marks)

(b) To improve the performance of the staff, the company launches a bonus scheme on staff performance in the coming four months. Two suggestions for the bonus scheme are listed below:

Suggestion I

Number of month with *good* performance	4	3	2	1	0
Bonus	\$ 5000	\$ 2500	\$ 1500	\$ 600	\$ 0

Suggestion II

Total number of lateness in these four months	Fewer than 5	Otherwise
Bonus	\$ 8000	\$ 0

Which one of the above suggestions is more favourable to Albert? Explain your answer.

(6 marks)

(c) The company also records the numbers of early leaves of its staff monthly. The number of early leaves of Albert in a month follows a Poisson distribution with a mean of λ . It is assumed that whether Albert is late and whether he leaves early are independent events.

(i) Express, in terms of e and λ, the probability that Albert is late for 2 times and does not leave early in a certain month.

(ii) Given that the sum of the number of lateness and the number of early leaves of Albert in a certain month is 2, the probability that Albert is late for 2 times and does not leave early in that month is 0.36 . Find λ .

(5 marks)

(2018 DSE-MATH-M1 Q10)

30. A department store issues a cash coupon to a customer spending at least \$500 in a transaction. The details are given in the following table:

Transaction amount (\$ x)	Cash coupon
$500 \le x < 1000$	\$50
$1000 \le x < 2000$	\$100
$x \ge 2000$	\$200

At the department store, 45%, 20% and 10% of the customers each gets one cash coupon of \$50 , \$100 and \$200 respectively in a transaction. Assume that the number of transactions per minute follows a Poisson distribution with a mean of 2 .

(a) Find the probability that there are at most 4 transactions at the department store in a certain minute.

(3 marks)

(b) Find the probability that there are exactly 3 transactions at the department store in a certain minute and cash coupons of total value \$200 are issued.

(3 marks)

(c) If there are exactly 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute.

(3 marks)

(d) Given that there are at most 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute.

(3 marks)

(2017 DSE-MATH-M1 Q10)

31. The number of customers buying tickets at cinema A in a minute can be modelled by a Poisson distribution with a mean of 3.2. The probability distribution of the number of tickets bought by a customer at cinema A is shown in the following table:

Number of tickets bought	1	2	3	4	5	6	≥ 7
Probability	0.12	0.7	0.08	0.04	0.03	0.02	0.01

(a) Find the probability that fewer than 4 customers buy tickets at cinema A in a certain minute.

(3 marks)

(b) Find the probability that the 8th customer buying tickets at cinema A is the 3rd customer who buys 2 tickets.

(2 marks)

(c) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and each of them buys 2 tickets.

(2 marks)

(d) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and they buy a total of 6 tickets.

(3 marks)

(e) Given that fewer than 4 customers buy tickets at cinema A in a certain minute, find the probability that they buy a total of 6 tickets.

(3 marks)

(2015 DSE-MATH-M1 Q10)

32. The number of delays in a day of a railway system follows the Poisson distribution with mean 4.8. Assume that the daily numbers of delays are independent.

(a) Find the probability that there are not more than 3 delays in a day.

(2 marks)

(b) Find the probability that, in 3 consecutive days, there are at most 2 days with not more than 3 delays in each day.

(2 marks)

(c) A day is called a *bad day* if there are more than 5 delays in that day; otherwise it is called a *good day*.

(i) Suppose today is a *bad day*. Find the mean number of *good days* between today and next *bad day*.

(ii) Find the probability that the last day of a week is the third *bad day* in that week.

(iii) Find the probability that there are at least 4 consecutive *bad days* in a week.

(7 marks)

(2014 DSE-MATH-M1 Q13)

33. A lift company provides a regular maintenance service for every lift in an estate at the beginning of each month. Assume that the number of breakdowns of a lift in a month follows the Poisson distribution with mean 1.9. Suppose there are totally 15 lifts in the estate, and the regular maintenance service of a lift in a month is regarded as unacceptable if there are more than 2 breakdowns in that month after the regular maintenance. Assume that the monthly numbers of breakdowns of lifts are independent.

(a) Find the probability that the regular maintenance service of a randomly selected lift in a certain month in the estate is unacceptable.

(2 marks)

(b) For a certain lift, find the probability that June of 2014 is the 3rd month in 2014 such that the regular maintenance service of that lift is unacceptable.

(2 marks)

(c) Find the expected total number of unacceptable regular maintenance services of all lifts in the estate for one year. **expected value**

(2 marks)

(d) In order to assure the quality of the maintenance service provided by the lift company, the

estate management office introduces the following term in the new maintenance contract for the 15 lifts, which will be effective on 1st January 2015.

> For each lift in the estate, if the regular maintenance services is unacceptable for 3 consecutive months in the new contract period, one warning letter will be immediately issued to the lift company, provided that no warning letter has been issued for that lift before.

(i) For a randomly selected lift, find the probability that a warning letter will be issued to the lift company on or before 30th April 2015.

(ii) Find the probability that 3 or more warning letters will be issued to the lift company on or before 30th April 2015.

(6 marks)

(2013 DSE-MATH-M1 Q13)

34. Drunk driving is against the law in a city. The police set up an inspection block at the entrance of a certain highway at night in order to arrest drunk drivers. From past experience, the number of drunk drivers arrested follows a Poisson distribution with mean 2.3 per hour.

(a) Find the probability that at least 2 drunk drivers are arrested in a certain hour.

(2 marks)

(b) Given that at least 2 drunk drivers are arrested in a certain hour, find the probability that not more than 4 drunk drivers are arrested.

(3 marks)

(c) In a certain week, the police sets up an inspection block for three nights, all at the same period from 1:00 am to 2:00 am. It is known that the numbers of drunk drivers arrested in different nights are independent.

(i) Find the probability that the third night is the first night to have at least 2 drunk drivers arrested.

(ii) Find the probability that at least 2 drunk drivers are arrested in each of the 3 nights and there are totally 10 drunk drivers arrested.

(5 marks)

(2012 DSE-MATH-M1 Q13)

35. There are 80 operators in an emergency hotline centre. Assume that the number of incoming calls for the operators are independent and the number of incoming calls for each operator is distributed as Poisson with mean 6.2 in a ten-minute time interval (TMTI). An operator is said to be *idle* if the number of incoming calls received is less than three in a certain TMTI.

(a) Find the probability that a certain operator is *idle* in a TMTI.

(3 marks)

(b) Find the probability that there are at most two *idle* operators in a TMTI.

(3 marks)

(c) A manager, Calvin, checks the numbers of incoming calls of the operators one by one in a TMTI. What is the least number of operators to be checked so that the probability of finding an *idle* operator is greater than 0.9 ?

(4 marks)

(SAMPLE DSE-MATH-M1 Q13)

36. A group of 5 members is waiting for a mini-bus to Mong Kok at a mini-bus station. It is known that there is one mini-bus every fifteen minutes and the number of empty seats on a mini-bus can be modelled by a Poisson distribution with mean λ . The probability that each of three consecutive mini-buses has at least one empty seat is 0.6465 . Assume the number of empty seats for each mini-bus is independent and the 5 members want to travel together.

(a) Find λ . Correct your answer to the nearest integer.

(2 marks)

(b) By using the λ corrected to the nearest integer, find the probability that

(i) the 5 members cannot get on the first arriving mini-bus together,

(ii) the 5 members will have to wait for more than two mini-buses.

(4 marks)

(c) After waiting for a long time, the 5 members decided to break up into a group of 2 members and a group of 3 members.

- All the 5 members will wait for the coming mini-buses if the mini-bus has less than two empty seats.
- The group of 2 members will get on a mini-bus if the mini-bus has exactly two empty seats and the group of 3 members will wait for the coming mini-buses.
- The group of 3 members will get on a mini-bus if the mini-bus has three or four empty seats and the group of 2 members will wait for the coming mini-buses.
- All the 5 members will get on a mini-bus if the mini-bus has at least five empty seats.

By using the λ corrected to the nearest integer, find the probability that

(i) the group of 2 members gets on the first arriving mini-bus and the group of 3 members gets on the next mini-bus,

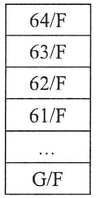
(ii) none of the members have to wait for more than two mini-buses,

(iii) the group of 2 members will go first given that some members have to wait for more than two mini-buses.

(9 marks)

(2013 ASL-M&S Q12)

37.



In a multi-storey office building as shown in Figure 4, there is a lift with maximum capacity of 6 persons that only serves G/F, 61/F – 64/F and operates on demand. The lift is said to be full when there are 6 persons in the lift. People waiting for the lift will enter the lift until it is full.

(a) In the morning, the lift only allows passengers from G/F to enter and travel to upper floors. The number of persons waiting at G/F can be modelled by a Poisson distribution with a mean of 4 persons. The probability that a person goes to each floor (61/F – 64/F) is the same.

(i) Find the probability that the lift is full at G/F.

(ii) Find the probability that there are exactly 4 persons getting into the lift at G/F and they will get off the lift at different floors.

(iii) Find the probability that at least 1 person will get off the lift at each floor (61/F – 64/F) in a single trip.

(7 marks)

(b) In the evening, the lift only takes passengers from upper floors (61/F – 64/F) to G/F and passengers are only allowed to exit the lift at G/F. The number of persons waiting at each floor (61/F – 64/F) can be modelled by a Poisson distribution with a mean of 3 persons.

(i) Find the probability that there are exactly 3 persons waiting for the lift and they are all from different floors.

(ii) Find the probability that there are exactly 2 persons waiting for the lift.

(iii) If there are exactly 3 persons waiting at 62/F, find the probability that all of them can get into the lift.

(8 marks)

(2011 ASL-M&S Q11)

38. Assume that the number of visitors arriving at each counter in an immigration hall is independent and follows a Poisson distribution with a mean of 3.9 visitors per minute. A counter is classified as busy if at least 4 visitors arriving at it in one minute.

(a) Find the probability that a counter is *busy* in a certain minute.

(3 marks)

(b) An officer checks 4 counters in a certain minute. Find the probability that at least one *busy* counter is found.

(2 marks)

(c) If 10 counters are open, find the probability that more than 7 of them are *busy* in a certain minute.

(3 marks)

(d) Suppose 10 counters are open and one of them is randomly selected. Find the probability that more than 7 of them are *busy* and the randomly selected counter is not busy in a certain minute.

(3 marks)

(e) The immigration hall is called *congested* if more than 90% of the open counters are *busy* in a minute. Suppose 15 counters in the hall are open. A senior officer checks the counters in a certain minute. It is given that more than 7 of the first 10 checked counters are *busy*. Find the probability that the hall is *congested*.

(4 marks)

(2008 ASL-M&S Q10)

39. There are many plants in a greenhouse and all of them are of the same species. Assume that the numbers of infected leaves on the plants in the greenhouse are independent and the number of infected leaves on each plant follows a Poisson distribution with a mean of 2.6 . A plant with at least 4 infected leaves is classified as *unhealthy*.

(a) Find the probability that a certain plant in the greenhouse is *unhealthy*.

(3 marks)

(b) A researcher, Teresa, inspects the plants one by one in the greenhouse. She finds that the Mth inspected plant is the first *unhealthy* plant.

(i) Find the probability that M is less than 5 .

(ii) Given that M is less than 5, find the probability that M is greater than 2 .

(iii) If Teresa inspects m plants in the greenhouse, find the least value of m so that the probability of finding an *unhealthy* plant is greater than 0.95 .

(9 marks)

(c) It is given that there are 150 plants in the greenhouse. The number of unhealthy plants in the greenhouse is recorded every Friday. Let N be the number of unhealthy plants recorded on a Friday. Find the mean and the variance of N.

(3 marks)

(2006 ASL-M&S Q12)

40. A bank customer service center records the number of incoming telephone calls in five-minute time intervals (FMTIs). The following table lists the number of calls in a sample of 50 FMTIs.

Number of calls	0	1	2	3	4	5	6	7 or more
Frequency	5	12	14	10	6	2	1	0

(a) Find the sample mean and the sample standard deviation of the data in the table.

(2 marks)

(b) The manager of the bank believes that the number of calls in a FMTI follows a Poisson distribution and its mean can be estimated by the sample mean obtained in (a).

(i) Find the probability that there are fewer than 4 calls in a FMTI.

(ii) Find the probability that there are fewer than 4 calls each in exactly 2 FMTIs out of 6 consecutive FMTIs.

(6 marks)

(c) Assume that model in (b) is adopted and it is known that 55% of the calls are from male customers and 45% of the calls are from female customers.

(i) If there are 3 calls in a FMTI, find the probability that exactly 2 calls are from male customers.

(ii) Find the probability that there are 2 calls in a FMTI and they are both from male customer.

(iii) Given that there are fewer than 4 calls in a FMTI, find the probability that there are at least 2 calls and all of there calls are from male customers.

(7 marks)

(2003 ASL-M&S Q10)

41. A building has only two entrances A and B. Within a 15-minute period, the numbers of persons who entered the building by using entrances A and B follow that Poisson distributions with means 3.2 and 2.7 respectively.

(a) Find the probability that, on a given 15-minute period,

(i) no one entered the building by using entrance A;

(ii) no one entered the building by using entrance B;

(iii) at least one person entered the building;

(iv) exactly two persons entered the building.

(7 marks)

(b) Let X be the number of persons who entered the building within a 15-minite period. Suppose X follows a Poisson distribution with mean λ and k is the most probability number of persons who entered the building within a 15-minute period.

(i) By considering $P(X=k-1)$, $P(X=k)$ and $P(X=k+1)$, show that $\lambda-1\le k\le\lambda$.

(ii) Suppose $\lambda=5.9$. For any 5 successive 15-minute periods, find the probability that the third time that exactly k persons entered the building within a 15-minute period will occur during the fifth 15-minute period.

(8 marks)

(2001 ASL-M&S Q11)

42. A bus company finds that the number of complaints received per day follows a Poisson distribution with mean 10. 40% of the complaints involve the time schedule, 35% involve the manner of drivers, 13% involve the routes and 12% involve other things. These four kinds of complaints are mutually exclusive and can be resolved to the passenger's satisfaction with probabilities 0.6 , 0.2 , 0.7 and 0.5 respectively.

(a) If a complaint cannot be resolved to the passenger's satisfaction, find the probability that this complaint involves the manner of drivers.

(4 marks)

(b) Find the probability that on a given day,

(i) there are 5 complaints,

(ii) there are 5 complaints and 3 of them involve the time schedule.

(4 marks)

(c) Find the probability that on a given day, there are n complaints and 9 of them involve the time schedule.

(2 marks)

(d) (i) Show that $\sum_{k=9}^{\infty}\frac{x^k}{(k-9)!}=x^9e^x$.

(ii) Find the probability that, on a given day, there are 9 complaints involving the time schedule

(5 marks)

(1999 ASL-M&S Q12

43. Suppose that the number of printing mistakes on each page of a 200-page Mathematics book is independent of that on other pages, and it follows a Poisson distribution with mean 0.2.

(a) Find the probability that there is no printing mistake on page 23.

(2 marks)

(b) Let page N be the first page which contains printing mistakes. Find

(i) the probability that N is less than or equal to 3 .

(ii) the mean and variance of N .

(7 marks)

(c) Let M be the number of pages which contain printing mistakes. Find the mean and variance of M .

(2 marks)

(d) Suppose there is another 200-page Statistics book and there are 40 printing mistakes randomly and independently scattered through it. Let Y be the number of printing mistakes on page 23.

(i) Which of the distributions – Bernoulli, binomial, geometric, Poisson or normal, does Y follow? Write down the parameter(s) of the distribution.

(ii) Find the probability that there is no printing mistake on page 23 .

(4 marks)

(1998 ASL-M&S Q11)

44. In city A, the occurrences of rainstorms are assumed to be independent. The number of occurrences may be modelled by a Poisson distribution with mean occurrence rate of 2 rainstorms per year.

(a) Find the probability of having more than two rainstorms in a particular year.

(3 marks)

(b) Last year, more than two rainstorms occurred. Estimate the number of years which will elapse before the next occurrence of more than two rainstorms in a year. Give the answer correct to the nearest integer.

(3 marks)

(c) Past experience suggests that the probability of having at least one serious landslide in a year depends on the number of rainstorms in that year as follows:

Number of rainstorms	0	1 or 2	3 or more
Probability of having at least one serious landslide	0.2	0.3	0.5

Find the probability that, in city A ,

(i) there is no serious landslide in a particular year;

(ii) no rainstorm has occurred if there is no serious landslide in a particular year;

(iii) there is no serious landslide for at most 2 out of 5 years.

(9 marks)

(1996 ASL-M&S Q13)

9. Binomial, Geometric and Poisson Distribution

1. (2017 DSE-MATH-M1 Q4)

(a) The required probability		
$=(1-0.6)^3(0.6)$	1M	for $(1-p)^3 p$, $0<p<1$
$=0.0384$	1A	
(b) $1-(1-0.6)^{10-k}>0.95$	1M	for $1-(1-q)^{10-k}$, $0<q<1$
$0.4^{10-k}<0.05$		
$\log(0.4^{10-k})<\log 0.05$	1M	
$k<6.730587608$		
Thus, the greatest value of k is 6 .	1A	
(c) The expected amount of money		
$=15\left(\frac{1}{0.6}\right)$	1M	for $15\left(\frac{1}{r}\right)$, $0<r<1$
$=\$25$	1A	
	-----------(7)	

(a)	Very good. Most candidates were able to write down a probability of geometric distribution but a few candidates wrongly wrote down $(0.6)^3(1-0.6)$ instead of $(1-0.6)^3(0.6)$.
(b)	Poor. Less than 10% of the candidates were able to set up the correct inequality $1-(1-0.6)^{10-k}>0.95$.
(c)	Good. Only some candidates were unable to find the expected amount of money correctly.

2. (2016 DSE-MATH-M1 Q3)

(a) The variance of the number of visitors entering the museum in a minute is 1.8 .	1A	
(b) The required probability		
$=\frac{e^{-3.6}3.6^3}{3!}$	1M+1M	1M for Poisson probability + 1M using mean 3.6
$=\frac{7.776}{e^{3.6}}$	1A	r.t. 0.2125
≈ 0.212469265		
≈ 0.2125		
The required probability		
$=2\left(\frac{e^{-1.8}1.8^0}{0!}\right)\left(\frac{e^{-1.8}1.8^3}{3!}\right)+2\left(\frac{e^{-1.8}1.8^1}{1!}\right)\left(\frac{e^{-1.8}1.8^2}{2!}\right)$	1M + 1M	1M for 4 cases + 1M for Poisson probability using mean 1.8
$=\frac{7.776}{e^{3.6}}$	1A	r.t. 0.2125
≈ 0.212469265		
≈ 0.2125		
(c) P(at most 3 visitors in a minute)		
$=\frac{e^{-1.8}1.8^0}{0!}+\frac{e^{-1.8}1.8^1}{1!}+\frac{e^{-1.8}1.8^2}{2!}+\frac{e^{-1.8}1.8^3}{3!}$	1M	
≈ 0.891291605		
≈ 0.8913		
The required probability		
$\approx(0.891291605)(1-0.891291605)^2$	1M	
≈ 0.010532851		
≈ 0.0105	1A	r.t. 0.0105
	-----------(7)	

(a)	Very good. A very high proportion of the candidates were able to write down the required variance.
(b)	Very good. More than 70% of the candidates were able to find the answer using a Poisson probability with a mean of 3.6 instead of a mean of 1.8 .
(c)	Good. Only a number of candidates made careless mistakes in finding the required probability.

3. (2015 DSE-MATH-M1 Q4)

Solution	Marks	Remarks
(a) The required probability		
$=(0.75)^3(1-0.75)$	1M	for $p^3(1-p)$, $0<p<1$
$=\dfrac{27}{256}$	1A	
$=0.10546875$		r.t. 0.1055
≈ 0.1055		
(b) The required probability		
$=1-\left((0.75)^4+4\left(\dfrac{27}{256}\right)\right)$	1M+1M	1M for $1-p+$ 1M for using (a)
$=\dfrac{67}{256}$	1A	
$=0.26171875$		r.t. 0.2617
≈ 0.2617		
The required probability		
$=(1-0.75)^4+C_1^4(1-0.75)^3(0.75)+C_2^4(1-0.75)^2(0.75)^2$	1M+1M	1M for the 3 cases + 1M for binomial probability
$=\dfrac{67}{256}$	1A	
$=0.26171875$		r.t. 0.2617
≈ 0.2617		
(c) The required probability		
$=\dfrac{(1-(0.75)^3)(1-0.75)}{0.26171875}$	1M	for denominator using (b)
$=\dfrac{37}{67}$	1A	
≈ 0.552238806		r.t. 0.5522
≈ 0.5522		
The required probability		
$=\dfrac{((1-0.75)^3+C_1^3(1-0.75)^2(0.75)+C_2^3(1-0.75)(0.75)^2)(1-0.75)}{0.26171875}$	1M	for denominator using (b)
$=\dfrac{37}{67}$	1A	
≈ 0.552238806		r.t. 0.5522
≈ 0.5522		
	----------(7)	

(a)	Very good. Most candidates were able to write a binomial probability while a few candidates wrongly wrote $(0.25)^3(1-0.25)$ instead of $(0.75)^3(1-0.75)$.
(b)	Very good. Most candidates were able to use the result of (a) while a few candidates wrongly wrote $1-\left((0.75)^4+\left(\dfrac{27}{256}\right)\right)$ instead of $1-\left((0.75)^4+4\left(\dfrac{27}{256}\right)\right)$.
(c)	Good. Some candidates failed to get the correct answer because they made a mistake in (b).

4. (2012 DSE-MATH-M1 Q7)

Solution	Marks
(a) $\dfrac{e^{-\lambda}}{0!}=0.1653$	
$\lambda=-\ln 0.1653$	
≈ 1.8	1A
(b) $\text{P(no. of goals in a match} <3)=\dfrac{e^{-1.8}}{0!}+\dfrac{e^{-1.8}(1.8)}{1!}+\dfrac{e^{-1.8}(1.8)^2}{2!}$	1M
≈ 0.7306	1A
(c) The number of goals scored in two matches by the team $\sim \text{Po}(3.6)$.	
$\therefore$ P(no. of goals in two matches <3)	
$=\dfrac{e^{-3.6}}{0!}+\dfrac{e^{-3.6}(3.6)}{1!}+\dfrac{e^{-3.6}(3.6)^2}{2!}$	1M
<u>Alternative Solution</u>	
P(no. of goals in two matches <3)	
$=\text{P}(0,0)+\text{P}(0,1)+\text{P}(1,0)+\text{P}(1,1)+\text{P}(0,2)+\text{P}(2,0)$	
$=\left(\dfrac{e^{-1.8}}{0!}\right)^2+2\left(\dfrac{e^{-1.8}}{0!}\right)\left[\dfrac{e^{-1.8}(1.8)}{1!}\right]+\left[\dfrac{e^{-1.8}(1.8)}{1!}\right]^2+2\left(\dfrac{e^{-1.8}}{0!}\right)\left[\dfrac{e^{-1.8}(1.8)^2}{2!}\right]$	1M
≈ 0.3027	1A
	(5)

(a)	Excellent.
(b)	Very good.
(c)	Poor. A few candidates used the Poisson distribution with mean 2λ. Many failed to consider all the events related to the required probability when using the Poisson distribution with mean λ.

5. (PP DSE-MATH-M1 Q8)

Solution	Marks	Remarks
(a) P(a box contains more than 1 rotten eggs)		
$=1-(0.96)^{30}-C_1^{30}(0.96)^{29}(0.04)$	1M+1M	1M for binomial prob 1M for correct cases
≈ 0.338820302		
≈ 0.3388	1A	
(b) (i) P(the 1^{st} box containing more than 1 rotten egg is the 6^{th} box inspected)		
$=(1-0.338820302)^5(0.338820302)$	1M	
≈ 0.0428	1A	
(ii) E(no. of boxes inspected until a box containing more than 1 rotten egg is found)		
$=\dfrac{1}{0.338820302}$	1M	
≈ 2.9514	1A	
	(7)	

(a)	良好。 少部分學生誤以爲所求概率是 $1-(0.96)^{30}-(0.96)^{29}(0.04)$ 。
(b) (i)	平平。 部分學生誤以爲所求概率是 $1-(0.3388)(0.3388)^5$ 。
(ii)	平平。 部分學生忘記期望値的公式。

6. (SAMPLE DSE-MATH-M1 Q8)

Solution	Marks	Remarks
(a) Let X be the number of traffic accidents occurred in this highway in the first quarter of this year. Therefore $X \sim \text{Po}(5.1)$.		
The required probability $= \text{P}(X \geq 4)$		
$= 1 - \left(\frac{e^{-5.1}5.1^0}{0!} + \frac{e^{-5.1}5.1^1}{1!} + \frac{e^{-5.1}5.1^2}{2!} + \frac{e^{-5.1}5.1^3}{3!} \right)$	1M+1M	1M for correct cases 1M for $\frac{e^{-5.1}5.1^x}{x!}$
≈ 0.748731735 ≈ 0.7487	1A	
(b) The required probability		
$\approx C_1^4(0.748731735)(1-0.748731735)^3$	1M+1M	1M for $C_r^n p^r (1-p)^{n-r}$ 1M for using (a)
≈ 0.047511545 ≈ 0.0475	1A	
	(6)	

7. (2013 ASL-M&S Q6)

Solution	Marks	Remarks
(a) $\text{E}(X) = 10\left(\frac{1}{4}\right) = 2.5$	1A	
$\text{Var}(X) = 10\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 1.875$	1A	
$\therefore\ (1+\theta)(2.5) = 2.5 + (0.1)(1.875)$ $\theta = 0.075$	1A	
(b) $\text{Var}(X) = 10p(1-p)$		
$= -10\left[p^2 - p + \left(\frac{1}{2}\right)^2 - \frac{1}{4}\right]$	1M	
$= -10\left(p - \frac{1}{2}\right)^2 + \frac{5}{2}$	1A	
Alternative Solution $\frac{d}{dp}\text{Var}(X) = 10(1-2p)$	1A	
$\therefore\ \frac{d}{dp}\text{Var}(X) = 0$ when $p = \frac{1}{2}$		
$\frac{d^2}{dp^2}\text{Var}(X) = -20 < 0$	1M	
Hence $\text{Var}(X)$ is greatest when $p = \frac{1}{2}$.	1	
(c) For Plan 1, $F = (1+0.075) \cdot 10\left(\frac{1}{2}\right) = 5.375$. For Plan 2, $F = 10\left(\frac{1}{2}\right) + 0.1 \times 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 5.25$.	1M	For both
Hence Plan 2 will give a lower game fee.	1	
	(8)	

Good.
In (b), some candidates were not able to present the proof well.

8. (2012 ASL-M&S Q4)

Solution	Marks
(a) Let X be the number of defective packs in a day.	
$\text{P}(X \geq 1) = 1 - \frac{e^{-\lambda}\lambda^0}{0!}$	1A
$\therefore\ 1 - e^{-2} = 1 - e^{-\lambda}$ i.e. $\lambda = 2$	1A
(b) P(the company will have to inspect the production line in a given day) $= \text{P}(X \geq 4)$	
$= 1 - e^{-2} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!} - \frac{e^{-2}2^3}{3!}$	1M
≈ 0.142876539 ≈ 0.1429	1A
(c) $(1-0.142876539)^n > 0.5$	1M
$n \ln(1-0.142876539) > \ln 0.5$ $n < 4.495896098$ i.e. the greatest integral value of n is 4.	1A
	(6)

Very good.
Nevertheless, some candidates were still not very competent in handling inequalities.

9. (2009 ASL-M&S Q5)

Solution	Marks	Remarks
(a) The required probability		
$= 1 - (0.64)^{15} - C_1^{15}(0.36)(0.64)^{14} - C_2^{15}(0.36)^2(0.64)^{13} - C_3^{15}(0.36)^3(0.64)^{12}$	1M+1A	
≈ 0.8469	1A	
(b) The required probability		
$= \frac{3 \times C_1^5(0.36)(0.64)^4 \times C_1^5(0.36)(0.64)^4 \times C_2^5(0.36)^2(0.64)^3}{C_4^{15}(0.36)^4(0.64)^{11}}$	1M+1A	OR $= \frac{C_1^3 \cdot \frac{4!}{2!1!1!} \cdot \frac{11!}{3!4!4!}}{\frac{15!}{5!5!5!}}$
$= \frac{50}{91}$	1A	OR 0.5495
	(6)	

Part (a) was well attempted although a number of candidates mistook the total number of customers to be 5 instead of 15. Many candidates still had difficulty in analysing the situation and hence could not exhaust and count the number of relevant outcomes.

10. (2002 ASL-M&S Q6)

Solution	Marks
Let N be the number of passengers arriving the bus stop in an hour and M be the number of male passengers.	
(a) $\text{P}(N=4) = \frac{5^4}{4!}e^{-5}$	1A
$\approx 0.17547 \approx 0.1755$	1A a−1 for r.t. 0.175
(b) $\text{P}(M=2 \text{ and } N=4)$	
$= C_2^4(0.65)^2(1-0.65)^2 \cdot 0.17547$	1M for binomial distribution 1M for multiplication rule
≈ 0.0545	1A a−1 for r.t. 0.055
	----------(5)

11. (2000 ASL-M&S Q7)

The probability that there is no people killed in a traffic accident

$= e^{-0.1}$ (p)

≈ 0.904837418 — **2A**

The required probability

$= p^5 + 5p^4(1-p)$ — **1M** binomial (at least 2 terms); **1M** cases 0 and 1

≈ 0.925477591

≈ 0.9255 — **1A** $a-1$ for r.t. 0.925

Alternatively,

The probability that there is at least 1 people killed in a traffic accident

$= 1 - e^{-0.1}$ (q)

≈ 0.095162582

The required probability

$= (1-q)^5 + 5(1-q)^4 q$

≈ 0.925477591

≈ 0.9255

(5)

12. (1999 ASL-M&S Q5)

Let X be the no. of passengers using Octopus in a compartment.

(a) $P(X=5) = C_5^{10}(0.6)^5(1-0.6)^5$ — **1A**

≈ 0.200658

≈ 0.2007 (p_1) — **1A** $a-1$ for r.t. 0.201

(b) $E(X) = np = 10 \times 0.6 = 6$ — **1A+1A**

The mean number of passengers using Octopus in a compartment is 6.

(c) The probability that the third compartment is the first one to have exactly 5 passengers using Octopus

$\approx (1-0.200658)^2(0.200658)$ — **1M** $(1-p_1)^2 p_1$

≈ 0.1282 — **1A** $a-1$ for r.t. 0.128

(6)

13. (1997 ASL-M&S Q6)

(a) Let X be the number of cars passing through the auto-toll in a minute, then $X \sim Po(5)$. — **1M**

$P(X>5)$

$= 1 - \sum_{x=0}^{5} \frac{5^x e^{-5}}{x!}$ — **1A**

≈ 0.3840 — **1A** a–1 for r.t. 0.384

(b) Out of the next 4 minutes, let Y be the number of minutes in which more than 5 cars will pass through the auto-toll, then $Y \sim B(4, 0.3840)$. — **1M**

$P(Y=3)$

$\approx C_3^4(0.3840)^3(1-0.3840)$ — **1M** For binomial formula

≈ 0.1395 (or 0.1396) — **1A** a–1 for r.t. 0.140

(6)

14. (1997 ASL-M&S Q7)

Let A_1 be the event that the original motor breaks down,
A_2 be the event that the backup motor breaks down and
W be the even that the machine is working.

(a) $P(A_1 A_2)$

$= 0.15 \times 0.24$ — **1A**

$= 0.036$ — **1A**

(b) $P(W) = 1 - P(A_1 A_2)$

$= 1 - 0.036$ — **1M**

$= 0.964$

Alternatively,

$P(W) = P(\overline{A_1}) + P(A_1\overline{A_2})$

$= 0.85 + 0.15 \times 0.76$ — **1M**

$= 0.964$

The probability that the machine is operated by the original motor

$= \frac{P(\overline{A_1})}{P(W)}$

$= \frac{0.85}{0.964}$ — **1M**

≈ 0.8817 — **1A** a–1 for r.t. 0.882

(c) The prob. that the 1st break down of the machine occurs on the 10th day

$= (0.036)(1-0.036)^{10-1}$ — **1M**

≈ 0.0259 — **1A** a–1 for r.t. 0.026

(7)

15. (1998 ASL-M&S Q7)

(a) Under Poisson (λ), $\frac{100\lambda^3 e^{-\lambda}}{3!} \approx 19.5$ — **1A**

and $\frac{100\lambda^4 e^{-\lambda}}{4!} \approx 19.5$

Therefore $\frac{100\lambda^3 e^{-\lambda}}{3!} \approx \frac{100\lambda^4 e^{-\lambda}}{4!}$ — **1M** can be omitted

$\lambda \approx 4$

Since λ is an integer, $\lambda = 4$. — **1A**

Alternatively,

By calculating the expected frequencies under Po(λ) when $\lambda = 1, 2, 3, \dots,$ — **1M**

Number of "over-weight" children	Expected frequency Po(1)	Po(2)	Po(3)	Po(4)
3	6.1	18.0	22.4	19.5
4	1.5	9.0	16.8	19.5
5	0.3	3.6	10.1	15.6

From the table above, $\lambda = 4$. — **2A** 1A for just writing $\lambda = 4$

(b) If $\lambda = np$, then $p = \frac{\lambda}{n}$

$= \frac{4}{50}$ — **1M**

$= 0.08$ — **1A**

(5)

Section B – Binomial and Geometric distribution

16. (2016 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) The required probability		
$=0.9+(1-0.9)(0.9)$	1M	
$=0.99$	1A	
	(2)	
(b) The required probability		
$=(0.9)(0.1)+(1-0.9)(0.9)(0.4)+(1-0.9)^2(1)$	1M	
$=0.136$	1A	
	(2)	
(c) The required probability		
$=C_2^6(1-0.136)^4(0.136)^2$	1M	
≈ 0.154605181		
≈ 0.1546	1A	r.t. 0.1546
	(2)	
(d) (i) The required probability		
$=(0.136)(0.7)^6+(1-0.136)(0.3)^6$	1M	
≈ 0.01663012		
≈ 0.0166	1A	r.t. 0.0166
(ii) The required probability		
$=(0.136)C_3^6(0.7)^3(0.3)^3+(1-0.136)C_2^6(0.7)^4(0.3)^2$	1M+1M	
≈ 0.30524256		
≈ 0.3052	1A	r.t. 0.3052
(iii) The required probability		
$=\dfrac{(0.136)C_3^6(0.7)^3(0.3)^3}{(0.136)C_3^6(0.7)^3(0.3)^3+(1-0.136)C_2^6(0.7)^4(0.3)^2}$	1M	1M for denominator using (d)(ii)
≈ 0.082524271		
≈ 0.0825	1A	r.t. 0.0825
	(7)	

(a)	Very good. More than 70% of the candidates were able to find the required probability.
(b)	Good. Some candidates missed the term $(1-0.9)^2(1)$ when finding the required probability.
(c)	Very good. Most candidates were able to formulate the required probability using binomial distribution.
(d) (i)	Good. About half of the candidates were able to find the required probability by using the result of (b). However, some candidates wrongly used 0.7 and 0.3 instead of $(0.7)^6$ and $(0.3)^6$ respectively in the required probability.
(ii)	Good. Many candidates were able to formulate the required probability by using an appropriate binomial probability
(iii)	Good. Many candidates were able to formulate the required conditional probability by using the result in (d)(ii).

17. (2013 ASL-M&S Q11)

Solution	Marks	Remarks
(a) (i) P(the air-conditioners are switched on for not more than one day on two consecutive school days) $=q^2+C_1^2q(1-q)$		OR $1-(1-q)^2$
$=2q-q^2$	1	
(ii) $2q-q^2=\dfrac{7}{16}$		
$16q^2-32q+7=0$		
$q=0.25$ or 1.75 (rejected)	1A	
	(2)	
(b) (i) P(the fifth week is the second week that the air-conditioners are *fully engaged*)		
$=C_1^4(0.75^5)(1-0.75^5)^3\cdot(0.75^5)$	1M+1M	1M for Binomial prob 1M for Geometric prob
≈ 0.0999	1A	
(ii) Expected number of consecutive weeks $=\dfrac{1}{0.75^5}-1$	1M	For $\dfrac{1}{0.75^5}$
$=3\dfrac{52}{243}$	1A	OR 3.2140
	(5)	
(c) (i) P(all conditioners are switched off) $=0.25^5$		
$=\dfrac{1}{1024}$	1A	OR 0.0010
(ii) P(exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air-conditioner being switched off)		
$=C_2^5(0.45)^2[0.25^3+C_1^3(0.25)^2(0.3)]$	1M+1A	
$=\dfrac{1863}{12800}$	1A	OR 0.1455
(iii) P(at least 1 classroom has no air-conditioners being switched off)		
$=\dfrac{\frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45)+C_2^5(0.45)^2(0.25)^3}{C_1^5(0.25)(0.3)^4+\frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45)+C_2^5(0.45)^2(0.25)^3}$	1M+1M+1A	1M for conditional prob 1M for cases in numerator 1A for numerator
$=\dfrac{85}{93}$	1A	OR 0.9140
	(8)	

(a)			Very good.
(b)	(i)		Good.
	(ii)		Satisfactory. Some candidates did not know the expression of the expected value of a geometric distribution, while some others did not minus one from the expected value, which would be the first occurrence of a *fully engaged* week.
(c)	(i)		Good.
	(ii)		Fair.
	(iii)		Poor. Many candidates were not able to analyse the given situation correctly and they were confused by the number of classrooms and the number of air-conditioners being switched off.

18. (2012 ASL-M&S Q12)

(a) (i) P(the centre needs to give out 2 or 3 coupons)
= P(10 or 11 customers show up)

$= C^{12}_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{2} + C^{12}_{11}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right)$ — 1M

$= \frac{10240}{59049}$ — 1A — OR 0.1734

(ii) P(every customer with booking who shows up can be assigned a trainer)
= P(at most 8 customers show up)

$= 1 - C^{12}_{9}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{3} - C^{12}_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{2} - C^{12}_{11}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{12}$ — 1M

$= \frac{107515}{177147}$ — 1A — OR 0.6069

(4)

(b) If the centre accepts 10 bookings, then
P(every customer who have made a booking can be assigned a trainer)

$= 1 - C^{10}_{9}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{10}$

≈ 0.8960 — 1A

> 0.8

If the centre accepts 11 bookings, then
P(every customer who have made a booking can be assigned a trainer)

$= 1 - C^{11}_{9}\left(\frac{2}{3}\right)^{9}\left(\frac{1}{3}\right)^{2} - C^{11}_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{11}$

≈ 0.7659 — 1A

< 0.8

Hence the centre can accepts 10 bookings at most. — 1A

(3)

(c) (i) The expected income in that evening

$= \$(0.5 \times 3800 + 0.3 \times 2800 + 0.2 \times 1800) \times 8$ — 1M

$= \$24800$ — 1A

(ii) P(the 8th customer is the first one to select Jade programs)

$= (0.8)^7(0.2)$

$= \frac{16384}{390625}$ — 1A — OR 0.0419

(iii) P(all programs are selected and exactly 3 are Diamond programs)

$= \frac{8!}{3!\,4!\,1!}(0.5)^3(0.3)^4(0.2)^1 + \frac{8!}{3!\,3!\,2!}(0.5)^3(0.3)^3(0.2)^2$

$+ \frac{8!}{3!\,2!\,3!}(0.5)^3(0.3)^2(0.2)^3 + \frac{8!}{3!\,1!\,4!}(0.5)^3(0.3)^1(0.2)^4$ — 1M+1A

OR $C^8_3(0.5)^3[(0.5)^5 - (0.3)^5 - (0.2)^5]$

$= 0.1995$ — 1A

(iv) The required probability

$= \frac{1}{0.1995}\left[\frac{8!}{3!\,4!\,1!}(0.5)^3(0.3)^4(0.2)^1 + \frac{8!}{3!\,3!\,2!}(0.5)^3(0.3)^3(0.2)^2\right]$ — 1M

OR $\frac{C^8_3(0.5)^3[C^5_1(0.3)^4(0.2) + C^5_2(0.3)^3(0.2)^2]}{0.1995}$

≈ 0.6632 — 1A

(8)

(a)	(i)		Satisfactory. Some candidates had difficulties in analysing the scenarios.
	(ii)		Poor. Many candidates were not able to come up with all the possible outcomes.
(b)			Very poor. Candidates were weak in calculating probabilities by counting the number of relevant outcomes followed by comparing with a given value.
(c)	(i)(ii)		Good.
	(iii)(iv)		Poor. The weakness of candidates was similar to that stated in (a)(ii).

19. (2010 ASL-M&S Q12)

(a) P(a tablet is contaminated)

$= 1-(1-0.6\%)(1-0.6\%)(1-0.1\%)$ — 1M+1M

≈ 0.012952036

≈ 0.0130 — 1A

(3)

(b) P(a bag is *unsafe*)

$= 1-(1-0.012952036)^{20}-20(1-0.012952036)^{19}(0.012952036)$ — 1M

≈ 0.027306899

≈ 0.0273 — 1A

(2)

(c) (i) P(the 10th bag is the first *unsafe* bag)

$\approx (1-0.027306899)^{10-1}(0.027306899)$ — 1M

≈ 0.0213 — 1A

(ii) P(the supply will be suspended in a certain week)

$\approx 1-(1-0.027306899)^{100}-C_1^{100}(1-0.027306899)^{99}(0.027306899)$

$-C_2^{100}(1-0.027306899)^{98}(0.027306899)^2$

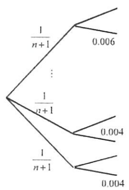
$-C_3^{100}(1-0.027306899)^{97}(0.027306899)^3-C_4^{100}(1-0.027306899)^{96}(0.027306899)^4$ — 1M+1A

≈ 0.1390 — 1A

(5)

(d) (i) P(the ingredient A is contaminated)

$= \dfrac{0.006+0.004n}{n+1}$ — 1M



(ii) P(the ingredient B is contaminated) $= \dfrac{0.006+0.004n}{n+1}$

$\therefore\ 1-\left(1-\dfrac{0.006+0.004n}{n+1}\right)\left(1-\dfrac{0.006+0.004n}{n+1}\right)(1-0.001)<0.01$ — 1A

$\left(1-\dfrac{0.006+0.004n}{n+1}\right)^2>\dfrac{110}{111}$

$1-\dfrac{0.006+0.004n}{n+1}>\sqrt{\dfrac{110}{111}}$ or $1-\dfrac{0.006+0.004n}{n+1}<-\sqrt{\dfrac{110}{111}}$ (rejected) — 1M

$n>2.885790831$ — 1A — OR $n>2.8858$

Hence the least number of n is 3. — 1A

(5)

(a)		Unsatisfactory. There were three sources of contamination and many candidates had difficulty in sorting out the situation. Many could not see that the required event was the complement of "a tablet completely free from contamination".
(b)		Very good.
(c)		Very good.
(d) (i)		Poor. Many candidates seemed to have difficulty in understanding the question.
(ii)		Very poor. Very few candidates were able to use the concept in (a) and most were weak in handling inequalities.

20. (2008 ASL-M&S Q12)

(a) The required probability

$= 1-[(1-0.01)^{40}+C_1^{40}(1-0.01)^{39}(0.01)+C_2^{40}(1-0.01)^{38}(0.01)^2]$ — 1M+1M (1M for cases correct; 1M for Binomial probability (Can be awarded in (d)(i)))

≈ 0.007497363

≈ 0.0075 — 1A

(3)

(b) The required probability

$\approx (1-0.007497363)^4(0.007497363)$ — 1M

≈ 0.0073 — 1A

(2)

(c) $C_1^k(0.007497363)(1-0.007497363)^{k-1}+C_2^k(0.007497363)^2(1-0.007497363)^{k-2}$

$+\cdots+C_{k-1}^k(0.007497363)^{k-1}(1-0.007497363)+(0.007497363)^k>0.05$

Alternative Solution

$0.007497363+(1-0.007497363)(0.007497363)+(1-0.007497363)^2(0.007497363)$

$+\ldots+(1-0.007497363)^{k-1}(0.007497363)>0.05$

$1-(1-0.007497363)^k>0.05$ — 1M

$0.992502636^k<0.95$

$k\ln 0.992502636<\ln 0.95$ — 1M

$k>\dfrac{\ln 0.95}{\ln 0.992502636}\approx 6.815832223$

Hence the least value of k is 7. — 1A

(3)

(d) (i) The required probability

$= 1-[(1-0.015)^{40}+C_1^{40}(1-0.015)^{39}(0.015)+C_2^{40}(1-0.015)^{38}(0.015)^2]$

≈ 0.022069897

≈ 0.0221 — 1A

(ii) The required probability

$= [C_0^8(1-0.007497363)^8(0.007497363)^0][C_2^{12}(1-0.022069897)^{10}(0.022069897)^2]$

$+[C_1^8(1-0.007497363)^7(0.007497363)][C_1^{12}(1-0.022069897)^{11}(0.022069897)]$

$+[C_2^8(1-0.007497363)^6(0.007497363)^2][C_0^{12}(1-0.022069897)^{12}(0.022069897)^0]$ — 1M+1M (1M for any 1 case correct; 1M for all cases correct)

≈ 0.037154780

≈ 0.0372 — 1A

(iii) The required probability

$\approx \dfrac{[C_0^8(1-0.007497363)^8(0.007497363)^0][C_2^{12}(1-0.022069897)^{10}(0.022069897)^2]}{0.037154780}$ — 1M+1M (1M for form correct; 1M for denominator using (ii))

≈ 0.6517 — 1A

(7)

(a)		Very good.
(b)		Very good.
(c)		Fair. Candidates were less skilful in handling inequalities.
(d)(i)		Good. Some candidates encountered difficulty in counting the number of relevant events.
(ii)(iii)		Fair. Some candidates had difficulty in counting the events.

21. (2007 ASL-M&S Q12)

Solution	Marks
(a) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$	1M for geometric probability
$=\frac{5124}{15625}$	1A
$=0.327936$	
≈ 0.3279	a–1 for r.t. 0.328 ----------(2)
(b) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)+\cdots$	1M must indicate infinite series and have at least 3 terms
$=\frac{\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)}{1-\left(\frac{4}{5}\right)^2}$	1M for summing geometric sequence
$=\frac{4}{9}$	1A
$=0.444444444$	
≈ 0.4444	a–1 for r.t. 0.444 ----------(3)
(c) The required probability	
$=\frac{\frac{4}{9}-\frac{5124}{15625}}{\frac{4}{9}}$	1M for numerator $=$ (b) $-$ (a) + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
$=0.262144$	
≈ 0.2621	a–1 for r.t. 0.262
The required probability	
$=1-\frac{\frac{5124}{15625}}{\frac{4}{9}}$	1M for complementary probability + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
$=0.262144$	
≈ 0.2621	a–1 for r.t. 0.262
	----------(3)

Solution	Marks
(d) (i) The required probability	
$=(\frac{1}{2})(\frac{3}{7})+(\frac{1}{2})(1)$	1M for either case
$=\frac{5}{7}$	1A
$=0.714285714$	
≈ 0.7143	a–1 for r.t. 0.714
The required probability	
$=1-(\frac{1}{2})(\frac{4}{7})$	1M for complementary probability
$=\frac{5}{7}$	1A
$=0.714285714$	
≈ 0.7143	a–1 for r.t. 0.714
(ii) The required probability	
$=1-\frac{5}{7}$	1M for $1-$(d)(i)
$=\frac{2}{7}$	1A
$=0.285714286$	
≈ 0.2857	a–1 for r.t. 0.286
The required probability	
$=(\frac{1}{2})(\frac{4}{7})(1)(1)$	1M for denominator $=(2)(7)$
$=\frac{2}{7}$	1A
$=0.285714286$	
≈ 0.2857	a–1 for r.t. 0.286
(iii) The required probability	
$=\frac{(\frac{4}{9})(\frac{2}{7})}{(\frac{4}{9})(\frac{2}{7})+(1-\frac{4}{9})(1-\frac{2}{7})}$	1M for $\frac{pq}{pq+(1-p)(1-q)}$ + 1M for $\begin{cases}p=\text{(b)}\\q=\text{(d)(ii)}\end{cases}$ or $\begin{cases}p=\text{(d)(ii)}\\q=\text{(b)}\end{cases}$
$=\frac{8}{33}$	1A
$=0.242424242$	
≈ 0.2424	a–1 for r.t. 0.242 ----------(7)

(a)		Good.
(b)		Good. Some candidates were not able to sum the infinite geometric series.
(c)		Fair. Some candidates could not work out the complementary probability.
(d)(i)		Good. Some candidates encountered difficulty in counting the number of relevant events.
(ii)		Fair. Many candidates did not realise this is the complementary event of d(i).
(iii)		Poor. Very few candidates attempted this part.

22. (2006 ASL-M&S Q11)

(a) The required probability	
$=1-\left((0.8)^5+C_1^5(0.8)^4(0.2)\right)$	1M for cases correct + 1M for binomial probability
$=\frac{821}{3125}$	1A
$=0.26272$	
≈ 0.2627	a–1 for r.t. 0.263
The required probability	
$=(0.2)^5+C_1^5(0.2)^4(0.8)+C_2^5(0.2)^3(0.8)^2+C_3^5(0.2)^2(0.8)^3$	1M for the 4 cases + 1M for binomial probability
$=\frac{821}{3125}$	1A
$=0.26272$	
≈ 0.2627	a–1 for r.t. 0.263
	----------(3)
(b) (i) The required probability	
$=(0.8)^6(0.2)$	1M for $p^6(1-p)$, where $0<p<1$
$=\frac{4096}{78125}$	1A
$=0.0524288$	
≈ 0.0524	a–1 for r.t. 0.052
(ii) The required probability	
$=\left(C_2^5(0.8)^4(0.2)^2\right)(0.8)+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$	1M for the 3 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a–1 for r.t. 0.324
The required probability	
$=C_2^6(0.8)^4(0.2)^2+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$	1M for the 2 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a–1 for r.t. 0.324
The required probability	
$=C_2^7(0.8)^5(0.2)^2+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)$	1M for the 2 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a–1 for r.t. 0.324
(iii) The required probability	
$=\frac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)}{0.3244032}$	1A for numerator 1M for denominator using (b)(ii)
$=\frac{13}{33}$	1A
≈ 0.3939393939	
≈ 0.3939	a–1 for r.t. 0.394

The required probability	
$=1-\frac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.8)}{0.3244032}$	1A for numerator 1M for denominator using (b)(ii)
$=\frac{13}{33}$	1A
≈ 0.3939393939	
≈ 0.3939	a–1 for r.t. 0.394
(iv) The required probability	
$=\frac{(0.8)^5(0.2)^2+C_1^5(0.8)^4(0.2)\left((0.2)^2+C_1^2(0.8)(0.2)\right)}{1-0.26272}$	1M (one term) + 1A for numerator 1M for denominator using (a)
$=\frac{49}{225}$	1A
≈ 0.2177777777	
≈ 0.2178	a–1 for r.t. 0.218
	----------(12)

(a)		Very good.
(b) (i)		Very good.
(ii)		Fair. Some candidates encountered difficulty in counting the number of relevant events.
(iii)		Fair. Some candidates encountered difficulty in counting the number of relevant events.
(iv)		Not satisfactory. Very few candidates attempted this part.

23. (2004 ASL-M&S Q10)

Solution	Marks
(a) The required probability	
$= (0.075)(0.94) + (1-0.075)(0.14)$	1M for $(p(0.94)+(1-p)(0.14))$ +1A
$= 0.2$	1A
	---------(3)
(b) The required probability	
$= \frac{(1-0.075)(0.14)}{0.2}$	1M for denominator using (a) +1A
$= 0.6475$	1A (accept $\frac{259}{400}$) a–1 for r.t. 0.648
	---------(3)
(c) (i) $P(M=3)$	
$= (1-0.2)^2(0.2)$	1M for $(1-(a))^2(a)$
$= 0.128$	1A
(ii) μ	
$= \frac{1}{0.2}$	1M for $\frac{1}{(a)}$
$= 5$	
σ	
$= \sqrt{\frac{1-0.2}{0.2^2}}$	1M for $\sqrt{\frac{1-(a)}{(a)^2}}$
$= \sqrt{20}$	1A for both correct
$= 2\sqrt{5}$	
(iii) Putting $k = 2\sqrt{5}$ in $P(-k\sigma \le M-\mu \le k\sigma) \ge 1-\frac{1}{k^2}$, we have	1A for $k=2\sqrt{5}$ or $k=\sqrt{20}$
$P(-2\sqrt{5}\sigma \le M-\mu \le 2\sqrt{5}\sigma) \ge 1-(\frac{1}{2\sqrt{5}})^2$.	
By (c)(ii), we have $P(-20 \le M-5 \le 20) \ge 0.95$.	
So, we have $P(-15 \le M \le 25) \ge 0.95$.	1M
Note that $P(-15 \le M < 1) = 0$.	1M for using $P(-l \le M < 1) = 0$ for any $l > 0$
Thus, we have	
$P(1 \le M \le 25)$	
$= P(-15 \le M \le 25) - P(-15 \le M < 1)$	
$= P(-15 \le M \le 25)$	
≥ 0.95	1 do not accept finding the value of $P(1 \le M \le 25)$ directly
	---------(9)

(a/b)		Very good.
(c) (i)		Good.
(ii)		Fair. Some candidates confused δ with δ^2.
(iii)		Poor. Many candidates did not have the confidence to try this unfamiliar question.

24. (2004 ASL-M&S Q11)

Solution	Marks
(a) The required probability	
$= (C_4^5(0.7)^4(0.3))(0.7)$	1M for binomial probability + 1M for multiplication rule
$= 0.252105$	1A
≈ 0.2521	a–1 for r.t. 0.252
	---------(3)
(b) Let X be the number of red coupons in the 10 packets of brand C potato chips.	
(i) The required probability	
$= P(X \ge 4)$	1M
$= 1-(0.7)^{10} - C_1^{10}(0.7)^9(0.3) - C_2^{10}(0.7)^8(0.3)^2 - C_3^{10}(0.7)^7(0.3)^3$	1A
[redacted]	
≈ 0.3504	1A a–1 for r.t. 0.350
(ii) The required probability	
$= P(4 \le X \le 5)$	1M
$= C_4^{10}(0.7)^6(0.3)^4 + C_5^{10}(0.7)^5(0.3)^5$	
[redacted]	
≈ 0.3030	1A a–1 for r.t. 0.303
(iii) The required probability	
$= P(4 \le X \le 5 \mid X \ge 4)$	
$= \frac{P(4 \le X \le 5)}{P(X \ge 4)}$	
$\approx \frac{0.3030402942}{0.3503892816}$	1M for numerator using (b)(ii) + 1M for denominator using (b)(i)
[redacted]	
≈ 0.8649	1A (accept 0.8647 and 0.8648) a–1 for r.t. 0.865
	---------(8)
(c) (i) The required probability	
$= (P(X \ge 4))^2$	
$\approx (0.3503892816)^2$ (by (b)(i))	1M for $((b)(i))^2$
[redacted]	
≈ 0.1228	1A a–1 for r.t. 0.123
(ii) The required probability	
$= 2P(X \ge 4)\,P(X=0)$	1M
$\approx 2(0.3503892816)(0.0282475249)$ (by (b)(i))	
[redacted]	
≈ 0.0198	1A a–1 for r.t. 0.020
	---------(4)

(a)		Fair. Many candidates wrongly adopted the geometric distribution.
(b/c)		Good.

25. (2003 ASL-M&S Q11)

(a) The required probability

$= (1)\left(\frac{1}{n}\right)$

$= \frac{1}{n}$ — 1A

> The required probability
>
> $= (n)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)$
>
> $= \frac{1}{n}$ — 1A

----------(1)

(b) (i) The required probability

$= p$ — 1A

(ii) $p + p + \frac{1}{n} = 1$ — 1M

$p = \frac{1}{2}(1 - \frac{1}{n})$ — 1A

(iii) $p \geq 0.46$ — 1A can be absorbed

$\frac{1}{2}(1 - \frac{1}{n}) \geq 0.46$ — 1M

$n \geq 12.5$

$\because$ n is a positive integer.

$\therefore$ the least value of n is 13. — 1A

----------(6)

(c) (i) The required probability

$= \left(\frac{5}{6}\right)^4 \frac{1}{6}$ — 1M for $\left(\frac{5}{6}\right)^k \frac{1}{6}$

$= \frac{625}{7776}$ — 1A

~~≈ 0.080375514~~

≈ 0.0804 — a–1 for r.t. 0.080

(ii) The required probability

$= \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \cdots$ — 1A must indicate infinite series and have at least 3 terms

$= \dfrac{\frac{1}{6}}{1 - \frac{25}{36}}$ — 1M for sum of GP

$= \frac{6}{11}$ — 1A

~~≈ 0.545454545~~

≈ 0.5455 — a–1 for r.t. 0.545

(iii) The required probability

$= \dfrac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right) + \cdots}{1 - \frac{6}{11}}$ — 1M for denominator using 1–(c)(ii) + 1A for numerator

$= \dfrac{\dfrac{\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5}{1 - \frac{25}{36}}}{\frac{5}{11}}$

$= \frac{625}{1296}$ — 1A

~~≈ 0.482253086~~

≈ 0.4823 — a–1 for r.t. 0.482

> The required probability
>
> $= \dfrac{\frac{5}{11} - \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)}{1 - \frac{6}{11}}$ — 1M for denominator using 1–(c)(ii) + 1A for numerator
>
> $= \dfrac{\frac{5}{11} - \frac{5}{36} - \frac{125}{1296}}{\frac{5}{11}}$
>
> $= \frac{625}{1296}$ — 1A
>
> ~~≈ 0.482253086~~
>
> ≈ 0.4823 — a–1 for r.t. 0.482

----------(8)

(a/b)		Not satisfactory. Many candidates were not able to identify the symmetry nature of '$P(A > B) = P(A < B)$'.
(c)		Satisfactory. When applying a geometric distribution, some candidates miscounted the number of dice throwing. Part (iii) was performed poorly.

26. (2001 ASL-M&S Q13)

Solution	Marks
Let X be the number of Grade A potatoes in the 8 selected potatoes.	
(a) $P(X \le 1 \mid p = 0.65) \approx 0.0002 + 0.0033$	1M
≈ 0.0035 $\boxed{0.0036}$	1A
	----------(2)
(b) (i) $P(X \le 3 \mid p = 0.65) \approx 0.0002 + 0.0033 + 0.0217 + 0.0808$	1M
≈ 0.1060 $\boxed{0.1061}$ (q)	1A
(ii) $P(X > 3 \mid p = 0.2)$	1M
$\approx 0.0459 + 0.0092 + 0.0011 + 0.0001 + 0.0000$	1M
$\boxed{\approx 1 - (0.1678 + 0.3355 + 0.2936 + 0.1468)}$	
≈ 0.0563	1A
	----------(5)
(c) The required probability	
$= C_2^3 q^2(1-q) + C_3^3 q^3$	1M for the 2 cases 1M for 1st term 1M for 2nd term
$\approx C_2^3(0.1060)^2(1-0.1060) + C_3^3(0.1060)^3$	1M+1M+1M
≈ 0.0313 $\boxed{0.0314}$	1A
	----------(4)
(d) The probability that the farmer will wrongly reject *the claim* is 0.1060 whereas the probability that his wife will wrongly reject *the claim* is 0.0313. Therefore the farmer will have a bigger chance of rejecting *the claim* wrongly.	1M
	----------(1)
(e) $P(X \le 2 \mid p = 0.65) \approx 0.0252$ $P(X \le 3 \mid p = 0.65) \approx 0.0252 + 0.0808 \approx 0.1060$	1M+1A 1M for 0.05 as a value between
Since $P(X \le 2 \mid p = 0.65) < 0.05 < P(X \le 3 \mid p = 0.65)$	
$\therefore$ $k = 2$.	1A independent
	----------(3)

27. (1995 ASL-M&S Q11)

Solution	Marks	Remarks
(a) Probability of acceptance, $p_a = (1-0.02)^5$	1A	
$= 0.9039$		
Probability of rejection, $p_r = 1 - p_a$	1M	
$= 0.0961$	1A	
(b) Let X be the number of cartons inspected by Madam Wong in a day, then $X \sim \text{Geom}(p_r)$.	1M	
$\therefore$ mean $= \dfrac{1}{p_r}$	1A	
$= 10.4$	1A	
(c) (i) Prob. that Madam Wong can achieve her target, p_t = P(All cartons are acceptable) + P(exactly 1 carton is not acceptable) + P(exactly 2 cartons are not acceptable)	1M	
$= (p_a)^{22} + \binom{22}{1} p_r (p_a)^{21} + \binom{22}{2} (p_r)^2 (p_a)^{20}$	1M + 1A	
$= 0.6445$	1A	Accept 0.6444 – 0.6445
Alternatively, p_t = P(the 1st 20 cartons are accepted) + P(1 is rejected in the 1st 20 cartons and the 21st carton is accepted) + P(2 is rejected in the 1st 21 cartons and the 22nd carton is accepted)	1M	
$= (p_a)^{20} + \binom{20}{1} p_r (p_a)^{20} + \binom{21}{2} (p_r)^2 (p_a)^{20}$	1M + 1A	
$= 0.6445$	1A	Accept 0.6444 – 0.6445
(ii) If Madam Wong can achieve her target, the prob. that she needs to inspect 20 cartons only		
$= \dfrac{(p_a)^{20}}{p_t}$	1M + 1A	
$= 0.2058$	1A	Accept 0.2057
(d) $(1 - r\%)^5 \ge 0.95$	1M	
$r\% \le 0.010206$		
$r \le 1.0206$		
$\therefore$ The greatest acceptable value of r is 1.0206.	1A	

28. (1994 ASL-M&S Q11)

(a) (i) Let X be the number of dry days in a week.

$X \sim \text{Bin}(7,\ 0.3)$ — 1M

$f_X(x) = \binom{7}{x}(0.3)^x(0.7)^{7-x}$ for $x = 0, 1, 2, \ldots, 7$ — 1A

The prob. of having exactly 3 dry days in a week is

$f_X(3) = \binom{7}{3}(0.3)^3(0.7)^4 = 0.2269$ — 1A

(ii) Let Y be the no. of days elapsed until the 1st humid day.

$Y \sim \text{Geom}(0.7)$ — 1M

$E(Y) = \dfrac{1}{0.7}$ — 1A

Hence the mean no. of dry days before the next humid day is

$E(Y) - 1 = \dfrac{1}{0.7} - 1 = 0.429$ — 1A

(iii) The prob. of having 2 or more humid days before the next dry day is

$1 - 0.3 - (0.7)(0.3)$ — 1A

$= 1 - 0.51$

$= 0.49$ — 1A

<u>Alternatively</u>

$\sum_{k=2}^{\infty} (0.3)(0.7)^k$ — 1A

$= (0.3)(0.7)^2[1 + 0.7 + (0.7)^2 + \ldots]$

$= (0.3)\dfrac{(0.7)^2}{1-0.7}$

$= 0.49$ — 1A

(b) Let a dry day and a humid day be denoted by D and H respectively.

(i) 19th-20th-21st : D-H-D

P(H on 20th, D on 21st | D on 19th)

$= (1-0.9)(1-0.8)$ — 1M

$= 0.02$ — 1A

(ii) 19th-20th-21st : D-H-D or D-D-D

P(D on 21st | D on 19th)

$= 0.02 + (0.9)(0.9)$ — 1M

$= 0.83$ — 1A

(iii) P(H on 20th | D on 19th and 21st)

$= \dfrac{0.02}{0.83}$ — 2M (1 for nominator, 1 for denominator)

$= 0.02410$ — 1A

Section B – Poisson distribution

29. (2017 DSE-MATH-M1 Q10)

(a) The required probability

$= \dfrac{2^0e^{-2}}{0!} + \dfrac{2^1e^{-2}}{1!} + \dfrac{2^2e^{-2}}{2!} + \dfrac{2^3e^{-2}}{3!} + \dfrac{2^4e^{-2}}{4!}$ — 1M+1M (1M for the 5 cases + 1M for Poisson probability)

[redacted]

≈ 0.9473 — 1A (r.t. 0.9473) ——(3)

(b) The required probability

$= \dfrac{2^3e^{-2}}{3!}(3(0.25)^2(0.1) + 3(0.25)(0.2)^2 + 3(0.45)^2(0.2))$ — 1M+1M (1M for Poisson probability + 1M for any one correct)

[redacted]

≈ 0.0307 — 1A (r.t. 0.0307) ——(3)

(c) The required probability

$= 4(0.25)^3(0.1) + 6(0.25)^2(0.2)^2 + (4)(3)(0.45)^2(0.2)(0.25) + (0.45)^4$ — 1M+1M (1M for any one correct + 1M for any three correct)

[redacted]

≈ 0.1838 — 1A (r.t. 0.1838) ——(3)

(d) The required probability

$\approx \dfrac{\left(\frac{2e^{-2}}{1!}\right)(0.1) + \left(\frac{2^2e^{-2}}{2!}\right)(2(0.25)(0.1) + (0.2)^2) + 0.030721109 + \left(\frac{2^4e^{-2}}{4!}\right)(0.18375625)}{0.947346982}$ — 1M+1M (1M for numerator using (b) or (c) + 1M for denominator using (a))

[redacted]

≈ 0.1042 — 1A (r.t. 0.1042) ——(3)

(a)	Very good. Over 85% of the candidates were able to write down all the five Poisson probabilities.
(b)	Very good. A few candidates were unable to use correct combinations in counting.
(c)	Good. Some candidates wrongly multiplied the Poisson probability to the required probability.
(d)	Good. Only some candidates were unable to consider all the possible cases that cash coupons of total value $200 are issued in a minute.

30. (2015 DSE-MATH-M1 Q10)

(a) The required probability

$= \frac{3.2^0 e^{-3.2}}{0!} + \frac{3.2^1 e^{-3.2}}{1!} + \frac{3.2^2 e^{-3.2}}{2!} + \frac{3.2^3 e^{-3.2}}{3!}$ — 1M+1M — 1M for the 4 cases + 1M for Poisson probability

≈ 0.602519724

≈ 0.6025 — 1A — r.t. 0.6025

----------(3)

(b) The required probability

$= C_2^7 (0.7)^2 (1-0.7)^5 (0.7)$ — 1M — for binomial probability

≈ 0.01750329

≈ 0.0175 — 1A — r.t. 0.0175

----------(2)

(c) The required probability

$= \frac{3.2^3 e^{-3.2}}{3!} (0.7)^3$ — 1M

≈ 0.076357282

≈ 0.0764 — 1A — r.t. 0.0764

----------(2)

(d) The required probability

$\approx 0.076357282 + \frac{3.2^3 e^{-3.2}}{3!} \left(3(0.12)^2(0.04) + 3!(0.12)(0.7)(0.08) \right)$ — 1M+1A — 1M for using (c) + 1A for any one correct

≈ 0.085717839

≈ 0.0857 — 1A — r.t. 0.0857

----------(3)

(e) The required probability

$\approx \frac{\left(\frac{3.2 e^{-3.2}}{1!}\right)(0.02) + \left(\frac{3.2^2 e^{-3.2}}{2!}\right)\left(2(0.12)(0.03) + 2(0.7)(0.04) + (0.08)^2\right) + 0.085717839}{0.602519724}$ — 1M+1M — 1M for numerator using (d) +1M for denominator using (a)

≈ 0.170703644

≈ 0.1707 — 1A — r.t. 0.1707

----------(3)

(a)	Very good. A few candidates missed the first case in the required sum of the Poisson probabilities.
(b)	Very good. A few candidates unnecessarily multiplied the Poisson probability to the required probability form.
(c)	Very good. A few candidates wrongly used $\frac{3.2^3 e^{-3.2}}{3!}(0.7)^2$ instead of $\frac{3.2^3 e^{-3.2}}{3!}(0.7)^3$ in the calculation.
(d)	Good. Some candidates failed to count the number of cases correctly, such as they wrongly multiplied 3 instead of 3! to the term $(0.12)(0.7)(0.08)$.
(e)	Good. Some candidates did not realize that a conditional probability is considered here. Some candidates did not consider the Poisson probabilities as a part of the joint probability in the numerator of the required conditional probability.

31. (2014 DSE-MATH-M1 Q13)

(a) P(not more than 3 delays in a day)

$= e^{-4.8}\left(1 + 4.8 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!}\right)$ — 1M

≈ 0.294229916

≈ 0.2942 — 1A

(2)

(b) P(at most 2 days with not more than 3 delays in a day in 3 consecutive days)

$\approx 1 - 0.294229916^3$ — 1M — OR $\sum_{r=0}^{2} C_r^3 p^r (1-p)^{3-r}$, where $p \approx 0.294229916$

≈ 0.9745 — 1A

(2)

(c) Denote P(*bad day*) by k.

(i) $k = 1 - e^{-4.8}\left(1 + 4.8 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!} + \frac{4.8^4}{4!} + \frac{4.8^5}{5!}\right)$

≈ 0.348993562 — 1A

$\therefore$ the mean number of *good days* between today and next *bad day*

$= \frac{1}{k} - 1$ — 1M

≈ 1.8654 — 1A

(ii) P(the last day in a week is the third *bad day* in that week)

$= C_2^6 k^2 (1-k)^4 k$ — 1M

≈ 0.1145 — 1A

(iii) P(there are at least 4 consecutive bad days in a week)

$= k^4 \cdot 1^3 + (1-k)k^4 \cdot 1^2 + 1(1-k)k^4 \cdot 1 + 1^2(1-k)k^4$ — 1M

Alternative Solution 1

$= [2k^4(1-k) + 2k^4(1-k)^2] + [2k^5(1-k) + k^5(1-k)^2] + 2k^6(1-k) + k^7$ — 1M

$= 2(k^4 - k^5 + k^4 - 2k^5 + k^6) + 2k^5 - 2k^6 + k^5 - 2k^6 + k^7 + 2k^6 - 2k^7 + k^7$

Alternative Solution 2

$= 4k^4(1-k)^3 + 9k^5(1-k)^2 + 6k^6(1-k) + k^7$ — 1M

$= 4(k^4 - 3k^5 + 3k^6 - k^7) + 9(k^5 - 2k^6 + k^7) + 6k^6 - 6k^7 + k^7$

$= 4k^4 - 3k^5$

≈ 0.0438 — 1A

(7)

(c)(iii)

S	M	T	W	T	F	S
B	B	B	B	/	/	/
G	B	B	B	B	/	/
/	G	B	B	B	B	/
/	/	G	B	B	B	B

(c)(iii) Alt Sol 1

S	M	T	W	T	F	S
B	B	B	B	G	/	/
G	B	B	B	B	G	/
/	G	B	B	B	B	G
/	/	G	B	B	B	B
B	B	B	B	B	G	/
G	B	B	B	B	B	G
/	G	B	B	B	B	B
B	B	B	B	B	B	G
G	B	B	B	B	B	B
B	B	B	B	B	B	B

(c)(iii) Alt Sol 2

S	M	T	W	T	F	S
B	B	B	B	G	G	G
G	B	B	B	B	G	G
G	G	B	B	B	B	G
G	G	G	B	B	B	B
B	B	B	B	B	G	G
G	B	B	B	B	B	G
G	G	B	B	B	B	B
B	B	B	B	G	B	G
B	B	B	B	G	G	B
G	B	B	B	B	G	B
B	G	B	B	B	B	G
B	G	G	B	B	B	B
G	B	G	B	B	B	B
B	B	B	B	B	B	G
B	B	B	B	B	G	B
B	B	B	B	G	B	B
B	B	G	B	B	B	B
B	G	B	B	B	B	B
G	B	B	B	B	B	B
B	B	B	B	B	B	B

(a)		Excellent. Some candidates missed the case of 3 delays in a day.
(b)		Good. Some candidates used incorrect expressions such as $1-(1-0.2942)^3$ to find the required probability.
(c)	(i)	Poor. Quite a number of candidates wrongly used $\frac{1}{\text{P(bad day)}}$ to find the required mean number.
	(ii)	Satisfactory. Some candidates used C_2^7 instead of C_2^6 in the calculation.
	(iii)	Very poor. Many candidates were able to write the related terms for the required probability, but assigned wrong coefficients to them.

32. (2013 DSE-MATH-M1 Q13)

(a) P(the regular maintenance service of a lift in a certain month in the estate is unacceptable)

$= 1 - e^{-1.9}\left(1 + \frac{1.9^1}{1!} + \frac{1.9^2}{2!}\right)$ 1M

≈ 0.296279646

≈ 0.2963 1A

(2)

(b) P(the maintenance service of a lift in June of 2014 is the 3rd month unacceptable)

$\approx C_2^5 (0.296279646)^2 (1 - 0.296279646)^3 \cdot (0.296279646)$ 1M

≈ 0.0906 1A

(2)

(c) The expected total number of unacceptable maintenance services of all lifts for one year

$\approx 15 \times 12 \times 0.296279646$ 1M

≈ 53.3303 1A

(2)

(d) (i) P(a warning letter will be issued for a lift on or before 30th April 2015)

$\approx (0.296279646)^3 + (1 - 0.296279646) \cdot (0.296279646)^3$ 1M+1M

≈ 0.044310205

≈ 0.0443 1A

(ii) P(3 or more warning letters will be issued on or before 30th April 2015)

$\approx 1 - (1 - 0.044310205)^{15} - C_1^{15}(0.044310205)(1 - 0.044310205)^{14}$

$- C_2^{15}(0.044310205)^2 (1 - 0.044310205)^{13}$ 1M+1M

≈ 0.0265 1A

(6)

(a)	Good. Some candidates missed out the term $e^{-1.9}\frac{1.9^2}{2!}$ in the expression $1 - e^{-1.9}\left(1 + \frac{1.9^1}{1!} + \frac{1.9^2}{2!}\right)$, while some others missed out the factor $e^{-1.9}$.
(b)	Satisfactory. Mistakes found were missing the factor C_2^5 or replacing it by C_3^6.
(c)	Poor. Some candidates missed out the factor 15 or 12, while some others used 1.9, the mean of the Poisson distribution given, instead of the probability found in (a).
(d)	Poor. Most candidates were not able to analyse the events correctly to calculate the probabilities.
(i)	Some candidates multiplied factors such as C_3^4, 15 or $\frac{1}{15}$ to the probability $(1-0.296279646)\cdot 0.296279646^3$, some multiplied 2 to 0.296279646^3, while some others wrote $2(1-0.296279646)\cdot 0.296279646^3$ without adding 0.296279646^4 to it.
(ii)	Some candidates used the probability found in (a) instead of that in (d)(i).

33. (2012 DSE-MATH-M1 Q13)

(a) P(at least 2 drunk drivers are prosecuted)

$= 1 - e^{-2.3} - e^{-2.3}(2.3)$ 1A

≈ 0.669145815

≈ 0.6691 1A

(2)

(b) P(≤ 4 drunk drivers are prosecuted | at least 2 drunk drivers are prosecuted)

$\approx \dfrac{e^{-2.3}\left(\frac{2.3^2}{2!} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!}\right)}{0.669145815}$ 1M+1M (1M for Poisson; 1M for conditional prob)

≈ 0.8748 1A

(3)

(c) (i) P(the third night was the 1st night to have ≥ 2 drunk drivers prosecuted)

$\approx (1 - 0.669145815)^2 (0.669145815)$ 1M

≈ 0.0732 1A

(ii) P(≥ 2 drunk drivers prosecuted in each night and totally 10 prosecuted)

$= C_2^3\left(e^{-2.3}\frac{2.3^2}{2!}\right)^2\left(e^{-2.3}\frac{2.3^6}{6!}\right) + 3!\left(e^{-2.3}\frac{2.3^2}{2!}\right)\left(e^{-2.3}\frac{2.3^3}{3!}\right)\left(e^{-2.3}\frac{2.3^5}{5!}\right)$

$+ C_2^3\left(e^{-2.3}\frac{2.3^2}{2!}\right)\left(e^{-2.3}\frac{2.3^4}{4!}\right)^2 + C_2^3\left(e^{-2.3}\frac{2.3^3}{3!}\right)^2\left(e^{-2.3}\frac{2.3^4}{4!}\right)$ 1M+1M (1M for any one case; 1M for all cases)

≈ 0.0471 1A

(5)

(a)		Excellent. However, a small number of candidates forgot the formula of Poisson probabilities.
(b)		Satisfactory. Some candidates failed to write all the terms needed in the numerator.
(c)	(i)	Satisfactory. Many candidates were able to apply the correct method, although some got wrong numerical answers.
	(ii)	Poor. Most candidates failed to identify all the events related to the probability required and some even used 4.6 instead of 2.3 as the mean of the Poisson distribution.

34. (SAMPLE DSE-MATH-M1 Q13)

Solution	Marks	Remarks
(a) The required probability		
$=\frac{6.2^0 e^{-6.2}}{0!}+\frac{6.2^1 e^{-6.2}}{1!}+\frac{6.2^2 e^{-6.2}}{2!}$	1M+1M	1M for correct cases 1M for Poisson prob
≈ 0.053617557		
≈ 0.0536	1A	
	(3)	
Let p be the probability obtained in (a).		
(b) The required probability		
$=(1-p)^{80}+{}_{80}C_1(1-p)^{79}p+{}_{80}C_2(1-p)^{78}p^2$		
$\approx(1-0.053617557)^{80}+80(1-0.053617557)^{79}(0.053617557)$		
$+3160(1-0.053617557)^{78}(0.053617557)^2$	1M+1M	1M for correct cases 1M for binomial prob
≈ 0.1908	1A	
	(3)	
(c) $p+(1-p)p+(1-p)^2p+\ldots+(1-p)^{m-1}p>0.9$	1M+1A	1M for geometric prob
$1-(1-p)^m>0.9$		
$(1-p)^m<0.1$		
$m\ln(1-0.053617557)<\ln(0.1)$	1M	
$m>41.78274367$		
Thus, the least number of operators to be checked is 42.	1A	
	(4)	

35. (2013 ASL-M&S Q12)

Solution	Marks	Remarks
(a) P(three consecutive mini-buses with at least one empty seat) = 0.6465		
$(1-e^{-\lambda})^3=0.6465$	1M	
$\lambda=-\ln(1-\sqrt[3]{0.6465})$		
≈ 2 (correct to the nearest integer)	1A	
	(2)	
(b) (i) P(the 5 members cannot get on the first arriving mini-bus together)		
$=e^{-2}+\frac{2e^{-2}}{1!}+\frac{2^2e^{-2}}{2!}+\frac{2^3e^{-2}}{3!}+\frac{2^4e^{-2}}{4!}$	1M	
$=7e^{-2}$	1A	OR 0.9473
(ii) P(the 5 members will have to wait for more than two mini-buses)		
$=(7e^{-2})^2$	1M	
$=49e^{-4}$	1A	OR 0.8975
	(4)	
(c) (i) P(the group of 2 gets on the first mini-bus and the group of 3 gets on the next mini-bus)		
$=\frac{2^2e^{-2}}{2!}\left[1-\left(e^{-2}+\frac{2e^{-2}}{1!}+\frac{2^2e^{-2}}{2!}\right)\right]$	1M	
$=2e^{-2}(1-5e^{-2})$	1A	OR 0.0875
(ii) P(none of the members have to wait for more than two mini-buses)		
$=\left(e^{-2}+\frac{2e^{-2}}{1!}\right)(1-7e^{-2})+2e^{-2}(1-5e^{-2})$		
$+\left(\frac{2^3e^{-2}}{3!}+\frac{2^4e^{-2}}{4!}\right)\left[1-\left(e^{-2}+\frac{2e^{-2}}{1!}\right)\right]+1-7e^{-2}$ by (b)(i) & (c)(i)	1M+1M	1M for using (c)(i) 1M for any other one case
$=1-37e^{-4}$	1A	OR 0.3223
(iii) P(the group of 2 go first ∣ some members have to wait for more than two mini-buses)		
$=\frac{\frac{2^2e^{-2}}{2!}\cdot e^{-2}\left(1+2+\frac{2^2}{2!}\right)+e^{-2}(1+2)\cdot\frac{2^2e^{-2}}{2!}+[e^{-2}(1+2)]^2\cdot\frac{2^2e^{-2}}{2!}+\cdots}{1-(1-37e^{-4})}$	1M+1A	1A for any one case
$=\frac{10e^{-4}+\frac{6e^{-4}}{1-3e^{-2}}}{37e^{-4}}$	1M	For sum of geometric series
$=\frac{2(8e^2-15)}{37(e^2-3)}$	1A	OR 0.5433
	(9)	

(a)			Satisfactory. Some candidates overlooked that the given probability is for three consecutive arriving mini-buses rather than for one only.
(b)			Good.
(c)	(i)		Satisfactory.
	(ii)		Fair. Many candidates had difficulty in counting and exhausting all the relevant outcomes.
	(iii)		Poor. Some candidates had difficulties in analysing the outcomes and combining different situations, while some failed to recognise that a conditional probability should be considered.

36. (2011 ASL-M&S Q11)

(a) (i) P(lift is full at G/F)

$$=1-e^{-4}\left(1+4+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right)$$ 1M

≈ 0.214869613

≈ 0.2149 1A

(ii) P(4 persons gets into the lift and it stops at each floor)

$$=\frac{e^{-4}4^4}{4!}\cdot 4!\left(\frac{1}{4}\right)^4$$ 1M

≈ 0.0183 1A

(iii) P(lift stops at each floor)

$$=\frac{e^{-4}4^4}{4!}\cdot\frac{4!}{4^4}+\frac{e^{-4}4^5}{5!}\cdot\frac{C_2^5\cdot 4!}{4^5}+0.214869613\cdot\frac{C_3^6\cdot 4!+C_2^6C_2^4\cdot\frac{4!}{2}}{4^6}$$ 1M+1M — 1M for using (i) and (ii); 1M for correct cases

≈ 0.1368 1A

(7)

(b) (i) P(3 persons from different floor waits for the lift)

$=C_3^4(e^{-3}3)^3(e^{-3})$ 1M

≈ 0.0007 1A

(ii) P(2 persons waits for the lift)

$$=C_1^4\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})^3+C_2^4(e^{-3}3)^2(e^{-3})^2$$ 1M — OR $\frac{e^{-12}12^2}{2!}$

≈ 0.0004 1A

(iii) Let the number of persons waiting above 62/F be X.

P(3 persons get into the lift at the 62/F | 3 persons wait at the 62/F)

$=\mathrm{P}(X=0)+\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)$

$$=(e^{-3})^2+C_1^2(e^{-3}3)(e^{-3})+\left[C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})+(e^{-3}3)(e^{-3}3)\right]$$
$$+\left[C_1^2\left(\frac{e^{-3}3^3}{3!}\right)(e^{-3})+C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3}3)\right]$$ 1M+1M+1M — 1M for 4 cases; 1M for the case $X=2$; 1M for the case $X=3$; OR $\sum_{k=0}^{3}\frac{e^{-6}6^k}{k!}$

≈ 0.1512 1A

(8)

(a) (i)		Satisfactory. Candidates seemed to have difficulty in understanding the situation described.
(ii)		Fair. Candidates were unable to master the rules of joint probabilities.
(iii)		Poor. Very few candidates were able to get through this part.
(b) (i) (ii)		Satisfactory.
(iii)		Fair. Candidates had difficulty in exhausting all relevant cases.

37. (2008 ASL-M&S Q10)

(a) The required probability

$$=1-\left(\frac{3.9^0e^{-3.9}}{0!}+\frac{3.9^1e^{-3.9}}{1!}+\frac{3.9^2e^{-3.9}}{2!}+\frac{3.9^3e^{-3.9}}{3!}\right)$$ 1M+1M — 1M for cases correct; 1M for Poisson probability

≈ 0.546753239

≈ 0.5468 1A

(3)

(b) The required probability

$=1-\mathrm{P}$(no *busy* counters are found after the 4th counter is checked)

$\approx 1-(1-0.546753239)^4$ 1M

≈ 0.9578 1A

Alternative Solution 1

The required probability

$\approx C_1^4(0.546753239)(1-0.546753239)^3+C_2^4(0.546753239)^2(1-0.546753239)^2$

$+C_3^4(0.546753239)^3(1-0.546753239)+(0.546753239)^4$ 1M — 1M for Binomial probability

≈ 0.9578 1A

Alternative Solution 2

The required probability

$\approx(0.546753239)+(1-0.546753239)(0.546753239)$

$+(1-0.546753239)^2(0.546753239)+(1-0.546753239)^3(0.546753239)$ 1M — 1M for Geometric probability

≈ 0.9578 1A

(2)

(c) The required probability

$\approx(0.546753239)^{10}+C_9^{10}(0.546753239)^9(1-0.546753239)$

$+C_8^{10}(0.546753239)^8(1-0.546753239)^2$ 1M+1M — 1M for cases correct; 1M for Binomial probability

≈ 0.096004444

≈ 0.0960 1A

(3)

(d) The required probability

$\approx C_8^{10}(0.546753239)^8(1-0.546753239)^2\times\frac{2}{10}$

$+C_9^{10}(0.546753239)^9(1-0.546753239)\times\frac{1}{10}+0$ 1M+1A — 1M for form correct

≈ 0.0167 1A

(3)

(e) The required probability

$$\approx\frac{(0.546753239)^{10}\times[(0.546753239)^5+C_4^5(0.546753239)^4(1-0.546753239)]+C_9^{10}(0.546753239)^9(1-0.546753239)\times(0.546753239)^5}{0.096004444}$$ 1M+1M+1A — 1M for denominator using (c); 1M for numerator form correct; 1A for numerator correct

Alternative Solution

The required probability

$$\approx\frac{(0.546753239)^{15}+C_{14}^{15}(0.546753239)^{14}(1-0.546753239)}{0.096004444}$$ 1M+1M+1A — 1M for denominator using (c); 1M for numerator form correct; 1A for numerator correct

≈ 0.0163 1A

(4)

(a)		Very good.
(b)		Very good.
(c)		Good.
(d)		Poor. Many candidates overlooked that a joint probability should be considered.
(e)		Fair. Many candidates were able to handle conditional probabilities but some were careless.

38. (2006 ASL-M&S Q12)

Solution	Marks
(a) The required probability	
$=1-\left(\frac{2.6^0 e^{-2.6}}{0!}+\frac{2.6^1 e^{-2.6}}{1!}+\frac{2.6^2 e^{-2.6}}{2!}+\frac{2.6^3 e^{-2.6}}{3!}\right)$	1M for cases correct + 1M for Poisson probability
≈ 0.263998355	
≈ 0.2640	1A a–1 for r.t. 0.264 ----------(3)
Let p be the probability described in (a).	
(b) (i) The required probability	
$=p+(1-p)p+(1-p)^2 p+(1-p)^3 p$	1M for the 4 cases + 1M for geometric probability
$=1-(1-p)^4$	
$\approx 1-(1-0.263998355)^4$	
≈ 0.70656282	
≈ 0.7066	1A a–1 for r.t. 0.707
(ii) The required probability	
$\approx \frac{(1-0.263998355)^2(0.263998355)+(1-0.263998355)^3(0.263998355)}{0.70656282}$	1M for numerator using (a) 1M for denominator using (b)(i)
≈ 0.351864771	
≈ 0.3514	1A (accept 0.3513) a–1 for r.t. 0.351
(iii) The integer m satisfies $\mathrm{P}(M \le m) > 0.95$.	
$p+(1-p)p+(1-p)^2 p+\cdots+(1-p)^{m-1} p > 0.95$	1M withhold 1M for bearing an equality sig
$1-(1-p)^m > 0.95$	
$(1-p)^m < 0.05$	
$(1-0.263998355)^m < 0.05$	
$m \ln(0.736001645) < \ln(0.05)$	1M for using log or trial and error
$m > 9.773273146$	
Thus, the least value of m is 10 .	1A ----------(9)
(c) Note that $N \sim \mathrm{B}(150, p)$.	
The mean of N	
$=150p$	
$\approx (150)(0.263998355)$	1M (either one)
≈ 39.59975325	
≈ 39.5998	1A (accept 39.6) a–1 for r.t. 39.600 (either one)
The variance of N	
$=150p(1-p)$	
$\approx (150)(0.263998355)(1-0.263998355)$	
≈ 29.14548353	
≈ 29.1455	1A (accept 29.1456) a–1 for r.t. 29.145 ----------(3)

(a)		Good. Some candidates overlooked the case of a plant without infected leaves.
(b) (i)		Good. Some candidates overlooked the case of $M=0$.
(ii)		Good. Many candidates could tackle this part on conditional probability.
(iii)		Not satisfactory. Only a few candidates were able to formulate the inequality correctly and simplify the expression to arrive at the conclusion.
(c)		Good. Many candidates could apply the binomial distribution although some candidates forgot the formulas for the mean and the variance of the distribution.

39. (2003 ASL-M&S Q10)

Solution	Marks
(a) Sample mean	
$=\frac{12(1)+14(2)+10(3)+6(4)+2(5)+1(6)}{5+12+14+10+6+2+1}$	
$=2.2$	1A
Sample Standard deviation	
$=\sqrt{\frac{12(1^2)+14(2^2)+10(3^2)+6(4^2)+2(5^2)+1(6)^2-(50)(2.2)^2}{5+12+14+10+6+2+1-1}}$	
$=\sqrt{2}$	1A
≈ 1.414213562	
≈ 1.4142	a–1 for r.t. 1.414 ----------(2)
(b) (i) The required probability	
$=\frac{2.2^0 e^{-2.2}}{0!}+\frac{2.2^1 e^{-2.2}}{1!}+\frac{2.2^2 e^{-2.2}}{2!}+\frac{2.2^3 e^{-2.2}}{3!}$	1M for the 4 cases + 1M for Poisson probability
≈ 0.819352421	
≈ 0.8194	1A (accept 0.8193) a–1 for r.t. 0.819
(ii) The required probability	
$\approx C_2^6(0.819352421)^2(1-0.819352421)^4$	1M for Binomial probability + 1M for using (b)(i)
≈ 0.010724111	
≈ 0.0107	1A a–1 for r.t. 0.011 ----------(6)
(c) (i) The required probability	
$=C_2^3(0.55)^2(0.45)$	1M for $(0.55)^2(0.45)$
$=0.408375$	1A
≈ 0.4084	a–1 for r.t. 0.408
(ii) The required probability	
$=\frac{2.2^2 e^{-2.2}}{2}(0.55^2)$	1M
≈ 0.081113452	
≈ 0.0811	1A a–1 for r.t. 0.081
(iii) The required probability	
$\approx \frac{\left(\frac{2.2^2 e^{-2.2}}{2!}\right)(0.55)^2+\left(\frac{2.2^3 e^{-2.2}}{3!}\right)(0.55)^3}{0.819352421}$	1M for numerator + 1M for denominator using (b)(i)
≈ 0.138925825	
≈ 0.1389	1A a–1 for r.t. 0.139 ----------(7)

(a)		Good. Candidates should have used '$n-1$' rather than 'n' when finding the sample standard deviation.
(b)		Good. Most candidates were able to apply the binomial distribution.
(c) (i)		Good. A few candidates forgot the binomial coefficient 'C_2^3'.
(ii)		Fair.
(iii)		Poor. Very few candidates were able to correctly obtain the required conditional probability.

40. (2001 ASL-M&S Q11)

Let X_A and X_B be the numbers of persons entered the building using entrances A and B respectively within a 15-minute period.

(a) (i) $P(X_A=0)=\dfrac{(3.2)^0e^{-3.2}}{0!}=e^{-3.2}$ $\boxed{0.0408}$ (p_1) — 1A $a-1$ for r.t. 0.041

(ii) $P(X_B=0)=\dfrac{(2.7)^0e^{-2.7}}{0!}=e^{-2.7}$ $\boxed{0.0672}$ (p_2) — 1A $a-1$ for r.t. 0.067

(iii) $P(X_A+X_B\ge 1)=1-P(X_A=0 \text{ and } X_B=0)$

$=1-P(X_A=0)P(X_B=0)$

$=1-e^{-3.2}e^{-2.7}$ — 1M $1-(p_1)(p_2)$

$=1-e^{-5.9}$ $\boxed{0.9973}$ — 1A $a-1$ for r.t. 0.997

(iv) $P(X_A+X_B=2)$

$=P(X_A=2)P(X_B=0)+P(X_A=1)P(X_B=1)+P(X_A=0)P(X_B=2)$ — 1M for the 3 cases

$=\dfrac{(3.2)^2e^{-3.2}}{2!}\cdot e^{-2.7}+\dfrac{3.2e^{-3.2}}{1!}\cdot\dfrac{2.7e^{-2.7}}{1!}+e^{-3.2}\cdot\dfrac{(2.7)^2e^{-2.7}}{2!}$ — 1A

$=17.405e^{-5.9}$ $\boxed{0.0477}$ — 1A $a-1$ for r.t. 0.048

---------(7)

(b) (i) Since k is the most probable number of persons entered the building within a 15-minute period,

$\therefore$ $P(X=k-1)\le P(X=k)$ and $P(X=k+1)\le P(X=k)$ — 1M+1M

Hence $\dfrac{\lambda^{k-1}e^{-\lambda}}{(k-1)!}\le\dfrac{\lambda^k e^{-\lambda}}{k!}$

$k\le\lambda$ — 1

and $\dfrac{\lambda^{k+1}e^{-\lambda}}{(k+1)!}\le\dfrac{\lambda^k e^{-\lambda}}{k!}$

$\lambda\le k+1$

$\lambda-1\le k$ — 1

(ii) From (b)(i), $k=5$. — 1A

The probability required

$=C_2^4[P(X=k)]^2[1-P(X=k)]^2[P(X=k)]$

$=C_2^4\left(\dfrac{(5.9)^5e^{-5.9}}{5!}\right)^2\left(1-\dfrac{(5.9)^5e^{-5.9}}{5!}\right)^2\left(\dfrac{(5.9)^5e^{-5.9}}{5!}\right)$ — 1M for binomial, 1M for all

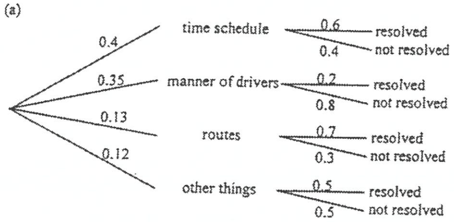
≈ 0.0183 — 1A $a-1$ for r.t. 0.018

---------(8)

41. (1999 ASL-M&S Q12)

Let N be the number of complaints received on a given day and X be the number of complaints involving the time schedule.

(a)



P(manner of drivers | not resolved)

$=\dfrac{0.35\times0.8}{0.4\times0.4+0.35\times0.8+0.13\times0.3+0.12\times0.5}$ $\left(\dfrac{p_1}{p_2}\right)$ — 1M+1A+1A (1A for p_1, 1A for p_2, 1M for $\dfrac{p_1}{p_2}$)

≈ 0.5195 — 1A $a-1$ for r.t. 0.519

(b) (i) $P(N=5)=\dfrac{10^5e^{-10}}{5!}$ — 1A

≈ 0.0378 (p_3) — 1A

(ii) $P(N=5 \text{ and } X=3)=\dfrac{10^5e^{-10}}{5!}\left(C_3^5(0.4)^3(0.6)^2\right)$ — 1M $p_3\left(C_3^5(0.4)^3(0.6)^2\right)$

≈ 0.0087 — 1A $a-1$ for r.t. 0.009

(c) $n\ge 9$. (or $P(N=n \text{ and } X=9)=0$ for $n<9$) — 1M

$P(N=n \text{ and } X=9)=\dfrac{10^n e^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$ — 1A

(d) (i) $\sum_{k=9}^{\infty}\dfrac{x^k}{(k-9)!}=x^9+\dfrac{x^{10}}{1!}+\dfrac{x^{11}}{2!}+\dfrac{x^{12}}{3!}+\cdots$

$=x^9\left(1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots\right)$ — 1A

$=x^9e^x$ — 1

(ii) $P(X=9)=\sum_{n=9}^{\infty}P(N=n \text{ and } X=9)$

$=\sum_{n=9}^{\infty}\dfrac{10^n e^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$ — 1M

$=\sum_{n=9}^{\infty}\dfrac{10^n e^{-10}}{n!}\cdot\dfrac{n!}{(n-9)!9!}(0.4)^9(0.6)^{n-9}$

$=\dfrac{e^{-10}(0.4)^9}{9!(0.6)^9}\sum_{n=9}^{\infty}\dfrac{6^n}{(n-9)!}$ — 1A

$=\dfrac{e^{-10}(0.4)^9}{9!(0.6)^9}6^9e^6$ (by (b)(i))

$=\dfrac{4^9e^{-4}}{9!}$ (or 0.0132) — 1A $a-1$ for r.t. 0.013

42. (1998 ASL-M&S Q11)

Solution	Marks
(a) Let X be the no. of printing mistakes on P.23, then $X \sim \text{Po}(0.2)$.	
$P(X=0) = e^{-0.2}$	1M+1A
≈ 0.8187	
(b) (i) Let p be the probability that there are printing mistakes on a page, then	
$p = 1 - e^{-0.2}$	1M
Hence $N \sim \text{Geometric}(p)$ and	1M
$P(N \le 3) = P(N=1) + P(N=2) + P(N=3)$	
$= p + p(1-p) + p(1-p)^2$	1M+1A
$= 1-(1-p)^3$	
$= 1-e^{-0.6}$	
≈ 0.4512	1A
(ii) Mean of $N = \frac{1}{p} = \frac{1}{1-e^{-0.2}} \approx 5.5167$	1A
Variance of $N = \frac{1-p}{p^2} = \frac{e^{-0.2}}{(1-e^{-0.2})^2} \approx 24.9168$	1A
(c) $M \sim \text{Binomial}(200, p)$ where $p = 1-e^{-0.2}$.	
Mean of $M = np = 200(1-e^{-0.2}) \approx 36.2538$	1A
Variance of $M = np(1-p) = 200e^{-0.2}(1-e^{-0.2}) \approx 29.6821$	1A
(d) (i) $Y \sim \text{Binomial}(40, \frac{1}{200})$.	1A+1A
(ii) $P(Y=0) = \left(1-\frac{1}{200}\right)^{40} \approx 0.8183$	1M+1A

43. (1996 ASL-M&S Q13)

Solution	Marks	Remarks
(a) Let X be the number of rainstorms in a year. $X \sim \text{Po}(2)$		
$P(X=x) = \frac{e^{-2}2^x}{x!}$, $x = 0, 1, 2, \ldots$		
$P(X \ge 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$	1M	
$= 1 - e^{-2}\left[1+2+\frac{4}{2}\right]$	1A	
$= 1-5e^{-2}$		
≈ 0.3233	1A	
(b) Let Y be the number of years which will elapse before the next occurence of more than two rainstorms in a year. $Y \sim \text{Geometric}(p=0.3233)$.	1M	
Number of years which will elapse $= \frac{1}{p} - 1$	1M	For $\frac{1}{p}$
$= 2.0929$		
≈ 2	1A	
(c) Let A be the event of having at least one serious landslide in city A.		
$P(A \mid X=0) = 0.2$		
$P(A \mid X=1, 2) = 0.3$		
$P(A \mid X \ge 3) = 0.5$		
(i) $P(\bar{A})$		
$= P(\bar{A} \mid X=0)P(X=0) + P(\bar{A} \mid X=1,2)P(X=1,2) + P(\bar{A} \mid X \ge 3)P(X \ge 3)$		
$= 0.8(e^{-2}) + 0.7(4e^{-2}) + 0.5(1-5e^{-2})$	1M+1A	
≈ 0.6489	1A	
<u>Alternatively,</u>		
$P(\bar{A}) = 1 - P(A)$	1M+1A	
$= 1-[0.2(e^{-2}) + 0.3(4e^{-2}) + 0.5(1-5e^{-2})]$		
≈ 0.6489	1A	
(ii) $P(X=0 \mid \bar{A}) = \frac{P(\bar{A} \mid X=0)P(X=0)}{P(\bar{A})}$		
$= \frac{0.8(e^{-2})}{0.6489}$	1M+1M	1A for the numerator 1M for the denominator
≈ 0.1669	1A	
(iii) The probability that there is no serious landslide for at most 2 out of 5 years		
$= C_0^5(1-0.6489)^5 + C_1^5(0.6489)(1-0.6489)^4 + C_2^5(0.6489)^2(1-0.6489)^3$	1M+1M	
≈ 0.2369	1A	
<u>Alternatively,</u>		
$1-[C_3^5(0.6489)^3(1-0.6489)^2 + C_4^5(0.6489)^4(1-0.6489) + C_5^5(0.6489)^5]$	1M+1M	
≈ 0.2369	1A	

10. Normal Distribution

Learning Unit	Learning Objective
Statistics Area	
Normal Distribution	
18. Basic definition and properties	18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution 18.2 recognise the concept and properties of the normal distribution
19. Standardisation of a normal variable and use of the standard normal table	19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution.
20. Applications of the normal distribution	20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu, \sigma^2)$ 20.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$ 20.3 use the normal distribution to solve problems

Section A

1. A factory manufactures a batch of marbles. The diameters of the marbles follow a normal distribution with a mean of 9 mm and a standard deviation of 0.125 mm . A marble is classified as *oversized* if its diameter is more than 9.16 mm .
   (a) Find the probability that a randomly selected marble from the batch is *oversized*.
   (b) The diameters of the marbles are measured one by one. Let X be the random variable representing the number of measurements taken when the first *oversized* marble is found. Find
   (i) $P(X \le 3)$,
   (ii) $E(X)$.

(6 marks) (2018 DSE-MATH-M1 Q3)

2. In a large farm, the weights of chickens follow a normal distribution with a mean of μ kg and a standard deviation of σ kg . It is given that the percentage of chickens being lighter than 1.83 kg is the same as the percentage of those being heavier than 3.43 kg. Moreover, 89.04% chickens weigh between 1.83 kg and 3.43 kg.
   (a) Find μ and σ.
   (b) If 9 chickens are selected randomly from the farm, find the probability that the mean of their weights lies between 2.5 kg and 3.1 kg.

(5 marks) (2017 DSE-MATH-M1 Q3)

3. Among the students sitting for a Mathematics test, 73% of them had revised before the test. For those who had revised, their scores are real numbers which can be modelled by $N(59, 10^2)$; and for those who had not revised, their scores are real numbers which can be modelled by $N(35.2, 12^2)$. Students who scored at least 43 passed the test.
   (a) Find the probability that a randomly selected student passed the test.
   (b) Given that a randomly selected student passed the test, find the probability that he had not revised before the test.
   (c) Ten students are randomly selected among those who passed the test. Find the probability that exactly four of them had not revised before the test.

(7 marks) (2012 DSE-MATH-M1 Q9)

4. The coach of a girls school basketball team recruits new members from the Form One students, of whom 11.7% are taller than 152 cm. Assume that their heights are normally distributed with a mean μ cm and a standard deviation of 5 cm.

(a) Find the value of μ .

(b) It is known that 20% of the Form One students taller than 152 cm do not apply to join the basketball team, while 10% of students shorter than 152 cm apply to join. If a Form One student is selected at random, find the probability that

(i) the student applies to join the basketball team;

(ii) the student is shorter than 152 cm given that she does not apply to join the basketball team.

(7 marks) (2010 ASL-M&S Q6)

5. The amount of money spent by a randomly selected customer of a jewellery shop is assumed to be normally distributed with a mean of \$ μ and a standard deviation of \$6 000. Suppose 24.2% of the customers spend more than \$30 000 in the shop.

(a) Find the value of μ .

(b) It is given that Mrs. Chan spends less than \$30 000 in the shop. Find the probability that she spends more than \$16 500 .

(6 marks) (2008 ASL-M&S Q5)

6. Some statistics from a survey on the monthly incomes (in thousands of dollars) of a group of university graduates are summarized as follows:

Minimum	8
Maximum	52
Lower quartile	10
Median	17
Upper quartile	20
Mean	17.94
Standard deviation	4.7

(a) Using the above information, construct a box-and-whisker diagram to describe the distribution of the monthly incomes.

(b) A student proposes to model the distribution of the monthly incomes of the group of university graduates by a normal distribution with mean and standard deviation given in the above table.

(i) Using the model proposed by the student, find the probability that the monthly income of a randomly selected university graduate from the group is less than \$ 17 000 .

(ii) Is the model proposed by the student appropriate? Explain your answer.

(6 marks) (2004 ASL-M&S Q5)

7. The amount of money involved in a business transaction follows a normal distribution with mean \$215 and standard deviation \$50 . Any transaction with an amount more than \$300 is classified as a Type A transaction.

(a) Find the probability that a transaction will be classified as Type A.

(b) Find the probability that in 7 randomly selected transactions, exactly 2 transactions will be classified as Type A.

(c) Find the probability that the 8th randomly selected transaction is the 3rd transaction which is classified as Type A.

(d) It is known that 64.8% of the transactions each exceeds $\$K$. Find K .

(7 marks) (2003 ASL-M&S Q6)

8. (a) Use the exponential series to find a polynomial of degree 6 which approximates $e^{\frac{-x^2}{2}}$ for x close to 0 .

Hence estimate the integral $\int_0^1 e^{\frac{-x^2}{2}} dx$.

(b) It is known that the area under the standard normal curve between $z=0$ and $z=a$ is $\int_0^a \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz$. Use the result of (a) and the normal distribution table to estimate, to 3 decimal places, the value of π .

(7 marks) (1994 ASL-M&S Q6)

Section B

9. A fruit wholesaler, John, grades a batch of apples according to their weights. The following table shows the classification of the apples, where a is a constant.

Weight of an apple (W g)	$W \le a$	$a < W \le 260$	$W > 260$
Classification	*small*	*medium*	*large*

The weights of the apples follow a normal distribution with a mean of μ g and a standard deviation of 16 g. It is known that 10.56% and 73.57% of the apples are *large* and *medium* respectively. Every 8 of the apples are packed in a box. A box of apples is regarded as *regular* if there are at least 6 *medium* apples in the box.

(a) Find μ and a .

(3 marks)

(b) Find the probability that a randomly chosen box of apples is *regular*.

(2 marks)

(c) John randomly chooses 3 boxes of apples.

(i) Find the probability that these 3 boxes of apples are *regular* and there are totally 21 *medium* apples and 3 *small* apples.

(ii) Given that these 3 boxes of apples are *regular*, find the probability that there are totally 21 *medium* apples and 3 *small* apples.

(iii) Given that there are totally 21 *medium* apples and 3 *small* apples in these 3 boxes of apples, find the probability that these 3 boxes of apples are *regular*.

(7 marks)

(2018 DSE-MATH-M1 Q9)

10. X and Y are two schools with the same number of students. The daily reading times (in minutes) of the students in each school are assumed to be normally distributed. In school X, 0.6% of the students read less than 40 minutes daily while 1.5% read more than 70 minutes. In school Y, 1.5% of the students read less than 48 minutes daily while 1. 7% read more than 72 minutes.

(a) Which school has less students reading more than 60 minutes daily? Explain your answer.

(6 marks)

(b) For the school that has less students reading more than 60 minutes daily, find the probability that the 4th randomly selected student is the 2nd one who reads more than 60 minutes daily.

(2 marks)

(c) Students reading T minutes or more daily will be awarded. What should the least value of T be so that no more than 10% of students are awarded in each school? Give your answer in integral minutes.

(4 marks)

(2016 DSE-MATH-M1 Q9)

11. Let $I=\int_1^4 \frac{1}{\sqrt{t}}e^{\frac{-t}{2}}\,dt$.

(a) (i) Use the trapezoidal rule with 6 sub-intervals to estimate I .

(ii) Is the estimate in (a)(i) an over-estimate or under-estimate? Justify your answer.

(7 marks)

(b) Using a suitable substitution, show that $I=2\int_1^2 e^{\frac{-x^2}{2}}\,dx$.

(3 marks)

(c) Using the above results and the Standard Normal Distribution Table, show that $\pi<3.25$.

(3 marks)

(2012 DSE-MATH-M1 Q10)

12. In a supermarket, there are two cashier counters: a regular counter and an express counter. It is known that 88% of customers pay at the regular counter. It is found that the waiting time for a customer to pay at the regular counter follows the normal distribution with mean 6.6 minutes and standard deviation 1.2 minutes.

(a) Find the probability that the waiting time for a customer to pay at the regular counter is more than 6 minutes.

(2 marks)

(b) Suppose 12 customers who pay at the regular counter are randomly selected. Assume that their waiting times are independent.

(i) Find the probability that there are more than 10 of the 12 customers each having a waiting time of more than 6 minutes.

(ii) Find the probability that the average waiting time of the 12 customers is more than 6 minutes.

(5 marks)

(b) It is found that the waiting time for a customer to pay at the express counter follows the normal distribution with mean μ minutes and standard deviation 0.8 minutes. It is known that exactly 21.19% of the customers at the regular counter wait less than k minutes, while exactly 3.59% of the customers at the express counter wait more than k minutes.

(i) Find k and μ.

(ii) Two customers are randomly selected. Assume that their waiting times are independent. Given that both of them wait more than μ minutes to pay, find the probability that exactly one of them pays at the regular counter.

(8 marks)

(PP DSE-MATH-M1 Q13)

13. The speeds of the vehicles (X km/h) on a highway follow a normal distribution with mean μ km/h and standard deviation σ km/h . It is known that 12.3% of vehicles have speeds more than 82.64 km/h and 24.2% of vehicles have speeds less than 75.2 km/h . A machine is used to detect the speeds of the vehicles at a spot on the highway. A notice will be issued to the driver if the speed of his/her vehicle is detected to be over 80 km/h .

(a) Find μ and σ .

(3 marks)

(b) (i) A vehicle passes the spot. What is the probability that a notice will be issued?

(ii) Suppose that 10 vehicles pass the spot on the highway. What is the probability that at most 2 notices will be issued?

(4 marks)

(c) On a certain day, the machine does not work properly and there is an error in detecting the speeds of the vehicles. The error (Y km/h) is defined as follows:

Y = speed detected – actual speed ,

and it can be modelled by the following probability distribution:

Error (Y)	2	$2+\theta$
Probability	0.5	0.5

where θ is a non-zero constant. A vehicle passes the spot.

(i) Find the probability that a notice will be issued but the speed of the vehicle is not over 80 km/h for the following two cases:

(1) $\theta=1$,

(2) $\theta=-3$.

(ii) Find the range of values of θ such that the probability that a notice will not be issued but the speed of the vehicle is over 80 km/h is at most 7.125% .

(8 marks)

(2013 ASL-M&S Q10)

14. A manufacturer produces batteries A and B for notebook computers. After fully charged, the operation times (in minutes) of batteries A are normally distributed with mean 168 minutes and standard deviation 32 minutes, and those of batteries B are normally distributed with mean μ minutes and standard deviation σ minutes. Past data revealed that 33% of batteries B have operation times longer than 188 minutes, while 87.7% have operation times shorter than 213.2 minutes.

(a) (i) Find the probability that a randomly chosen battery A has an operation time shorter than 152 minutes or longer than 184 minutes.

(ii) If the probability that a randomly chosen battery A has an operation time longer than k minutes is 5%, find the value of k.

(iii) Find the values of μ and σ.

(iv) Find the probability of a randomly chosen battery B having an operation time shorter than 146 minutes.

(7 marks)

(b) The manufacturer produces 1500 batteries per day. One-third of them are A and the rest are B. A battery is regarded as 'faulty' when the operation time is shorter than 104 minutes. Let λ_A and λ_B be respectively the mean numbers of 'faulty' batteries of A and B produced per day. Assume that the numbers of 'faulty' batteries A and B produced per day can be approximately modelled by Poisson distributions with means λ_A and λ_B.

(i) Find λ_A and λ_B correct to 1 decimal place.

(ii) Find the probability that the number of 'faulty' batteries A produced on a certain day is between 4 and 6 inclusively.

(iii) Given that the total number of 'faulty' batteries A and B produced on a certain day is 10 and the number of 'faulty' batteries A produced is between 4 and 6 inclusively, find the probability that the number of 'faulty' batteries B produced is more than 4.

(8 marks)

(2012 ASL-M&S Q10)

15. In a scoring game, a player will roll a ball at a starting point along a long horizontal track. When the ball comes to rest, let Y cm be the distance of the ball having travelled. The scoring system is shown in the following table.

Range of Y	$154 \le Y < 160$	$160 \le Y < K$	$K \le Y < 174$	Otherwise
Score	20	50	30	0

It is known that Y can be modelled by a normal distribution with mean 165 and variance 16. It is also known that 78.88% of the players score 50 in a game. A game in which the player scores 50 is called "Bingo". Assume that the games are independent.

(a) Find the value of K.

(2 marks)

(b) Find the probability that a player will score 30 in a game.

(2 marks)

(c) Find the probability that the 6th game is the 3rd "Bingo".

(2 marks)

(d) If the variance of the number of "Bingo" in n games is at most 2.3, determine the largest value of n.

(2 marks)

(e) A player will win a prize if his average score in 4 games is at least 40.

(i) Find the probability that a player will win the prize.

(ii) Find the probability that he wins the prize and his average score in the first 2 games is at least 40.

(iii) Given that a player wins the prize, find the probability that his average score in the first 2 games is less than 40.

(7 marks)

(2011 ASL-M&S Q10)

16. A construction company proposes to use the daily rainfall precipitation to determine the effect of rainfall on a construction project. The following table shows the classification system.

Daily Rainfall Precipitation (Y mm)	$Y < 100$	$100 \le Y < 150$	$150 \le Y < 200$	$Y \ge 200$
Effect Level of the Day	Low	Medium	High	Severe

Assume that the daily rainfall precipitation recorded follows a normal distribution with mean μ mm and standard deviation σ mm. From past record, 12.10% of the days are classified as Low and 9.18% of the days are classified as Severe.

(a) Find the values of μ and σ.

(3 marks)

(b) Find the probability that a day is classified as High.

(1 mark)

(c) It is given that, in a certain rainy day, the rainfall precipitation exceeds 100mm. Find the probability that the day is classified as High.

(2 marks)

(d) In a construction site, the numbers of days that a project is postponed under the precipitation levels Medium, High and Severe of a rainy day follow Poisson distributions with means 1, 3 and 6 respectively. The project will not be postponed if a day is classified as Low. Given that during the construction of the project, there is exactly 1 rainy day with precipitation exceeding 100 mm.

(i) Find the probability that the project will NOT be postponed.

(ii) Find the probability that the project will be postponed for exactly 1 day.

(iii) Given that the project is postponed for at least 3 days, find the probability that the rainy day is classified as High.

(9 marks)

(2011 ASL-M&S Q12)

17. Suppose the width of the tongues of normal new born babies can be modelled by a normal distribution with mean μ cm and standard deviation 0.4 cm. It is known that 24.2% of the normal babies will have their tongue widths less than 2.22 cm . If babies have inherited a certain genetic disease A , their tongues will be wider. It is known that 5% of new born babies have inherited disease A and the width of their tongues can be modelled by a normal distribution with mean $(\mu + 0.3)$ cm and standard deviation 0.2 cm . A diagnostic test is proposed such that if the width of the tongue of a baby is wider than $(\mu + 0.5)$ cm , he/she is diagnosed to have inherited disease A.

(a) Find the value of μ .

(1 mark)

(b) (i) What is the probability that a normal baby is diagnosed as having inherited disease A ?

(ii) What is the probability of a wrong diagnosis?

(iii) Given that a baby is diagnosed as NOT having inherited disease A , what is the probability that the baby has actually inherited the disease?

(8 marks)

(c) A group of 20 babies are going to take the test one by one.

(i) Given that exactly 4 babies are diagnosed wrongly among the 20 babies, what is the probability that exactly 3 babies are diagnosed wrongly in the first 8 tests?

(ii) Given that at most 4 babies are diagnosed wrongly among the 20 babies, what is the probability that the 8th baby to take the test is the 3rd baby who is diagnosed wrongly?

(6 marks)

(2009 ASL-M&S Q11)

18. A manager of a maintenance centre launches an appraisal system to assess the performance of technicians in terms of the time spent to complete a task. A technician can get 2 points if he takes less than 2 hours to complete a task, 1 point if he takes between 2 and 4.6 hours, and 0 point if he takes longer than 4.6 hours.
Assume the time for a technician to complete a task is normally distributed with a mean of 3 hours and a standard deviation of 0.8 hour, and the number of tasks assigned to a technician follows a Poisson distribution with a mean of 1.8 tasks per day.

(a) Find the probability that a technician is assigned not more than 4 tasks on a certain day.

(3 marks)

(b) Let p_i be the probability of a technician getting i point(s) upon completing a task, where $i = 0, 1, 2$. Find the values of p_0 , p_1 and p_2 .

(3 marks)

(c) Find the probability that a technician gets exactly 4 points on a certain day under each of the following conditions:

(i) 3 tasks are assigned,

(ii) 4 tasks are assigned.

(5 marks)

(d) It is given that a technician is assigned fewer than 5 tasks on a certain day. Find the probability that the technician gets exactly 4 points.

(4 marks)

(2008 ASL-M&S Q11)

19. The manager, Teresa, of a superstore launches a promotion plan to increase the sales volume. The number of customers shopping at the superstore in a minute can be modelled by a Poisson distribution with a mean of 2.4 customers per minute. The expense of customers in the superstore are assumed to be independent and follow a normal distribution with a mean of \$ 375 and a standard deviation of \$ 125. A customer who spends more than \$ 300 but less than \$ 600 in the superstore can enter lucky draw X in which the probability of winning a gift is 0.25. A customer who spends \$ 600 or more in the superstore can enter lucky draw Y in which the probability of winning a gift is 0.8. Assume that each customer enters at most one lucky draw for each visit.

(a) Find the probability that there are more than 2 customers shopping at the superstore in a certain minute.

(3 marks)

(b) Find the probability that a randomly selected customer shopping at the superstore can enter lucky draw X.

(2 marks)

(c) Find the probability that a randomly selected customer shopping at the superstore wins a gift.

(2 marks)

(d) Find the probability that there are exactly 3 customers shopping at the superstore in a certain minute and each of them wins a gift.

(2 marks)

(e) Given that there are more than 2 customers shopping at the superstore in a certain minute, find the probability that there are fewer than 5 customers shopping at the superstore in this minute and each of them wins a gift.

(3 marks)

(f) If Teresa wants to revise that least expense of a customer for entering lucky draw Y so that 33% of the customers shopping at the superstore could enter lucky draw Y, what should be the revised least expense?

(3 marks)

(2007 ASL-M&S Q10)

20. A factory produces brand D coffee beans which are packed into boxes of 30 cans each. The net weight of each can of coffee beans follows a normal distribution with a mean of 300g and a standard deviation of 7.5 g. A can of coffee beans with net weight less than 283.5 g or more than 316.5 g is classified as *exceptional*.

(a) Find the probability that a randomly selected can of brand D coffee beans is *exceptional*.

(2 marks)

(b) The manager of the factory randomly selects a box of brand D coffee beans and inspects every can in the box one by one.

(i) Find the probability that the 12th inspected can is the 1st *exceptional* can of coffee beans in the box.

(ii) Find the probability that there is exactly 1 *exceptional* can of coffee bean in the box.

(iii) Find the probability that there is at most 1 *exceptional* can of coffee beans in the box.

(8 marks)

(c) The shopkeeper of a coffee shop buys one box of brand D coffee beans. The shopkeeper regards a can of coffee beans as *unacceptable* if the net weight of the can is less than 283.5g.

(i) Find the probability that in the box there is exactly 1 *exceptional* can of coffee beans which is *unacceptable*.

(ii) Given that in the box there is at most 1 *exceptional* can of coffee beans, find the probability that there is exactly 1 *unacceptable* can of coffee beans in the box.

(5 marks)

(2007 ASL-M&S Q11)

21. A researcher models the number of cars entering a roundabout in five-second time intervals (FSTIs) by a Poisson distribution with a mean of 4.7 cars per FSTI, and the speed of a car entering the roundabout by a normal distribution with a mean of 42.8km/h and a standard deviation of 12 km/h. A car is *speeding* if the speed of the car is over 50 km/h.

(a) Find the probability that fewer than 6 cars enter the roundabout in a certain FSTI.

(3 marks)

(b) Find the probability that a car entering the roundabout is *speeding*.

(2 marks)

(c) Find the probability that the 6th car entering the roundabout is the 1st *speeding* car.

(3 marks)

(d) The roundabout is *hazardous* in a certain FSTI if at least 4 cars enter the roundabout in that FSTI and more than 2 of them are *speeding*.

(i) If exactly 4 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(ii) Given that fewer than 6 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(7 marks)

(2006 ASL-M&S Q10)

22. In a city, the number of cars entering a filling station for petrol per hour can be modelled by a Poisson distribution with a mean of 6.2 cars per hour.

(a) Find the probability that there are fewer than 5 cars entering the filling station for petrol in a certain hour.

(3 marks)

(b) The manager of the filling station models the amount of petrol for refuelling a car by a normal distribution with a mean of 23.2 litres and a standard deviation of 6 litres.

(i) Find the probability that the amount of petrol for refuelling a car is at least 25 litres.

(ii) Find the probability that the 9th car entering the filling station for petrol is the 3rd car which has been refuelled with at least 25 litres.

(iii) Find the probability that there are exactly 3 cars entering the filling station for petrol in a certain hour and each of them will be refuelled with at least 25 litres.

(iv) If there are exactly 4 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(v) Given that there are fewer than 5 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(12 marks)

(2005 ASL-M&S Q10)

23. Every school day, Peter leaves home at 7:00 a.m. to go to the train station to take a train to his school. The time needed for him to go to the train station platform follows a normal distribution with a mean of 17.5 minutes and a standard deviation of 2 minutes.

The following table shows the departure times for trains A , B and C and the probabilities that Peter to be late when taking trains A , B and C respectively:

Train	Departure time	Probability for Peter to be late
A	7:13 a.m.	0.02
B	7:19 a.m.	0.15
C	7:22 a.m.	0.35

Peter takes the earliest departing train when he arrives at the train station platform. Assume that the time needed for him to get on the train from the platform is negligible. It is certain that he will be late if he cannot catch any one of the trains A, B and C .

(a) Find the probability that Peter takes train B to the school on a certain morning.

(2 marks)

(b) Find the probability that Peter is late on a certain morning.

(3 marks)

(c) Given that Peter is late on a certain morning, find the probability that Peter takes train B to the school on this morning.

(2 marks)

(d) Find the probability that Peter is late on exactly 2 mornings in a certain week of 5 school days.

(2 marks)

(e) Given that Peter is late on exactly 2 mornings in a certain week of 5 school days, find the probability that he takes train B to the school only on these 2 mornings.

(3 marks)

(f) If Peter tries to leave home earlier so that the probability of his getting on train A is at least 0.95, what is the latest time that he should leave home? Give your answer correct to the nearest minute.

(3 marks)

(2005 ASL-M&S Q11)

24. A customer who spends \$300 or more in a store during a visit is classified as a 'valuable' customer. The expenses of customers in the store are assumed to be independent and follow a normal distribution with a mean of \$428 and a standard deviation of \$100. The number of customers visiting the store in a minute can be modeled by a Poisson distribution with a mean of 4 customers per minute.

(a) Find the probability that a randomly selected customer of the store is a 'valuable' customer.

(2 marks)

(b) Find the probability that there are at least 2 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day.

(3 marks)

(c) Find the probability that there are exactly 3 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day and exactly 2 of them are 'valuable' customers.

(3 marks)

(d) Given that there are 2 or 3 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day, find the probability that exactly 2 of them are 'valuable' customers.

(3 marks)

(e) A customer who spends \$600 or more in the store during a visit will receive a gift. If the probability of the store giving out gifts is at least 0.99, find the smallest number of customers visiting the store.

(4 marks)

(2004 ASL-M&S Q12)

25. A teacher randomly selected 7 students from a class of 13 boys and 17 girls to form a group to take part in a flag-selling activity.

(a) Find the probability that the group consists of at least 1 boy and 1 girl.

(3 marks)

(b) Given that the group consists of at least 1 boy and 1 girl, find the probability that there are more than 2 girls in the group.

(3 marks)

(c) A group of 3 boys and 4 girls is formed. It is known that the amount of money collected by a boy and a girl in the activity can be modelled respectively by normal distributions with the following means and standard deviation.

Student	Mean	Standard deviation
Boy	\$673	\$100
Girls	\$708	\$100

Any student who collects more than \$800 receives a certificate.

(i) Find the probability that a particular boy in the group will receive a certificate.

(ii) Find the probability that exactly 1 boy and 1 girl in the group will receive certificates.

(iii) Given that the group has received 2 certificates, find the probability that exactly 1 boy and 1 girl received the certificates.

(9 marks)

(2003 ASL-M&S Q12)

26. The weight of each bag of self-raising flour in a batch produced by a factory follows a normal distribution with mean 400 g and standard deviation 10 g . A bag of flour with weight less than 376 g is **underweight**, and more than 424 g is **overweight**.

(a) Find the probability that a randomly selected bag of flour

(i) is **underweight**;

(ii) is **overweight**.

(3 marks)

(b) If a bag of flour is either **underweight** or **overweight**, it will be classified as a **substandard** bag by the director of the factory. The director randomly selects 50 bags as a sample from the batch.

(i) Find the probability that there is no **substandard** bag of flour in the sample.

(ii) Find the probability that there are no more than 2 **substandard** bags of flour in the sample.

(5 marks)

(c) A wholesaler is only concerned about the number of bags of flour which are **underweight** . The wholesaler re-analyses the sample of 50 bags of flour in (b).

(i) Find the probability that in the sample there is only 1 **substandard** bag and it is not **underweight**.

(ii) Find the probability that there are no more than 2 **substandard** bags in the sample and no **underweight** bag of flour in the sample.

(iii) Given that in the sample there are no more than 2 **substandard** bags, find the probability that there is no **underweight** bag in the sample.

(7 marks)

(2002 ASL-M&S Q13)

27. Suppose the number of customers visiting a supermarket per minute follows a Poisson distribution with mean 6 .

(a) Find the probability that the number of customers visiting the supermarket in one minute is more than 2 .

(3 marks)

(b) Suppose the amount $\$X$ spent by a customer in the supermarket follows a normal distribution $N(\mu, \sigma^2)$.

Probability distribution of the amount spent by a customer

Amount spent ($\$X$)	Probability *
$X < 100$	0.063
$100 \le X < 200$	0.364
$200 \le X < 300$	a_1
$300 \le X < 400$	a_2
$X \ge 400$	0.006

* Correct to 3 decimal places.

(i) Using the probabilities provided in the above table, find the values of μ and σ correct to 1 decimal place.
Hence find the values of a_1 and a_2 correct to 3 decimal places.

(ii) What is the median of the normal distribution?

(iii) Given that a customer spends less than \$200, find the probability that the customer spends more than \$50 .

(iv) Find the probability that there are 5 customers visiting the supermarket in a minute and exactly 2 of them each spends less than \$200 .

(12 marks)

(2002 ASL-M&S Q14)

28. The table gives the probability distributions of the lifetimes of two brands of compact fluorescent lamps (CFLs). The lifetime of a Brand X CFL follows a normal distribution with mean μ hours and standard deviation 400 hours. The lifetime of a Brand Y CFL follows another normal distribution with mean 8 800 hours and standard deviation σ hours.

Probability distributions of the lifetimes of brand X and Y CFLs

Lifetime of a CFL (in hours)	Probability *	
	Brand X: $N(\mu, 400^2)$	Brand Y: $N(8, \sigma^2)$
Under 8 200	0.0808	0.1587
8 200 to 8 600	0.2638	b_1
8 600 to 9 000	a_1	b_2
9 000 to 9 400	0.2195	b_3
Over 9 400	a_2	0.1587

* Correct to 4 decimal places.

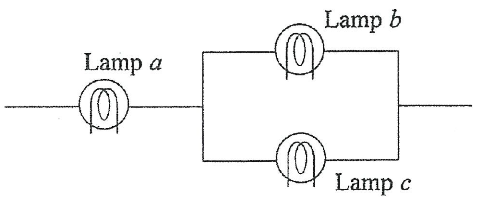
(a) Using the probabilities provided in the table, find μ and σ.
Hence find the values of a_1, a_2, b_1, b_2, b_3 in the table.

(5 marks)

(b) Based on the results of (a), which brand of CFL would you choose to buy? Explain.

(1 mark)

(c) The figure shows a lighting system formed by three lamps. The system will work only if lamp a works and either lamp b or lamp c works.



(i) Suppose all the lamps in the system are brand X CFLs.

(I) Find the probability that the lifetime of the lighting system is more than 8200 hours.

(II) It is known that the lifetime of the lighting system is less than 8 200 hours. Find the probability that only the lifetime of lamp a is less than 8200 hours.

(ii) Suppose the lighting system is formed by 2 brand X and 1 brand Y CFLs. In order for the system to have a better chance of having a lifetime of more than 8 200 hours, where would you put the brand Y CFL in the system? Explain.

(9 marks)

(2001 ASL-M&S Q12)

29. The milk produced by Farm A has been contaminated by dioxin. The amount of dioxin presented in each bottle of milk follows a normal distribution with mean 20 ng $(1ng = 10^{-6} g)$ and standard deviation 5 ng. Bottles which contain more than 12 ng of dioxin are classified as *risky*, and those which contain more than 27 ng are *hazardous*.

(a) Suppose a bottle of milk from Farm A is randomly chosen.

(i) Find the probability that it is *risky* but not *hazardous*.

(ii) If it is *risky*, find the probability that it is *hazardous*.

(6 marks)

(b) A distributor purchases bottles of milk from both Farm A and Farm B and sells them under the same brand name 'Healthy'. It is known that 60% of the milk is from Farm A and the rest from Farm B. A bottle of milk from Farm B has a probability of 0.058 of being *risky* and 0.004 of being *hazardous*.

(i) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is from Farm B.

(ii) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is a *hazardous* bottle from Farm B.

(iii) The Health Department inspects 5 randomly chosen bottles of 'Healthy' milk. If 2 or more bottles of milk in the batch are *risky*, the distributor's license will be suspended immediately. Find the probability that the license will be suspended.

(9 marks)

(2000 ASL-M&S Q12)

30. A criminologist has developed a questionnaire for predicting whether a teenager will become a delinquent. Scores on the questionnaire can range from 0 to 100, with higher values indicating a greater criminal tendency. The criminologist sets a critical level at 75, i.e., a teenager scores more than 75 will be classified as a potential delinquent (PD). Extensive studies have shown that the scores of those considered non-PDs follow a normal distribution with a mean of 65 and standard deviation of 5. The scores of those considered PDs follow a normal distribution with a mean of 80 and standard deviation of 5.

(a) Find the probability that

(i) a PD will be misclassified,

(ii) a non-PD will be misclassified.

(4 marks)

(b) What is the probability that out of 10 PDs, not more than 2 will be misclassified?

(3 marks)

(c) If a sociologist wants to ensure that only 1 in 100 PDs should be misclassified, what critical level of score should be used?

(3 marks)

(d) It is known that 10% of all teenagers are PDs. Will the probability of teenagers misclassified by the sociologist in (c) be greater than that misclassified by the criminologist? Explain.

(5 marks)

(1999 ASL-M&S Q10)

31. The weight of each box of washing powder produced by a factory follows a normal distribution with mean 500 g and variance 25 g^2 . The weights of boxes of washing powder are independent of each other. Every thirty minutes, a test consists of one or two parts will be performed as follows:

First part of the test

A randomly selected box of washing powder is weighed. If the weight of this box is greater than 510 g or less than 490 g , a black signal will be generated.

Second part of the test

(*Performed only when the weight of the box in the first part is greater than 508 g or less than 492 g and no black signal has been generated.*)

Another randomly selected box of washing powder is weighed.

(I) A black signal will be generated if the weight of this box is greater than 510 g or less than 490 g .

(II) A red signal will be generated if the weights of the two boxes in the first and second parts are **both between** 508 g and 510 g , or **both between** 490 g and 492 g.

(a) Find the probability that a black signal will be generated in the first part of a test.

(2 marks)

(b) Find the probability that the second part has to be performed in a test.

(3 marks)

(c) Find the probability that a black signal will be generated in a test.

(3 marks)

(d) Given that the second part has to be performed in a test, find the probability that the weights of the two boxes selected are both between 508 g and 510 g.

(3 marks)

(e) Given that the second part has to be performed in a test, find the probability that a red signal is generated.

(2 marks)

(f) Find the probability that a red signal will be generated in a test.

(2 marks)

(1998 ASL-M&S Q13)

32. The number of fire insurance claims (FICs) received by an insurance company is modelled by a Poisson distribution with mean 4 claims per day. The company found that 60% of the FICs are related to house fires.

(a) Find the probability that no FICs are received on a particular day.

(2 marks)

(b) If 5 FICs are received on a certain day, find the probability that at least 2 of them are related to house fires.

(3 marks)

(c) It is known that the amounts of FICs related and not related to house fires can be modelled respectively by normal distributions with the following means and standard deviations:

FICs	Mean	Standard deviation
Related to house fires	\$100 000	\$50 000
Not related to house fires	\$150 000	\$20 000

If the amount of a FIC is greater than \$ 200 000, the FIC is said to be *large*.

(i) Find the probability that a certain FIC is *large*.

(ii) Given that a FIC is *large*. Find the probability that the FIC is related to a house fire.

(iii) Find the probability that on a particular day, the company receives 5 FICs and at least 2 of them are *large*.

(10 marks)

(1997 ASL-M&S Q11)

33. Every morning, Mr. Wong wears a necktie to work. If the length of the front portion of his necktie is between 44 cm and 45 cm, he regards it to be a *perfect tying*. Otherwise, he has to tie it again until he gets the *perfect tying*. Suppose that the length of the front portion of his necktie can be modelled by a normal distribution with mean 44.6 cm and standard deviation 1.2 cm.

(a) Find the probability that Mr. Wong gets a *perfect tying* in one trial.

(3 marks)

(b) Find the mean number of trials to be taken by Mr. Wong to get the first *perfect tying*.

(2 marks)

(c) Find the probability that Mr. Wong gets the *perfect tying* in not more than 3 trials.

(2 marks)

(d) Mr. Wong will have to go to work by taxi only if he doesn't get the *perfect .tying* in the first 3 trials in any morning.

(i) Find the probability that Mr. Wong will have to go to work by taxi in less than 2 out of 6 days.

(ii) Given that Mr. Wong has to go to work by taxi on a certain morning, find the probability that he could not get the *perfect tying* until the 5th trial.

(iii) Find the probability that in a certain week of 6 working days (Monday to Saturday), Mr. Wong will have to go to work by taxi on 2 consecutive mornings and he will not have to take a taxi on the other 4 mornings.

(8 marks)

(1997 ASL-M&S Q13)

34. A machine discharges soda water once for each cup of soda water purchased. The amount of soda water in each discharge is independently normally distributed with mean 210 ml and standard deviation 15 ml.

(a) Find the probability that the amount of a cup of soda water is between 200 ml and 220 ml.

(2 marks)

(b) Suppose cups of capacity 240 ml each are used.

(i) Find the probability that a discharge will overflow.

(ii) What is the probability that there will be exactly 1 overflow out of 30 discharges?

(iii) If Sam buys a cup of soda water from the machine every day starting on 1st July, find the probability that he will get the second overflow on 31st July.

(5 marks)

(c) The vendor has decided to use cups of capacity 220 ml each and to repair the machine so that, on the average, 80 in 100 cups contain more than 205 ml of soda water in each and only 1 in 100 discharges overflows. The amount of soda water in each discharge is still independently normally distributed.

(i) What will the new mean and standard deviation of the amount of soda water in each discharge be? Give the answers correct to 1 decimal place.

(ii) If a discharge from the repaired machine overflows, find the probability that the amount of soda water in this discharge exceeds 225 ml. Give the answer correct to 2 decimal places.

(8 marks)

(1996 ASL-M&S Q11)

35. A test is used to diagnose a disease. For people *with* the disease, it is known that the test scores follow a normal distribution with mean 70 and standard deviation 5. For people *without* the disease, the test scores follow another normal distribution with mean μ and the same standard deviation 5. It is known that 33 % of those people *without* the disease will achieve a test score over 63.2.

(a) Find μ .

(3 marks)

(b) It is estimated that 15 % of the population of a city has the disease. A doctor has proposed that a person be classified as having the disease if the person's test score exceeds 66, otherwise the person will be classified as not having the disease.

If a person is randomly selected from the population to take the test,

(i) what is the probability that this person will be classified as having the disease?

(ii) find the probability that this person will be misclassified.

(12 marks)

(1995 ASL-M&S Q12)

36. Batches of screws are produced by a manufacturer under two different sets of conditions, favourable and unfavourable. If screws are produced under favourable conditions, the diameters of the screws will follow a normal distribution with mean 10 mm and standard deviation 0.4 mm. If screws are produced under unfavourable conditions, the diameters of the screws will follow a normal distribution with mean 12.3 mm and standard deviation 0.6 mm. A batch of screws is examined by measuring the diameter X mm of a screw randomly selected from the batch.

(a) The batch is classified as acceptable by the manufacturer if $X < c_1$ and as unacceptable if otherwise. The value c_1 satisfies $P(X < c_1) = 0.95$ under favourable conditions. Determine the value of c_1 .

(3 marks)

(b) The buyer uses a different criterion instead. He classifies the batch as acceptable if $X < c_2$ and as unacceptable if otherwise. The value c_2 satisfies $P(X < c_2) = 0.01$ under unfavourable conditions. Determine the value of c_2 .

(3 marks)

(c) For a batch of screws produced under favourable conditions and based on the same measurement of a screw, find the probability that the batch will be classified as unacceptable by the manufacturer but acceptable by the buyer.

(4 marks)

(d) After some negotiation, the manufacturer and the buyer agree to use a common cut-off point c_3 such that $P(X < c_3)$ under favourable conditions is equal to $P(X \ge c_3)$ under unfavourable conditions. Determine the value of c_3 .

(3 marks)

(e) The manufacturer and the buyer later agree that a batch will be rejected in the future if $X \ge 10.8$ (too thick) or $X < 9.4$ (too thin). If the population mean μ mm of the diameters of the screws produced can be modified by adjusting the machine, find μ so that the probability of rejection, $P(X < 9.4 \text{ or } X > 10.8)$, is minimized.

(2 marks)

(1994 ASL-M&S Q13)

2021 DSE Q9

The weight of each potato in a large farm follows a normal distribution with a mean of 200 grams and a standard deviation of σ grams. The classification of the potatoes is as follows:

Weight of a potato (W grams)	$W < 180$	$180 \le W < 230$	$W \ge 230$
Classification	*small*	*medium*	*big*

It is given that 21.19% of the potatoes in the farm are *small*.

(a) Find the percentage of *medium* potatoes in the farm. (3 marks)

(b) The potatoes in the farm are now inspected one by one. Find the probability that the 4th potato inspected is the 2nd *big* potato inspected. (3 marks)

(c) From the farm, 5 potatoes are randomly selected.

(i) Find the probability that there are exactly 1 *big* potato and 2 *small* potatoes.

(ii) Given that there is exactly 1 *big* potato, find the probability that there are at least 2 *small* potatoes.

(5 marks)

10. Normal Distribution

Section A

1. (2018 DSE-MATH-M1 Q3)

2. (2017 DSE-MATH-M1 Q3)

(a) μ

$= \dfrac{1.83+3.43}{2}$

$= 2.63$ 1A

$P\left(\dfrac{1.83-2.63}{\sigma} < Z < \dfrac{3.43-2.63}{\sigma}\right) = 0.8904$ 1M

$P\left(\dfrac{-0.8}{\sigma} < Z < \dfrac{0.8}{\sigma}\right) = 0.8904$

$P\left(0 < Z < \dfrac{0.8}{\sigma}\right) = 0.4452$

$\dfrac{0.8}{\sigma} = 1.6$

$\sigma = 0.5$ 1A

(b) The required probability

$= P\left(\dfrac{2.5-2.63}{\frac{0.5}{\sqrt{9}}} < Z < \dfrac{3.1-2.63}{\frac{0.5}{\sqrt{9}}}\right)$ 1M

$= P(-0.78 < Z < 2.82)$

$= 0.2823 + 0.4976$

$= 0.7799$ 1A

----------(5)

(a)	Very good. Most candidates were able to find the required mean μ and standard deviation σ .
(b)	Good. Some candidates mistook σ as the standard deviation of the sample mean.

3. (2012 DSE-MATH-M1 Q9)

(a) Let X be the score of a student who had revised.

$P(X \geq 43) = P\left(Z \geq \dfrac{43-59}{10}\right)$

$= P(Z \geq -1.6)$

≈ 0.9452 1A

Let Y be the score of a student who had not revised.

$P(Y \geq 43) = P\left(Z \geq \dfrac{43-35.2}{12}\right)$

$= P(Z \geq 0.65)$

≈ 0.2578

(Either one)

$\therefore$ P(pass the test) $\approx 0.73 \times 0.9452 + 0.27 \times 0.2578$ 1M

$= 0.759602$ 1A OR 0.7596

(b) P(a student had not revised for the test | he passed the test)

$= \dfrac{0.27 \times 0.2578}{0.759602}$ 1M

≈ 0.091634829

≈ 0.0916 1A

(c) P(4 students had not revised for the test among 10 passed students)

$\approx C_6^{10}(0.091634829)^4(1-0.091634829)^6$ 1M

≈ 0.0083 1A

(7)

(a)	Good. Nevertheless, some candidates did not figure out that the required probability was $0.73\,P(X \geq 43) + 0.27\,P(Y \geq 43)$, and some failed to use the standard normal distribution table.
(b)	Satisfactory. Many candidates were able to apply the correct method, although some got wrong numerical answers.
(c)	Fair. Some candidates wrote a binomial probability but did not use the result of (b).

4. (2010 ASL-M&S Q6)

Solution	Marks	Remarks
(a) $P\left(Z > \frac{152-\mu}{5}\right) = 0.117$	1A	
$P\left(0 < Z < \frac{152-\mu}{5}\right) = 0.383$	1M	
$\frac{152-\mu}{5} \approx 1.19$ $\mu \approx 146.05$	1A	
(b) (i) The required probability $= 0.117 \times (1-0.2) + (1-0.117) \times 0.1$ $= 0.1819$	1A	
(ii) The required probability $= \frac{(1-0.117)\times(1-0.1)}{1-0.1819}$	1M+1M	
$= \frac{883}{909}$	1A	OR 0.9714
	(7)	

Very good. Candidates performed well in simple application of normal and conditional probabilities.

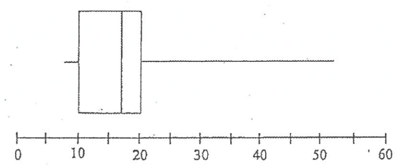
5. (2008 ASL-M&S Q5)

Solution	Marks	Remarks
(a) Let $\$X$ be the amount of money spent by a randomly selected customer. $P(X > 30000) = 0.242$ $P\left(Z > \frac{30000-\mu}{6000}\right) = 0.242$	1M	For standardization
$\therefore P\left(0 < Z \le \frac{30000-\mu}{6000}\right) = 0.258$ $\therefore \frac{30000-\mu}{6000} = 0.7$	1A	
i.e. $\mu = 25800$	1A	
(b) The required probability $= \frac{P(16500 < X < 30000)}{P(X < 30000)}$	1A	For $P(16500 < X < 30000)$
$= \frac{P\left(\frac{16500-25800}{6000} < Z < \frac{30000-25800}{6000}\right)}{1 - P(X \ge 30000)}$ $= \frac{P(-1.55 < Z < 0.7)}{1-0.242}$	1A	For denominator
$= \frac{0.4394 + 0.258}{0.758}$ ≈ 0.9201	1A	
	(6)	

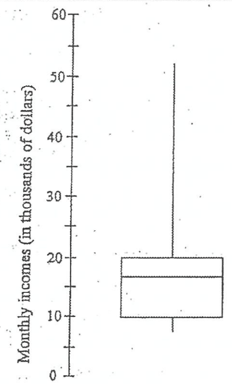
Fair. Some candidates were not aware that a conditional probability is required.

6. (2004 ASL-M&S Q5)

(a)



1A for any correct box-and-whisker diagram
1A for correct scale
pp−1 for omitting the title



1A for any correct box-and-whisker diagram
1A for correct scale
pp−1 for omitting the title

Solution	Marks	Remarks
(b) (i) Let $\$X$ be the monthly income of a randomly selected university graduate from the group. Then, we have $X \sim N(17940, 4700^2)$. The required probability $= P(X < 17000)$ $= P(Z < \frac{17000-17940}{4700})$ $= P(Z < -0.2)$	1A	
$= 0.4207$	1A	$a-1$ for r.t. 0.421
(ii) Since the distribution is skewed to the right side,	1M	accept skewed to one side or not symmetrical
the model proposed by the student is not appropriate.	1M	
	(6)	

Good. Many candidates did not construct the box-and-whisker diagram to the scale which is necessary for correctly describing the distribution of data.

7. (2003 ASL-M&S Q6)

Let $\$X$ be the amount of a business transaction. Then, $X \sim N(215, 50^2)$.

(a) The required probability

$= P(X > 300)$

$= P\left(Z > \frac{300-215}{50}\right)$ — 1A accept $P\left(Z \geq \frac{300-215}{50}\right)$

$= P(Z > 1.7)$

$= 0.0446$ — 1A a−1 for r.t. 0.045

(b) The required probability

$= C_2^7 (0.0446)^2 (1-0.0446)^5$ — 1M for Binomial probability

≈ 0.033251802

≈ 0.0333 — 1A a−1 for r.t. 0.033

(c) The required probability

$\approx (0.033251802)(0.0446)$ (by (b))

≈ 0.001483030369

≈ 0.0015 — 1A a−1 for r.t. 0.001

(d) $P(X > K) = 64.8\%$

$P\left(Z > \frac{K-215}{50}\right) = 0.648$

$\frac{K-215}{50} = -0.38$ — 1M accept $\frac{K-215}{50} = 0.38$

$K = 196$ — 1A

----------(7)

Very good. Most candidates were able to make use of the normal distribution table.

8. (1994 ASL-M&S Q6)

(a) Since $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ for $x \approx 0$,

$e^{-\frac{x^2}{2}} \approx 1 + (-\frac{x^2}{2}) + \frac{1}{2}(-\frac{x^2}{2})^2 + \frac{1}{6}(-\frac{x^2}{2})^3$ — 1M

$\approx 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}$ for $x \approx 0$. — 1A

$\int_0^1 e^{-\frac{x^2}{2}} dx \approx \int_0^1 (1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}) dx$

$= \left[x - \frac{x^3}{6} + \frac{x^5}{40} - \frac{x^7}{336}\right]_0^1$ — 1M

$= 1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336}$

$= 0.8554$ — 1A

(b) From the normal distribution table,

$\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{x^2}{2}} dx = 0.3413$ — 1A

Hence $\frac{1}{\sqrt{2\pi}} \times 0.8554 = 0.3413$ — 1M

$\therefore \quad \pi = \frac{0.8554^2}{2 \times 0.3413^2} = 3.141$ — 1A 3.140 for using exact value of (a)

7

Section B

9. (2016 DSE-MATH-M1 Q9)

Let J minutes and K minutes be the random variables representing the daily reading times of the students in schools X and Y respectively.

(a) Let μ_1 minutes and σ_1 minutes be the mean and the standard deviation of the daily reading times of the students in school X respectively, while μ_2 minutes and σ_2 minutes be the mean and the standard deviation of the daily reading times of the students in schools Y respectively.

$$\begin{cases} \frac{40-\mu_1}{\sigma_1} = -2.51 \\ \frac{70-\mu_1}{\sigma_1} = 2.17 \end{cases}$$

$$\begin{cases} \frac{48-\mu_2}{\sigma_2} = -2.17 \\ \frac{72-\mu_2}{\sigma_2} = 2.12 \end{cases}$$

1M+1A either one

Solving, we have

$\mu_1 = \frac{4375}{78}, \sigma_1 = \frac{250}{39}$ — 1A for both

$\mu_1 \approx 56.08974359, \sigma_1 \approx 6.41025641$

$\mu_1 \approx 56.0897, \sigma_1 \approx 6.4103$ — r.t. $\mu_1 \approx 56.0897, \sigma_1 \approx 6.4103$

$\mu_2 = \frac{8600}{143}, \sigma_2 = \frac{800}{143}$ — 1A for both

$\mu_2 \approx 60.13986014, \sigma_2 \approx 5.594405594$

$\mu_2 \approx 60.1399, \sigma_2 \approx 5.5944$ — r.t. $\mu_2 \approx 60.1399, \sigma_2 \approx 5.5944$

P(students reading more than 60 minutes daily in school X)

$= P(J > 60)$

$= P\left(Z > \frac{60 - \frac{4375}{78}}{\frac{250}{39}}\right)$

$= P(Z > 0.61)$

$= 0.2709$

P(students reading more than 60 minutes daily in school Y)

$= P(K > 60)$

$= P\left(Z > \frac{60 - \frac{8600}{143}}{\frac{800}{143}}\right)$

$= P(Z > \frac{-1}{40})$

$> P(Z > 0)$

$= 0.5$

> 0.2709

1M either one

Thus, there are less students reading more than 60 minutes daily in school X. — 1A f.t.

----------(6)

Solution	Marks	Remarks
(b) The required probability		
$= C_1^3(0.2709)(1-0.2709)^2(0.2709)$	1M	
≈ 0.11703438		
≈ 0.1170	1A	r.t. 0.1170
	----------(2)	
(c) For school X,		
$P(J \geq T) \leq 0.1$		
$\dfrac{T-\frac{4375}{78}}{\frac{250}{39}} \geq 1.29$	1M+1A	1A for 1.29 (either one)
$T \geq \frac{2510}{39}$		
$T \geq 64.35897436$		
$T \geq 65$	1A	accept $T = 65$ (either one)
For school Y,		
$P(K \geq T) \leq 0.1$		
$\dfrac{T-\frac{8600}{143}}{\frac{800}{143}} \geq 1.29$		
$T \geq \frac{9632}{143}$		
$T \geq 67.35664336$		
$T \geq 68$		
Thus, the least value of T should be 68 .	1A	f.t.
	----------(4)	

(a)	Good. Many candidates were able to formulate the corresponding equations in means and standard deviations, but some candidates were unable to give the numerical answers either in an exact fraction or correct to 4 decimal places.
(b)	Good. Many candidates were able to apply the result of (a) .
(c)	Fair. About half of the candidates were unable to use inequality to formulate the problem. Besides, many candidates used 1.28 instead of 1.29 in the inequality.

10. (2012 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) (i) $I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$		
$= \frac{1}{2} \cdot \frac{4-1}{6} \left[\frac{1}{\sqrt{1}} e^{\frac{-1}{2}} + \frac{1}{\sqrt{4}} e^{\frac{-4}{2}} + 2\left(\frac{1}{\sqrt{1.5}} e^{\frac{-1.5}{2}} + \frac{1}{\sqrt{2}} e^{\frac{-2}{2}} + \frac{1}{\sqrt{2.5}} e^{\frac{-2.5}{2}} + \frac{1}{\sqrt{3}} e^{\frac{-3}{2}} + \frac{1}{\sqrt{3.5}} e^{\frac{-3.5}{2}} \right) \right]$	1M	
≈ 0.692913377		
≈ 0.6929	1A	
(ii) $\frac{d}{dt}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}} \right) = \frac{-1}{2} t^{\frac{-3}{2}} e^{\frac{-t}{2}} + t^{\frac{-1}{2}} \cdot \frac{-1}{2} e^{\frac{-t}{2}}$	1M+1A	
$= \frac{-1}{2} e^{\frac{-t}{2}} \left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}} \right)$		
$\frac{d^2}{dt^2}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}} \right) = \frac{-1}{2}\left[e^{\frac{-t}{2}} \left(\frac{-3}{2} t^{\frac{-5}{2}} + \frac{-1}{2} t^{\frac{-3}{2}} \right) + \frac{-1}{2} e^{\frac{-t}{2}} \left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}} \right) \right]$	1M+1A	
$= \frac{1}{4} e^{\frac{-t}{2}} \left(3t^{\frac{-5}{2}} + 2t^{\frac{-3}{2}} + t^{\frac{-1}{2}} \right)$		
> 0 for $1 \leq t \leq 4$.		
Hence the estimation in (i) is an over-estimate.	1	
	(7)	
(b) Let $t = x^2$.	1M	
$dt = 2x dx$	} 1A	
When $t = 1$, $x = 1$; when $t = 4$, $x = 2$.		
$I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$		
$= \int_1^2 \frac{1}{x} e^{\frac{-x^2}{2}} 2x dx$		
$= 2\int_1^2 e^{\frac{-x^2}{2}} dx$	1	
	(3)	
(c) $2\int_1^2 e^{\frac{-x^2}{2}} dx < 0.692913377$	1M	
$2\sqrt{2\pi} \int_1^2 \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx < 0.692913377$		
$2\sqrt{2\pi}(0.4772 - 0.3413) < 0.692913377$	1A	For 0.4772 and 0.3413
$\pi < 3.249593152$		
$\therefore \pi < 3.25$	1	
	(3)	

(a)	(i)	Good. Many candidates applied the trapezoidal rule correctly.
	(ii)	Poor. Many candidates used $\frac{d}{dt}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}} \right)$ instead of $\frac{d^2}{dt^2}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}} \right)$ to determine whether the estimate in (i) is an over-estimate or under-estimate.
(b)		Fair. Many candidates used wrong substitutions.
(c)		Very poor. Only a few candidates attempted this part. Among them, some wrote $I \approx 0.692913377$ instead of $I < 0.692913377$.

11. (PP DSE-MATH-M1 Q13)

Let X_r minutes and X_e minutes be the waiting times for a customer in the regular and express counter respectively.

(a) $\mathrm{P}(X_r > 6) = \mathrm{P}\left(Z > \frac{6-6.6}{1.2}\right)$ 1M

$= \mathrm{P}(Z > -0.5)$

≈ 0.6915 1A

(2)

(b) (i) P(more than 10 from 12 customers with $X_r > 6$)

$= C_{11}^{12}(0.6915)^{11}(1-0.6915) + (0.6915)^{12}$ 1M+1M

≈ 0.0759 1A

(ii) Let Y minutes be the average waiting time of the 12 customers

$Y \sim \mathrm{N}\left(6.6, \frac{1.2^2}{12}\right) = \mathrm{N}(6.6, 0.12)$ 1A

$\mathrm{P}(Y > 6) = \mathrm{P}\left(Z > \frac{6-6.6}{\sqrt{0.12}}\right)$

$\approx \mathrm{P}(Z > -1.73)$ OR $\mathrm{P}(Z > -1.732)$

≈ 0.9582 1A OR 0.9584

(5)

(c) (i) $\mathrm{P}(X_r < k) = 0.2119$

$\mathrm{P}\left(Z < \frac{k-6.6}{1.2}\right) = 0.2119$ 1M

$\frac{k-6.6}{1.2} = -0.8$

$k = 5.64$ 1A

$\mathrm{P}(X_e > k) = 0.0359$

$\mathrm{P}\left(Z > \frac{5.64-\mu}{0.8}\right) = 0.0359$ 1M

$\frac{5.64-\mu}{0.8} = 1.8$

$\mu = 4.2$ 1A

(ii) $\mathrm{P}(X_r > \mu) = \mathrm{P}\left(Z > \frac{4.2-6.6}{1.2}\right)$

≈ 0.9772 1A

P(1 customer pays at regular counter | 2 customers wait more than μ min)

$\approx \frac{2(0.88)(0.9772)(0.12)(0.5)}{[(0.88)(0.9772)+(0.12)(0.5)]^2}$ 1M+1M (1M for numerator, 1M for denominator)

≈ 0.1219 1A

(8)

(a)	良好。
(b) (i)	良好。
(ii)	平平。 部分學生不知道平均等候時間服從的正態分佈的標準差。
(c) (i)	甚差。 很多學生沒有完全明白題目所描述的特快櫃台付款與普通櫃台付款的分別。
(ii)	甚差。 很少學生嘗試答這部分。

12. (2013 ASL-M&S Q10)

Let X be the speed of a randomly selected vehicle.

(a) $\mathrm{P}(X > 82.64) = 0.123$ and $\mathrm{P}(X < 75.2) = 0.242$

$\frac{82.64-\mu}{\sigma} = 1.16$ and $\frac{75.2-\mu}{\sigma} = -0.7$ 1M

Dividing the equations, we have $\frac{82.64-\mu}{75.2-\mu} = \frac{1.16}{-0.7}$.

i.e. $\mu = 78$ 1A

$\therefore\ \sigma = 4$ 1A

(3)

(b) (i) P(a notice will be issued) $= \mathrm{P}(X > 80)$ 1M

$= \mathrm{P}\left(Z > \frac{80-78}{4}\right)$

$= \mathrm{P}(Z > 0.5)$

≈ 0.3085 1A

(ii) P(at most 2 notices will be issued for the 10 vehicles)

$\approx (1-0.3085)^{10} + C_1^{10}(0.3085)(1-0.3085)^9 + C_2^{10}(0.3085)^2(1-0.3085)^8$ 1M

≈ 0.3604 1A

(4)

(c) (i) (I) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = 1$)

= P(speed of vehicle $\leq$ 80 and (speed of vehicle + error) > 80)

$= \mathrm{P}(78 < X \leq 80 \mid Y = 2)\mathrm{P}(Y = 2) + \mathrm{P}(77 < X \leq 80 \mid Y = 3)\mathrm{P}(Y = 3)$ 1M+1A (1A for either term)

$= \mathrm{P}(0 < Z \leq 0.5)(0.5) + \mathrm{P}(-0.25 < Z \leq 0.5)(0.5)$

$\approx (0.1915)(0.5) + (0.0987 + 0.1915)(0.5)$

≈ 0.2409 1A (Accept 0.24085)

(II) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = -3$)

= P(speed of vehicle $\leq$ 80 and (speed of vehicle + error) > 80)

$= \mathrm{P}(78 < X \leq 80 \mid Y = 2)\mathrm{P}(Y = 2) + 0 \cdot \mathrm{P}(Y = -1)$ 1A

$= \mathrm{P}(0 < Z \leq 0.5)(0.5)$

$\approx (0.1915)(0.5)$

≈ 0.0958 1A (Accept 0.09575)

(ii) We need $2 + \theta < 0$ for the scenario happens.

P(a notice will not be issued but the speed of vehicle is over 80) ≤ 0.07125

P(speed of vehicle > 80 and (speed of vehicle + error) $\leq$ 80) ≤ 0.07125 1M

$\mathrm{P}(80 < X \leq 80 - (2+\theta) \mid Y = 2+\theta)\,\mathrm{P}(Y = 2+\theta) \leq 0.07125$

$\mathrm{P}\left(0.5 < Z \leq \frac{78-\theta-78}{4}\right)(0.5) \leq 0.07125$

$\mathrm{P}\left(0 < Z \leq \frac{-\theta}{4}\right) \leq 0.1425 + 0.1915$

$\frac{-\theta}{4} \leq 0.97$ 1A (For 0.97)

$\theta \geq -3.88$

Hence the range is $-3.88 \leq \theta < -2$. 1A

(8)

(a)		Good. When making use of the normal distribution table, some candidates equated $\frac{75.2-\mu}{\sigma}$ to 0.7 rather than to -0.7 .
(b)		Good.
(c)	(i)	Fair. Some candidates wrongly assumed that the variable was discrete.
	(ii)	Poor.

13. (2012 ASL-M&S Q10)

Let A and B be the operation time of a randomly chosen battery A and B respectively.

(a) (i) $P(A<152 \text{ or } A>184)$

$=P\left(Z<\frac{152-168}{32} \text{ or } Z>\frac{184-168}{32}\right)$

$=P(Z<-0.5 \text{ or } Z>0.5)$

$=0.617$ — 1A

(ii) $P(A>k)=0.05$

$\frac{k-168}{32}=1.645$ — 1M — Accept 1.64 or 1.65

$k=220.64$ — 1A — Accept 220.48 or 220.8

(iii) $P(B>188)=0.33$ and $P(B<213.2)=0.877$

$P\left(Z>\frac{188-\mu}{\sigma}\right)=0.33$ and $P\left(Z<\frac{213.2-\mu}{\sigma}\right)=0.877$

$\frac{188-\mu}{\sigma}=0.44$ and $\frac{213.2-\mu}{\sigma}=1.16$ — 1M

Solving, $\mu=172.6$ and $\sigma=35$. — 1A+1A

(iv) $P(B<146)=P\left(Z<\frac{146-172.6}{35}\right)$

$=P(Z<-0.76)$

$=0.2236$ — 1A

(7)

(b) (i) $\lambda_A=1500\times\frac{1}{3}\times P(A<104)$ — 1M (Either one)

$=500\times P\left(Z<\frac{104-168}{32}\right)$

$=500\times P(Z<-2)$

$=11.4$ (correct to 1 d.p.) — 1A (Either one)

$\lambda_B=1500\times\frac{2}{3}\times P(B<104)$

$=1000\times P\left(Z<\frac{104-172.6}{35}\right)$

$=1000\times P(Z<-1.96)$

$=25.0$ (correct to 1 d.p.) — 1A — Accept 25

(ii) $P(4\le$ number of 'faulty' batteries A produced $\le 6)$

$=\frac{e^{-11.4}11.4^4}{4!}+\frac{e^{-11.4}11.4^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}$ — 1M

≈ 0.0600 — 1A

(iii) The required probability

$$=\frac{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}}{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}\times\frac{e^{-25}25^4}{4!}}$$ — 1M+1M — 1M for numerator, 1M for denominator

≈ 0.8815 — 1A

(8)

(a) (i)(ii)		Very good.
(iii)(iv)		Good.
(b) (i)		Fair. Some candidates were not able to make use of the given information of 1500 batteries.
(ii)		Very good.
(iii)		Poor. Many candidates had difficulty in counting the number of outcomes and considering all the relevant ones. Some candidates failed to recognise that a conditional probability should be considered.

14. (2011 ASL-M&S Q10)

(a) $P(160\le Y<K)=78.88\%$

$P\left(\frac{160-165}{4}\le Z<\frac{K-165}{4}\right)=0.7888$

$0.3944+P\left(0\le Z<\frac{K-165}{4}\right)=0.7888$ — 1M

$\frac{K-165}{4}=1.25$

$K=170$ — 1A

(2)

(b) $P(\text{score } 30)=P(170\le Y<174)$

$=P(1.25\le Z<2.25)$ — 1M

$=0.4878-0.3944$

$=0.0934$ — 1A

(2)

(c) $P(\text{6th game is the 3rd Bingo})=C_2^5(0.2112)^3(0.7888)^3$ — 1M

≈ 0.0462 — 1A

(2)

(d) The number of "Bingo" in n games $\sim B(n, 0.7888)$.

$\therefore\ n(0.7888)(0.2112)\le 2.3$ — 1M

$n\le 13.80597302$

Thus the largest value of n is 13 . — 1A

(2)

(e) (i) $P(\text{score } 20)=P(-2.75\le Z<-1.25)=0.1026$ — 1A

$\therefore P(\text{win a prize})$

$=P(\text{total score in 4 games}\ge 160)$

$=(0.7888)^4+C_1^4(0.7888)^3(0.0934+0.1026)+C_2^4(0.7888)^2(0.0934)^2$ — 1M

≈ 0.804490478

≈ 0.8045 — 1A

(ii) $P(\text{win a prize and average score in the first 2 games}\ge 40)$

$=P(\text{total score in 4 games}\ge 160 \text{ and total score in first 2 games}\ge 80)$

$=(0.7888)^4+C_1^4(0.7888)^3(0.0934)+C_1^2(0.7888)^3(0.1026)$

$+(C_2^4-1)(0.7888)^2(0.0934)^2$ — 1M

> Alternative Solution
>
> $=P(\text{total score in 4 games}\ge 160)-P(\text{total score in 4 games}\ge 160$ and
>
> total score in first 2 games $<80)$
>
> $\approx 0.804490478-(0.7888)^2(0.0934)^2-C_1^2(0.7888)^3(0.1026)$ — 1M

≈ 0.698351364

≈ 0.6984 — 1A

(iii) $P(\text{average score in the first 2 games}<40 \mid \text{win a prize})$

$\approx\frac{0.804490478-0.698351364}{0.804490478}$ — 1M

≈ 0.1319 — 1A

(7)

(a)(b)(c)		Very good.
(d)		Good. Some candidates were not familiar with the variance of a binomial distribution.
(e) (i) (ii)		Fair. Many candidates were unable to exhaust all relevant outcomes.
(iii)		Fair. Many candidates were unable to fully understand the rules of the game described in the question.

15. (2011 ASL-M&S Q12)

(a) $P(Y<100)=0.121$

$P\left(\frac{100-\mu}{\sigma}\le Z<0\right)=0.379$

$\frac{100-\mu}{\sigma}=-1.17$ ---------------- (1) **1A**

$P(Y\ge 200)=0.0918$

$P\left(0<Z<\frac{200-\mu}{\sigma}\right)=0.4082$

$\frac{200-\mu}{\sigma}=1.33$ ---------------- (2) **1A**

Solving (1) and (2), we get $\mu=146.8$ and $\sigma=40$ **1A**

(3)

(b) P(High level)

$=P(150\le Y<200)$

$=P(0.08\le Z<1.33)$

$\approx 0.4082-0.0319$

$=0.3763$ **1A**

(1)

(c) P(High | rainfall exceeds 100 mm)

$=\frac{0.3763}{1-0.121}$ **1M** For conditional probability

≈ 0.428100113

≈ 0.4281 **1A**

(2)

(d) (i) P(Severe | rainfall exceeds 100 mm)

$=\frac{0.0918}{1-0.121}$

≈ 0.10443686 **1A**

P(Medium | rainfall exceeds 100 mm)

$=\frac{1-0.121-0.0918-0.3763}{1-0.121}$

≈ 0.467463026 **1A**

P(job will NOT be postponed | rainfall exceeds 100 mm)

$=(0.467463026)e^{-1}+(0.428100113)e^{-3}+(0.10443686)e^{-6}$ **1M**

≈ 0.193542759

≈ 0.1935 **1A**

(ii) P(job will be postponed for 1 day | rainfall exceeds 100 mm)

$=(0.467463026)\cdot e^{-1}1+(0.428100113)\cdot e^{-3}3+(0.10443686)\cdot e^{-6}6$ **1M**

≈ 0.237464824

≈ 0.2375 **1A**

(iii) P(job will be postponed for 2 days | rainfall exceeds 100 mm)

$=(0.467463026)\cdot\frac{e^{-1}1^2}{2!}+(0.428100113)\cdot\frac{e^{-3}3^2}{2!}+(0.10443686)\cdot\frac{e^{-6}6^2}{2!}$

≈ 0.186557057 **1A**

P(High level | job will be postponed for at least 3 days)

$=\frac{0.428100113\left(1-e^{-3}-e^{-3}3-\frac{e^{-3}3^2}{2!}\right)}{1-0.193542759-0.237464824-0.186557057}$ **1M**

≈ 0.6457 **1A**

(9)

(a) (b) (c)		Good.
(d) (i) (ii)		Satisfactory. The given condition in the stem of (d) was overlooked by some candidates.
(iii)		Fair. Many candidates were unable to analyse the situation and exhaust all relevant cases.

16. (2009 ASL-M&S Q11)

Solution	Marks	Remarks
Let X_N cm and X_D cm be the widths of the tongue of a normal baby and a baby having inherited disease A respectively.		
(a) $P(X_N < 2.22) = 0.242$		
$\frac{2.22-\mu}{0.4} = -0.7$		
$\mu = 2.5$	1A	
	(1)	
(b) (i) The required probability		
$= P(X_N > 2.5 + 0.5)$		
$= P\left(Z > \frac{3-2.5}{0.4}\right)$	1M	
$= 0.5 - 0.3944$		
$= 0.1056$	1A	
(ii) The required probability		
$= 0.05 \times P(X_D < 2.5 + 0.5) + 0.95 \times P(X_N > 2.5 + 0.5)$	1M+1A	
$= 0.05 \times P\left(Z < \frac{0.2}{0.2}\right) + 0.95 \times 0.1056$		
$= 0.05 \times 0.8413 + 0.95 \times 0.1056$		
$= 0.142385$	1A	OR 0.1424
(iii) The required probability		
$= \frac{0.05(0.8413)}{0.05(0.8413) + 0.95(1-0.1056)}$	1M+1A	
≈ 0.0472	1A	
	(8)	
(c) (i) The required probability		
$= \frac{C_3^8 C_1^{12} (0.142385)^4 (1-0.142385)^{16}}{C_4^{20} (0.142385)^4 (1-0.142385)^{16}}$	1M	
$= \frac{224}{1615}$	1A	OR 0.1387
(ii) The required probability		
$= \frac{C_2^7 (0.142385)^3 (1-0.142385)^{17} + C_2^7 C_1^{12} (0.142385)^4 (1-0.142385)^{16}}{(1-0.142385)^{20} + C_1^{20}(0.142385)(1-0.142385)^{19} + C_2^{20}(0.142385)^2(1-0.142385)^{18} + C_3^{20}(0.142385)^3(1-0.142385)^{17} + C_4^{20}(0.142385)^4(1-0.142385)^{16}}$	1M+1M+1A	1M for numerator 1M for denominator
≈ 0.0156	1A	
	(6)	

(a)		Fair. Candidates were not familiar with the use of normal tables especially in determining whether the z-value is positive or negative.
(b) (i)		Good. Except for the wrong answer carried forward from part (a).
(ii)		Fair. Many candidates could not formulate the problem.
(iii)		Fair. Many candidates did not fully understand the question and hence could not work out the conditional probability.
(c) (i)		Fair. Many candidates were affected by the wrong answers obtained in the previous parts.
(ii)		Very poor. Very few candidates got to attempt this part.

17. (2008 ASL-M&S Q11)

Solution	Marks	Remarks
(a) The required probability		
$= \frac{1.8^0 e^{-1.8}}{0!} + \frac{1.8^1 e^{-1.8}}{1!} + \frac{1.8^2 e^{-1.8}}{2!} + \frac{1.8^3 e^{-1.8}}{3!} + \frac{1.8^4 e^{-1.8}}{4!}$	1M+1M	1M for $P(X \le 4)$ 1M for Poisson probability with correct λ
≈ 0.963593339		
≈ 0.9636	1A	
	(3)	
(b) $p_0 = P\left(Z > \frac{4.6-3}{0.8}\right) = P(Z > 2) = 0.5 - 0.4772 = 0.0228$	1M	For standardization
$p_2 = P\left(Z < \frac{2-3}{0.8}\right) = P(Z < -1.25) = 0.5 - 0.3944 = 0.1056$	1A	For any one correct
$p_1 = 1 - p_0 - p_2 = 1 - 0.0228 - 0.1056 = 0.8716$	1A	For all correct
	(3)	
(c) (i) The required probability		
$= C_1^3 p_2 p_1^2 + C_1^3 p_2^2 p_0$	1M	1M for form correct
$= 3(0.1056)(0.8716)^2 + 3(0.1056)^2(0.0228)$		
≈ 0.241431455		
≈ 0.2414	1A	
(ii) The required probability		
$= C_2^4 p_2^2 p_0^2 + \frac{4!}{1!2!1!} p_2 p_1^2 p_0 + p_1^4$	1M	1M for form correct
$= 6(0.1056)^2(0.0228)^2 + 12(0.1056)(0.8716)^2(0.0228) + (0.8716)^4$	1A	
≈ 0.599107436		
≈ 0.5991	1A	
	(5)	
(d) The required probability		
$\approx \frac{\frac{1.8^2 e^{-1.8}}{2!}(0.1056)^2 + \frac{1.8^3 e^{-1.8}}{3!}(0.241431455) + \frac{1.8^4 e^{-1.8}}{4!}(0.599107436)}{0.963593339}$	1M+1M+1A	1M for any 2 cases 1M for denominator using (a) 1A for all correct
≈ 0.0883	1A	
	(4)	

(a)		Very good.
(b)		Good.
(c) (i) (ii)		Fair. Some candidates did not do the counting right and missed some of the eligible events.
(d)		Poor. Many candidates had difficulty in identifying the joint probabilities required for the numerator of the conditional probability.

18. (2007 ASL-M&S Q10)

(a) The required probability

$=1-\left(\frac{2.4^0 e^{-2.4}}{0!}+\frac{2.4^1 e^{-2.4}}{1!}+\frac{2.4^2 e^{-2.4}}{2!}\right)$

1M for complemeantary events
+ 1M for Poisson probability

≈ 0.430291254

≈ 0.4303

1A $a-1$ for r.t. 0.430
----------(3)

(b) Let $\$X$ be the expense of a customer.
Then, $X \sim N(375, 125^2)$.

The required probability

$= P(300 < X < 600)$

$= P(\frac{300-375}{125} < Z < \frac{600-375}{125})$

1M (accept $P(\frac{300-375}{125} \le Z \le \frac{600-375}{125})$)

$= P(-0.6 < Z < 1.8)$

$= 0.2257 + 0.4641$

$= 0.6898$

1A $a-1$ for r.t. 0.690
----------(2)

(c) The required probability

$= (0.25)(0.6898) + (0.8)(0.5 - 0.4641)$

1M for $0.25(b) + 0.8p$, $0 < p < 0.5$

≈ 0.20117

≈ 0.2012

1A $a-1$ for r.t. 0.201
----------(2)

(d) The required probability

$\approx \frac{2.4^3 e^{-2.4}}{3!}(0.20117)^3$

1M for $\frac{2.4^3 e^{-2.4}}{3!}(c)^3$

≈ 0.00170163

≈ 0.0017

1A $a-1$ for r.t. 0.002
----------(2)

(e) The required probability

$\approx \frac{0.00170163 + (0.20117)^4 \left(\frac{2.4^4 e^{-2.4}}{4!}\right)}{0.430291254}$

1M for numerator using (c) and (d)
+ 1M for denominator using (a)

≈ 0.00443193

≈ 0.0044

1A $a-1$ for r.t. 0.004
----------(3)

(f) Suppose that the revised least expense is $\$x$.
Then, we have $P(X \ge x) = 0.33$. 1M

So, we have $P(Z \ge \frac{x-375}{125}) = 0.33$.

Therefore, we have $\frac{x-375}{125} = 0.44$. 1A

Hence, we have $x = 430$. 1A

Thus, the revised least expense is $\$430$.

----------(3)

(a)		Very good.
(b)		Very good.
(c)		Good.
(d)		Good. Some candidates overlooked the given condition that *each* of the three customers wins a gift.
(e)		Fair. Many candidates were able to handle conditional probabilities but some were not able to identify the compound events and get the numerator right.
(f)		Fair. A number of candidates could not establish the inequality and some could not solve for the required value.

19. (2007 ASL-M&S Q11)

Let X g be the net weight of a can of brand D coffee beans.
Then, $X \sim N(300, 7.5^2)$.

(a) The required probability

$= P(X < 283.5 \text{ or } X > 316.5)$

$= P(Z < \frac{283.5-300}{7.5} \text{ or } Z > \frac{316.5-300}{7.5})$

1M (accept $P(Z \le \frac{283.5-300}{7.5} \text{ or } Z \ge \frac{316.5-300}{7.5})$)

$= P(Z < -2.2 \text{ or } Z > 2.2)$

$= 2(0.0139)$

$= 0.0278$

1A $a-1$ for r.t. 0.028
----------(2)

(b) (i) The required probability

$= (1-0.0278)^{11}(0.0278)$

1M for $(1-p)^{11}p$

≈ 0.020387152

≈ 0.0204

1A $a-1$ for r.t. 0.020

(ii) The required probability

$= C_1^{30}(1-0.0278)^{29}(0.0278)$

1M for $C_1^{30}(1-p)^{29}p$ ------ 1M for $p = ($ (either one

≈ 0.368195889

≈ 0.3682

1A $a-1$ for r.t. 0.368

(iii) The required probability

$\approx (1-0.0278)^{30} + 0.368195889$

1M for $((1-p)^{30} + q)$ + 1M for $q = (b)(ii)$

≈ 0.797404576

≈ 0.7974

1A $a-1$ for r.t. 0.797
----------(8)

(c) (i) The required probability

$\approx \frac{1}{2}(0.368195889)$

1M for $\frac{1}{2}((b)(ii))$

≈ 0.184097944

≈ 0.1841

1A $a-1$ for r.t. 0.184

(ii) The required probability

$\approx \frac{0.184097944}{0.797404575}$

1M for numerator using (c)(i)
+ 1M for denominator using (b)(iii)

≈ 0.230871443

≈ 0.2309

1A $a-1$ for r.t. 0.231
----------(5)

(a)		Very good.
(b)		Good.
(c) (i)		Poor. Many candidates could not apply the well known multiplication rule: $P(A \cap B) = P(A)P(B \mid A)$. In this particular case, $P(A)$ is from b(ii) and $P(B \mid A) = \frac{1}{2}$.
(ii)		Satisfactory.

20. (2006 ASL-M&S Q10)

(a) The required probability

$$=\frac{4.7^0e^{-4.7}}{0!}+\frac{4.7^1e^{-4.7}}{1!}+\frac{4.7^2e^{-4.7}}{2!}+\frac{4.7^3e^{-4.7}}{3!}+\frac{4.7^4e^{-4.7}}{4!}+\frac{4.7^5e^{-4.7}}{5!}$$

1M for the 6 cases + 1M for Poisson probability

≈ 0.668438485

≈ 0.6684

1A $a-1$ for r.t. 0.668
----------(3)

(b) Let X km/h be the speed of a car entering the roundabout.
Then, $X \sim \text{N}(42.8, 12^2)$.
The required probability

$=\text{P}(X>50)$

$=\text{P}(Z>\frac{50-42.8}{12})$

1M (accept $\text{P}(Z\geq\frac{50-42.8}{12})$)

$=\text{P}(Z>0.6)$

$=0.2743$

1A $a-1$ for r.t. 0.274
----------(2)

(c) The required probability

$=(1-0.2743)^5(0.2743)$

1M for $(1-p)^5p$ + 1M for $p=$ (b)

≈ 0.055209196

≈ 0.0552

1A $a-1$ for r.t. 0.055
----------(3)

(d) (i) The required probability

$=C_1^4(0.2743)^3(1-0.2743)+(0.2743)^4$

1M for the 2 cases + 1M for binomial probability

≈ 0.065570471

≈ 0.0656

1A $a-1$ for r.t. 0.066

(ii) The required probability

$$\approx\frac{0.065570471\left(\frac{(4.7)^4e^{-4.7}}{4!}\right)+\left((0.2743)^5+C_1^5(0.2743)^4(1-0.2743)+C_2^5(0.2743)^3(1-0.2743)^2\right)\left(\frac{(4.7)^5e^{-4.7}}{5!}\right)}{0.668438485}$$

1M + 1M for numerator +
1M for denominator using (a)

≈ 0.052151265

≈ 0.0522

1A $a-1$ for r.t. 0.052
----------(7)

(a)		Very good.
(b)		Good. However, some candidates did not define the notation X when they used it to denote a random variable.
(c)		Good. A number of candidates could not adopt the geometric distribution.
(d) (i)		Fair. Some candidates mistook the required probability to be a conditional probability.
(ii)		Not satisfactory. Many candidates were not able to count the number of relevant events and clearly formulate the required probability.

21. (2005 ASL-M&S Q10)

(a) The required probability

$$=\frac{6.2^0e^{-6.2}}{0!}+\frac{6.2^1e^{-6.2}}{1!}+\frac{6.2^2e^{-6.2}}{2!}+\frac{6.2^3e^{-6.2}}{3!}+\frac{6.2^4e^{-6.2}}{4!}$$

1M for the 5 cases + 1M for Poisson probability

≈ 0.259177368

≈ 0.2592

1A $a-1$ for r.t. 0.259
----------(3)

(b) (i) Let X litres be the amount of the petrol for refuelling a car.
Then, $X \sim \text{N}(23.2, 6^2)$.
The required probability

$=\text{P}(X\geq 25)$

$=\text{P}(Z\geq\frac{25-23.2}{6})$

1M (accept $\text{P}(Z>\frac{25-23.2}{6})$)

$=\text{P}(Z\geq 0.3)$

$=0.3821$

1A $a-1$ for r.t. 0.382

(ii) The required probability

$=C_2^8(0.3821)^2(1-0.3821)^6(0.3821)$

1M for $C_2^8p^2(1-p)^6p$
+1M for $p=$ (b)(i)

≈ 0.086935732

≈ 0.0869

1A $a-1$ for r.t. 0.087

(iii) The required probability

$=\frac{6.2^3e^{-6.2}}{3!}(0.3821)^3$

1M for $\frac{6.2^3e^{-6.2}}{3!}p^3$ --------either one

≈ 0.004497064778

≈ 0.0045

1A $a-1$ for r.t. 0.004

(iv) The required probability

$=C_3^4(0.3821)^3(1-0.3821)+(0.3821)^4$

1M for $C_3^4p^3(1-p)+p^4$

≈ 0.159198667

≈ 0.1592

1A $a-1$ for r.t. 0.159

(v) The required probability

$$\approx\frac{0.004497064+0.159198667\left(\frac{6.2^4e^{-6.2}}{4!}\right)}{0.259177368}$$

1M for numerator using (b)(iii) and (b)(iv)
+1M for denominator using (a)

≈ 0.094100184

≈ 0.0941

1A $a-1$ for r.t. 0.094
----------(12)

(a)		Very good.
(b)(i)		Very good.
(ii)		Very good.
(iii)		Fair. Some candidates mistook the required probability to be a conditional probability.
(iv)		Not satisfactory. Many candidates mistook the required probability to be a conditional probability.
(v)		Fair. Many candidates were unable to correctly work out the numerator.

22. (2005 ASL-M&S Q11)

Let X minutes be the time needed for Peter to go to the train station platform.
Then, $X \sim N(17.5, 2^2)$.

(a) The required probability
$= P(13 < X \le 19)$
$= P(\frac{13-17.5}{2} < Z \le \frac{19-17.5}{2})$ — 1M (accept $P(\frac{13-17.5}{2} \le Z < \frac{19-17.5}{2})$)
$= P(-2.25 < Z \le 0.75)$
$= 0.4878 + 0.2734$
$= 0.7612$ — 1A a–1 for r.t. 0.761
---------(2)

(b) The required probability
$= (0.02)(0.0122) + (0.15)(0.7612) + (0.35)(0.2144) + (1)(0.0122)$ — 1M for $(0.02)p_1 + (0.15)p_2 + (0.35)p_3$ + 1M for $(1)(1 - p_1 - p_2 - p_3)$
$= 0.201664$ — 1A
≈ 0.2017 — a–1 for r.t. 0.202
---------(3)

(c) The required probability
$\approx \frac{(0.15)(0.7612)}{0.201664}$ — 1M for $\frac{(0.15)(a)}{(b)}$
≈ 0.566189305
≈ 0.5662 — 1A (accept 0.5661) a–1 for r.t. 0.566
---------(2)

(d) The required probability
$= C_2^5 (0.201664)^2 (1 - 0.201664)^3$ — 1M for $C_2^5 (b)^2 (1-(b))^3$
≈ 0.206925443
≈ 0.2069 — 1A (accept 0.2070) a–1 for r.t. 0.207
---------(2)

(e) The required probability
$\approx \frac{C_2^5((0.15)(0.7612))^2((0.0122)(1-0.02)+(0.2144)(1-0.35))^3}{0.206925443}$ — 1M for $\frac{C_2^5 p^2 q^3}{(d)}$ + 1A
≈ 0.002182834
≈ 0.0022 — 1A a–1 for r.t. 0.002

The required probability $\approx (0.566189305)^2 \left(\frac{(0.0122)(1-0.02)+(0.2144)(1-0.35)}{1-0.201664}\right)^3$	1M for $(c)^2 r^3$ + 1A
≈ 0.002182834 ≈ 0.0022	1A a–1 for r.t. 0.002

---------(3)

(f) Suppose Peter leaves home t minutes before 7:00 a.m.
Then, we have $P(X \le 13 + t) \ge 0.95$. — 1M withhold 1M for equality or strict inequality
So, we have $P(Z \le \frac{13 + t - 17.5}{2}) \ge 0.95$.
Therefore, we have $\frac{t - 4.5}{2} \ge 1.645$. — 1A (accept $\frac{t-4.5}{2} \ge z$, $1.64 \le z \le 1.65$)
Hence, we have $t \ge 7.79$.
Thus, the required time is 6:52 a.m. — 1A
---------(3)

(a)		Fair. Some candidates were not able to express the required probability.
(b)		Fair. Many candidates overlooked the case that Peter cannot catch any one of the three trains.
(c)		Fair. Some candidates got the numerator wrong and forgot that the required probability should be a joint probability.
(d)		Very good.
(e)		Poor. Many candidates were not able to count the number of relevant events and hence they were unable to correctly work out the numerator.
(f)		Poor. Many candidates could not formulate the problem using the correct inequality.

23. (2004 ASL-M&S Q12)

Let $\$X$ be the amount of money spent by a customer. Then, $X \sim N(428, 100^2)$.
Also let Y be the number of customers visiting the store in a minute. Then, $X \sim P_0(4)$.

(a) The required probability
$= P(X \ge 300)$
$= P(Z \ge \frac{300-428}{100})$
$= P(Z \ge -1.28)$ — 1A accept $P(Z > -1.28)$
$= 0.8997$ — 1A a–1 for r.t. 0.900
---------(2)

(b) The required probability
$= 1 - P(Y = 0) - P(Y = 1)$ — 1M for complementary probability
$= 1 - \frac{4^0 e^{-4}}{0!} - \frac{4^1 e^{-4}}{1!}$ — 1A
$= 1 - 5e^{-4}$ — 1A
[redacted]
≈ 0.9084 — a–1 for r.t. 0.908
---------(3)

(c) The required probability
$= P(Y = 3)(C_2^3 (0.8997)^2 (1 - 0.8997))$ — 1M for $C_2^3 (a)^2 (1-(a))$ + 1M for multiplication rule
$= (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1 - 0.8997))$
[redacted]
≈ 0.0476 — 1A a–1 for r.t. 0.048
---------(3)

(d) The required probability
$= \frac{(\frac{4^2 e^{-4}}{2!})(C_2^2 (0.8997)^2) + (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1 - 0.8997))}{\frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!}}$ — 1A for denominator + 1M for numerator
$\approx \frac{0.16619104895}{0.34189192592}$
[redacted]
≈ 0.4861 — 1A a–1 for r.t. 0.486
---------(3)

(e) $P(X \ge 600)$
$= P(Z \ge \frac{600-428}{100})$
$= P(Z \ge 1.72)$
$= 0.0427$ — 1A
Let n be the number of customers visiting the store. Then, we have
$1 - (1 - 0.0427)^n \ge 0.99$ — 1M accept $(1 - 0.0427)^n \le 0.01$ withhold 1M for using equality or strict inequality
$(0.9573)^n \le 0.01$
$n \ln 0.9573 \le \ln 0.01$ — 1M for using ln or trial and error
$n \ge \frac{\ln 0.01}{\ln 0.9573}$
$n \ge 105.5300874$
Thus, the smallest number of customers visiting the store is 106. — 1A
---------(4)

(a)		Good. The skill is straightforward but some candidates did not understand the question and were unable to correctly find the probability.
(b)		Good.
(c)		Good. Some candidates were not capable of applying the multiplication rule.
(d)		Fair.
(e)		Poor. Very few candidates managed to establish the inequality that the stated probability ≥ 0.99.

24. (2003 ASL-M&S Q12)

(a) The required probability

$= 1 - \frac{C_7^{17} + C_7^{13}}{C_7^{30}}$ — 1M for counting cases + 1A for correctness of probability

$= \frac{38743}{39150}$ — 1A

≈ 0.989604086

≈ 0.9896 — $a-1$ for r.t. 0.990

---------(3)

(b) The required probability

$= \frac{\frac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30}}}{\frac{38743}{39150}}$ — 1M for denominator using (a) + 1A for numerator

$= \frac{1498}{2279}$ — 1A

≈ 0.657305835

≈ 0.6573 — $a-1$ for r.t. 0.657

The required probability $= \frac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30} - C_7^{17} - C_7^{13}}$	1M for denominator using (a) + 1A for numerator
$= \frac{1498}{2279}$	1A
≈ 0.657305835	
≈ 0.6573	$a-1$ for r.t. 0.657

---------(3)

(c) Let $\$X$ be the amount of money collected by a boy and $\$Y$ be the amount of money collected by a girl.
Then, $X \sim N(673, 100^2)$ and $Y \sim N(708, 100^2)$.

(i) The required probability

$= \mathrm{P}(X > 800)$

$= \mathrm{P}\left(Z > \frac{800-673}{100}\right)$ — 1M accept $Z \geq \frac{800-708}{100}$

$= \mathrm{P}(Z > 1.27)$

$= 0.102$ — 1A accept 0.1020

(ii) $\mathrm{P}(Y > 800)$

$= \mathrm{P}\left(Z > \frac{800-708}{100}\right)$

$= \mathrm{P}(Z > 0.92)$

$= 0.1788$ — 1A

The required probability

$= \left(C_1^3(0.102)(0.898)^2\right)\left(C_1^4(0.1788)(0.8212)^3\right)$ — 1M for Binomial probability + 1M for Binomial × Binomial

≈ 0.097734619

≈ 0.0977 — 1A $a-1$ for r.t. 0.098

(iii) The required probability

$\approx \frac{0.097734619}{0.097734619 + C_2^3(0.102)^2(0.898)(0.8212)^4 + (0.898)^3 C_2^4(0.1788)^2(0.8212)^2}$ — 1M for numerator +1M for denominator

≈ 0.478730045

≈ 0.4787 — 1A (accept 0.4786) $a-1$ for r.t. 0.479

---------(9)

(a/b)		Good. Most candidates successfully managed to count the number of combinations.
(c)		Parts (i) and (ii) were well attempted. Part (iii) was more demanding and most candidates were unable to obtain the probability of getting two certificates.

25. (2002 ASL-M&S Q13)

Let Xg be the weight of a bag of self raising flour in the batch.

(a) (i) P(a bag of flour is underweight) $= P(X < 376)$

$= P(\frac{X-400}{10} < \frac{376-400}{10})$ — 1M (either one)

$= P(Z < -2.4)$

≈ 0.0082 — 1A (either one)

(ii) P(a bag of flour is overweight) $= P(X > 424)$

$= P(\frac{X-400}{10} > \frac{424-400}{10})$

$= P(Z > 2.4)$

≈ 0.0082 — 1A

------(3)

(b) (i) P(a bag of flour is substandard)

$= P(X < 376) + P(X > 424)$

$\approx 0.0082 + 0.0082 = 0.0164$ — 1A

Let Y be the number of substandard bags in the sample.

P(there is no substandard bags in the sample) $= P(Y=0)$

$= C_0^{50}\ 0.0164^0 \times (1-0.0164)^{50}$ — 1M

$= 0.9836^{50} \approx 0.4374$ — 1A (0.43745) a–1 for r.t. 0.437

(ii) $P(Y \le 2)$

$= P(Y=0) + P(Y=1) + P(Y=2)$

$= C_0^{50} 0.0164^0 \times 0.9836^{50} + C_1^{50}\ 0.0164 \times 0.9836^{49} + C_2^{50}\ 0.0164^2 \times 0.9836^{48}$ — 1M

$\approx 0.43745 + 0.36469 + 0.14897$

≈ 0.9511 — 1A (0.95111) a–1 for r.t. 0.951

------(5)

(c) Let W be the number of underweight bags in the sample.

(i) $P(W=0, Y=1)$

$= P(W=0 \mid Y=1) \cdot P(Y=1)$

$= \frac{1}{2} \times C_1^{50} (0.0164)(0.9836)^{49}$ — 1M + 1M for $\frac{1}{2}$ and cond. prob.

≈ 0.1823 — 1A a–1 for r.t. 0.182

(ii) The required probability is $P(W=0, Y \le 2)$

$= P(W=0, Y=0) + P(W=0, Y=1) + P(W=0, Y=2)$ — 1M

$= P(Y=0) + P(W=0, Y=1) + P(W=0 \mid Y=2) \cdot P(Y=2)$

$\approx 0.43745 + 0.18235 + \left(\frac{1}{2}\right)^2 \cdot C_2^{50} (0.0164)^2 (0.9836)^{48}$ — 1M for the last term

≈ 0.6570 — 1A (0.65704) (Accept 0.6569) a–1 for r.t. 0.657

(iii) The required probability is $P(W=0 \mid Y \le 2)$

$= \frac{P(W=0, Y \le 2)}{P(Y \le 2)}$

$\approx \frac{0.65704}{0.95111}$ — 1M

≈ 0.6908 — (Accept 0.6907)

------(7)

26. (2002 ASL-M&S Q14)

(a) Let N be the number of customers visiting the supermarket in one minute.

$P(N \le 2) = \sum_{k=0}^{2} \frac{6^k}{k!} e^{-6}$ — 1A

$= e^{-6} + \frac{6}{1!} e^{-6} + \frac{6^2}{2!} e^{-6}$

$\approx 0.002479 + 0.01487 + 0.04462$

≈ 0.0620

$\therefore\ P(N > 2) = 1 - P(N \le 2) \approx 0.9380$ — 1M+1A a–1 if 0.938

------(3)

(b) (i) $X \sim N(\mu, \sigma^2)$

$P(X < 100) = 0.063$

$P(Z < \frac{100-\mu}{\sigma}) = 0.063$

$\frac{100-\mu}{\sigma} \approx -1.53$ .... (1) — 1A

$P(X \ge 400) = 0.006$

$p(Z \ge \frac{400-\mu}{\sigma}) = 0.006$

$\frac{400-\mu}{\sigma} \approx 2.51$ .... (2) — 1A (Accept $\frac{200-\mu}{\sigma} \in [-0.185, -0.18]$)

Solving (1) and (2), we get

$\mu \approx 213.6$ — 1A a–1 for more than 1 d.p. (Accept $\mu \in [213.3, 213.8]$)

$\sigma \approx 74.26 \approx 74.3$ — 1A a–1 for more than 1 d.p. (Accept $\sigma \in [74.1, 74.3]$)

$a_1 = P(200 \le X < 300)$

$= p(Z < \frac{300-213.6}{74.3}) - P(Z < \frac{200-213.6}{74.3})$

≈ 0.4484

≈ 0.448 — 1A a–1 for more than 3 d.p. (Accept $a_1 \in [0.448, 0.453]$)

$a_2 = P(300 \le X < 400)$

≈ 0.117 — 1A a–1 for more than 3 d.p. (Accept $a_2 \in [0.115, 0.119]$)

(ii) For normal distribution, median = mean = 213.6 — 1M

(iii) $P(X > 50 \mid X \le 200)$

$= \frac{P(50 \le X < 200)}{P(X < 100) + P(100 \le X < 200)}$ — 1M

$= \frac{P(-2.20 \le Z < -0.18)}{0.063 + 0.364}$

$\approx \frac{0.4861 - 0.0714}{0.427}$

≈ 0.9712 — 1A a–1 for more than 4 d.p. (Accept probability $\in [0.9620, 0.9749]$)

$P(X > 50 \mid X \le 200)$

$= \dfrac{P(50 \le X < 200)}{P(X < 100) + P(100 \le X < 200)}$ 1M

$= \dfrac{P(X < 200) - P(X \le 50)}{P(X \le 200)}$

$= \dfrac{P(X < 200) - P(Z \le -2.20)}{P(X \le 200)}$

$= \dfrac{0.427 - 0.0139}{0.427}$

≈ 0.9674 1A $a-1$ for more than 4 d.p. (Accept probability $\in [0.9620, 0.9749]$)

(iv) The required probability

$= C_2^5\, P(X < 200)^2 (1 - P(X < 200))^3 \cdot P(N = 5)$

$= 10(0.063 + 0.364)^2 (1 - (0.063 + 0.364))^3 \cdot \dfrac{6^5}{5!} e^{-6}$ 1M for Binomial/Poisson probability; 1M for the multiplication rule (Binomial × Poisson)

$\approx 10(0.1823)(0.1881)\cdot(0.1606)$

≈ 0.0551 1A $a-1$ for r.t. 0.055 (Accept probability $\in [0.0550, 0.0552]$)

----------(12)

27. (2001 ASL-M&S Q12)

Let E_X and E_Y be the lifetimes of brand X and brand Y CFLs respectively.

(a) $P(E_X < 8200) = 0.1151 \Rightarrow P\left(\dfrac{E_X - \mu}{400} < \dfrac{8200 - \mu}{400}\right) = 0.0808$

$\Rightarrow \dfrac{8200 - \mu}{400} = -1.4$ 1A for either

$\Rightarrow \mu = 8760$ 1A

$P(E_Y < 8200) = 0.1587 \Rightarrow P\left(\dfrac{E_Y - 8800}{\sigma} < \dfrac{8200 - 8800}{\sigma}\right) = 0.1587$

$\Rightarrow \dfrac{8200 - 8800}{\sigma} = -1.00$

$\Rightarrow \sigma = 600$ 1A

$a_1 = 0.3811$, $a_2 = 0.0548$ 1A

$b_1 = 0.2120$, $b_2 = 0.2586$, $b_3 = 0.2120$ 1A $b_1 = b_3 \in [0.2101, 0.2120]$, $b_2 \in [0.2586, 0.2624]$

[$b_1 = 0.2109$, $b_2 = 0.2608$, $b_3 = 0.2109$]

----------(5)

(b) The mean of the lifetimes of the 2 brands only differ a little but the standard deviation of the lifetimes of brand X CFLs is significantly smaller than that of brand Y.

I shall choose brand X because the lifetimes of its CFLs are more reliable. 1M

[I shall choose brand Y because there will be a bigger chance of getting a long life CFL.] [1M]

[I shall choose brand Y because the mean lifetime is larger.] [1M]

----------(1)

(c) (i) Let X_a, X_b and X_c be the lifetimes of lamps a, b and c resp.

(I) The required probability

$= P(X_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$ 1M

$= [1 - P(E_X < 8200)]\{1 - [P(E_X < 8200)]^2\}$ 1M

$\approx (1 - 0.0808)(1 - 0.0808^2)$ 1M

[$\approx 2(0.9192)^2(1 - 0.9192) + (0.9192)^3$] [1M]+[1M]+[1M]

≈ 0.9132 1A

(II) The required probability

$= \dfrac{P(X_a < 8200)\,P(X_b > 8200)\,P(X_c > 8200)}{1 - 0.9132}$ 1M for numerator; 1M for denominator

$= \dfrac{0.0808(1 - 0.0808)^2}{1 - 0.9132}$

$= 0.7865$ 1A

(ii) Note that $P(E_X < 8200) \approx 0.0808$

and $P(E_Y < 8200) \approx 0.1578$.

Since a brand X CFL is less likely than a brand Y CFL to have a lifetime less than 8200 hours, and lamp a is the most critical lamp for the lighting system to work (according to the result of (c)(i)(II)), 1

$\therefore$ Lamp a should be a brand X CFL.

Hence I will put the brand Y CFL as lamp b or c. 1A with explanation

[Let X_a and Y_a be the lifetimes of lamp a when using brand X CFL and brand Y CFL respectively. Similar notations are used for the other two lamps.

$P(Y_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$

$= (1 - 0.1587)(1 - 0.0808^2)$

$= 0.8358$

$P(X_a > 8200)[P(Y_b > 8200 \text{ or } X_c > 8200)]$

$= (1 - 0.0808)[(1 - 0.0808) + (1 - 0.1587) - (1 - 0.0808)(1 - 0.1587)]$

$= 0.9074$ } [1]

Hence putting the brand Y CFL as lamp b or c will yeild a better system.] [1A] with explanation

----------(9)

28. (2000 ASL-M&S Q12)

(a) Let $X \sim N(20, 5^2)$ and $Z \sim N(0, 1)$.

(i) P(risky but not hazardous | A)

$= P(12 < X < 27)$

$= P(\frac{12-20}{5} < Z < \frac{27-20}{5})$ — 1A

$= P(-1.6 < Z < 1.4)$

$\approx 0.4452 + 0.4192$ — 1M

≈ 0.8644 — 1A *a*−1 for r.t. 0.864

(ii) P(risky | A) $= P(X > 12)$

$= P(Z > -1.6)$

$\approx 0.4452 + 0.5$ — 1A

≈ 0.9452

P(hazardous | A) $= P(X > 27)$

$= P(Z > 1.4)$

$\approx 0.5 - 0.4192$ — 1A

≈ 0.0808

$\therefore$ P (a risky bottle is hazardous | A) $\approx \frac{0.0808}{0.9452} \approx 0.0855$ — 1M

(b) (i) P(risky) $= 0.6$ P(risky | A) $+ 0.4$ P(risky | B)

$\approx 0.6(0.9452) + 0.4(0.058)$ — 1M

≈ 0.59032

≈ 0.5903 (p) — 1A *a*−1 for r.t. 0.590

P(B and risky | risky) $= \frac{\text{P(risky | B)P(B)}}{\text{P(risky)}}$

$\approx \frac{(0.058)(0.4)}{0.59032}$ — 1A numerator; 1M Bayes' theorem

≈ 0.0393

(ii) P(B and hazardous | risky) $= \frac{\text{P(hazardous | B)P(B)}}{\text{P(risky)}}$

$\approx \frac{(0.004)(0.4)}{0.59032}$ — 1A numerator; 1M Bayes' theorem

≈ 0.00271

≈ 0.0027

(iii) P(license suspended) $= 1-(1-p)^5 - 5p(1-p)^4$ — 1M binomial; 1M complement of cases 0 & 1; 1M p from b(i)

$\approx 1-(1-0.59032)^5 - 5(0.59032)(1-0.59032)^4$

≈ 0.9053

29. (1999 ASL-M&S Q10)

Let X be the score on the questionnaire.

(a) (i) P(classify as non-PD | PD)

$= P(X < 75 \mid X \sim N(80, 5^2))$

$= P(Z < \frac{75-80}{5})$ — 1A

$= P(Z < -1)$

$\approx 0.5 - 0.3413$

$= 0.1587$ — 1A

(ii) P(classify as PD | non-PD)

$= P(X > 75 \mid X \sim N(65, 5^2))$

$= P(Z > \frac{75-65}{5})$ — 1A

$= P(Z > 2)$

$\approx 0.5 - 0.4772$

$= 0.0228$ — 1A

(b) The probability that out of 10 PDs, not more than 2 will be misclassified

$\approx (1-0.1587)^{10} + C_1^{10}(0.1587)(1-0.1587)^9 + C_2^{10}(0.1587)^2(1-0.1587)^8$ — 1M+1M (1M for 2nd or 3rd term; 1M for all)

≈ 0.7971 — 1A

(c) Let x_0 be the required critical level of score.

$P(X < x_0 \mid X \sim N(80, 5^2)) = 0.01$

$P(Z < \frac{x_0 - 80}{5}) = 0.01$ — 1A

$\frac{x_0 - 80}{5} \approx -2.3267$ — 1M (accept −2.325 to −2.33, for the case '$Z <$ ...' only)

$x_0 \approx 68.3665$ — 1A (accept 68.35 to 68.375)

(d) If a teenager is classified by the sociologist, then

P(classify as PD | non-PD)

$= P(X > 68.3665 \mid X \sim N(65, 5^2))$

$= P(Z > 0.6733)$ — 1M (accept 68.35 to 68.375; accept 0.67 to 0.675)

$\approx 0.5 - 0.2496$

$= 0.2504$

$\therefore$ P(misclassified) $\approx (0.01)(0.1) + (0.2504)(0.9)$ — 1M (for either) (accept 0.2498 to 0.2514)

≈ 0.2264 — 1A (accept 0.2258 to 0.2273)

If a teenager is classified by the criminologist, then

P(misclassified) $\approx (0.1587)(0.1) + (0.0228)(0.9)$

≈ 0.0364 — 1A

$\because$ $0.2264 > 0.0364$

$\therefore$ The probability of teenagers miscalssified by the sociologist is greater than that by the criminologist. — 1

30. (1998 ASL-M&S Q13)

Solution	Marks	Remarks
Let X, Y be the weights of the randomly selected boxes in parts 1 and 2 of a test respectively.		
(a) $P(X<490 \text{ or } X>510)$		
$=1-P(\frac{490-500}{5}\le Z\le\frac{510-500}{5})$	1A	deduct 1 mark once for the whole question for any wrong inequality sign
$=1-P(-2\le Z\le 2)$		
$\approx 1-2\times 0.4772$		
≈ 0.0456	1A	
(b) $P(490\le X<492)+P(508<X\le 510)$	1A	
$=P(\frac{490-500}{5}\le Z<\frac{492-500}{5})+P(\frac{508-500}{5}<Z\le\frac{510-500}{5})$	1A	
$=P(-2\le Z<-1.6)+P(1.6<Z\le 2)$		
$\approx(0.4772-0.4452)\times 2$		
≈ 0.0640	1A	
Alternatively,		
$P(X<492)+P(X>508)-$ P(a black signal is generated in the first part)	1A	
$=P(Z<\frac{492-500}{5})+P(Z>\frac{508-500}{5})-0.0456$	1A	
$\approx 0.0548+0.0548-0.0456$		
≈ 0.0640	1A	
(c) P(black)		
= P(black in part 1) + P(black in part 2)		
$\approx 0.0456+0.0640\times 0.0456$	1M+1M	
≈ 0.0485	1A	
(d) $P(508<X\le 510 \text{ and } 508<Y\le 510 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$=\frac{P(508<X\le 510)P(508<Y\le 510)}{P(490\le X<492)+P(508<X\le 510)}$		
$\approx\frac{0.0320\times 0.0320}{0.0320+0.0320}$	1M+1M	
≈ 0.0160	1A	
(e) P(red | part 2)		
$=P(508<X\le 510 \text{ and } 508<Y\le 510 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$+P(490\le X<492 \text{ and } 490\le X<492 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$\approx 2\times 0.0160$	1M	
≈ 0.0320	1A	
(f) P(red) = P(red | part 2) P(part 2)		
$\approx 0.0320\times 0.0640$	1M	
≈ 0.0020	1A	
Alternatively,		
P(red) = $P(508<X\le 510 \text{ and } 508<Y\le 510)$		
$+P(490\le X<492 \text{ and } 490\le Y<492)$		
$=0.0320^2\times 2$	1M	
$=0.0020$	1A	

31. (1997 ASL-M&S Q11)

Solution	Marks
(a) Let X be the number of FICs per day, then $X\sim Po(4)$.	
$P(X=0)=\frac{4^0e^{-4}}{0!}$	1M
≈ 0.0183	1A
(b) Let Y be the number of FICs which are related to house fires in 5 FICs, then $Y\sim B(5, 0.6)$.	
$P(Y\ge 2)=1-P(Y=0)-P(Y=1)$	
$=1-C_0^5(0.4)^5-C_1^5(0.6)(0.4)^4$	1M+1A
≈ 0.9130	1A
(c) Let H and L be the events of "a FIC is related to a house fire" and "a FIC is *large*". Let A be the amount of a FIC.	
(i) $P(L\mid H)=P(A>20\,000)$	
$=P(Z>\frac{200\,000-100\,000}{50\,000})$	1M
$=P(Z>2)$	
≈ 0.0228	1A
$P(L\mid \overline{H})=P(A>20\,000)$	
$=P(Z>\frac{200\,000-150\,000}{20\,000})$	
$=P(Z>2.5)$	
≈ 0.0062	1A
$P(L)=P(L\mid H)P(H)+P(L\mid\overline{H})P(\overline{H})$	
$\approx 0.0228(0.6)+0.0062(0.4)$	1M
≈ 0.0162	1A
(ii) $P(H\mid L)=\frac{P(L\mid H)P(H)}{P(L)}$	
$\approx\frac{0.0228\times 0.6}{0.0162}$	1M
≈ 0.8444	1A
(iii) P(5 FICs and at least 2 of them are *large*)	
= P(2 or more out of 5 FICs are *large*)P(X=5)	
$\approx[1-(1-0.0162)^5-5(0.0162)(1-0.0162)^4]\frac{e^{-4}4^5}{5!}$	1M+1A
≈ 0.0004	1A

32. (1997 ASL-M&S Q13)

Let L cm be the length of the front portion of Mr. Wong's necktie.

Solution	Marks	Remarks
(a) $P(44 < L < 45)$		
$= P(\frac{44-44.6}{1.2} < Z < \frac{45-44.6}{1.2})$	1M	
$\approx P(-0.5 < Z < 0.3333)$		
$\approx 0.1915 + 0.1293$ (or $0.1915 + 0.1306$)	1A	For either
≈ 0.3208 (or 0.3221)	1A	
(b) Let Y be the number of trials that Mr. Wong gets the first *perfect tying*, then $Y \sim \text{Geometric}(p)$, where		
$p \approx 0.3208$ (or 0.3221)		
$E(Y) = \frac{1}{p}$	1M	
≈ 3.1172 (or 3.1046)	1A	
(c) P(not more than 3 trials)		
= P(1 trial) + P(2 trials) + P(3 trials)		
$= p + p(1-p) + p(1-p)^2$	1M	or $1-(1-p)^3$
≈ 0.6867 (or 0.6885)	1A	
(d) Let T be the event that Mr. Wong has to go to work by taxi.		
(i) $P(T) \approx 1 - 0.6867$ (or $1 - 0.6885$)		
$= 0.3133$ (or 0.3115)		
P(less than $2T$ out of 6 days)		
$\approx C_0^6(0.6867)^6 + C_1^6(0.6867)^5(0.3133)$	1M+1A	
(or $C_0^6(0.6885)^6 + C_1^6(0.6885)^5(0.3115)$)		
≈ 0.3919 (or 0.3957)	1A	
(ii) $P(Y=5 \mid T) \approx \frac{(1-p)^4 p}{P(T)}$	1M	
≈ 0.2179 (or 0.2184)	1A	
(iii) Probability required		
$\approx 5(0.3133)^2(0.6867)^4$ (or $5(0.3115)^2(0.6885)^4$)	1M+1A	
≈ 0.1091 (or 0.1090)	1A	

33. (1996 ASL-M&S Q11)

Let X ml be the amount of soda water in each discharge. $X \sim N(210, 15^2)$.

Solution	Marks	Remarks
(a) $P(200 < X < 220)$		
$= P(\frac{200-210}{15} < Z < \frac{220-210}{15})$	1M	
$\approx P(-0.6667 < Z < 0.6667)$		
≈ 0.4972	1A	Accept value in [0.494, 0.4972]
(b) (i) $P(X > 240)$		
$= P(Z > \frac{240-210}{15})$	1M	
$= P(Z > 2)$		
≈ 0.0228	1A	
(ii) The probability that there is exactly 1 overflow out of 30 discharges is		
$C_1^{30}(0.0228)(0.9772)^{29}$	1M	
≈ 0.3504	1A	
(iii) The probability that Sam will get the second overflow on 31st July is		
0.3504×0.0228		
≈ 0.0080	1M	
(c) (i) $\because P(X > 205) = 0.8$		
$\therefore P(Z > \frac{205-\mu}{\sigma}) = 0.8$		
$\frac{205-\mu}{\sigma} = -0.84$ ……….(1)	1M+1A	Accept value in [−0.845, −0.84]
$\because P(X > 220) = 0.01$		
$\therefore P(Z > \frac{220-\mu}{\sigma}) = 0.01$		
$\frac{220-\mu}{\sigma} = 2.33$ ……….(2)	1A	Accept value in [2.32, 2.33]
Solving (1) & (2) :		
$\begin{cases} \sigma = 4.7 \\ \mu = 209.0 \end{cases}$	1A+1A	
(ii) $P(X > 225)$		
$= P(Z > \frac{225-209}{4.7})$		
$\approx P(Z > 3.4042)$		
≈ 0.0003	1A	
Probability required		
$= \frac{0.0003}{0.01}$	1M	
$= 0.03$	1A	

34. (1995 ASL-M&S Q12)

Let X denote the test score and D the event that a person has the disease.

(a) $P(X>63.2|D') = 0.33$

$P(Z>\frac{63.2-\mu}{5}) = 0.33$ — 1A

From the normal distribution table,

$\frac{63.2-\mu}{5} = 0.44$ — 1A

$\therefore \quad \mu = 61$ — 1A

(b) (i) $P(X>66|D) = P(Z>\frac{66-70}{5})$ — 1A

$= P(Z>-0.8)$

$= 0.7881$ — 1A

and $P(X>66|D') = P(Z>\frac{66-61}{5})$

$= P(Z>1)$

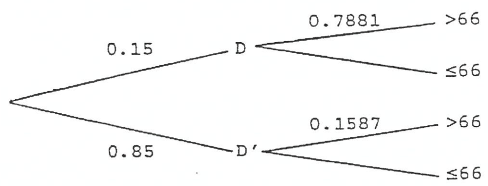
$= 0.1587$ — 1A

$\therefore$ P(the person will be classified as having the disease)

$= 0.15\times0.7881+(1-0.15)\times0.1587$ — 1M + 1A

$= 0.2531$ — 1A

(ii)



1M

$P(X\le66|D) = 1 - 0.7881$ — 1M

$= 0.2119$ — 1A

$\therefore$ P(the person will be misclassified)

$= 0.15\times0.2119+(1-0.15)\times0.1587$ — 1M + 1A

$= 0.1667$ — 1A

35. (1994 ASL-M&S Q13)

(a) $\because \quad P(Z<\frac{c_1-10}{0.4}) = 0.95$ — 1M

$\therefore \quad \frac{c_1-10}{0.4} = 1.645$ — 1A — Accept 1.64-1.65

$c_1 = 10.658$ — 1A — Accept 10.656-10.66

(b) $\because \quad P(Z<\frac{c_2-12.3}{0.6}) = 0.01$ — 1M

$\therefore \quad \frac{c_2-12.3}{0.6} = -2.327$ — 1A — -2.33 to -2.32

$c_2 = 10.9038$ — 1A — 10.902 to 10.908

(c) Given the batch is produced under the favourable condition, the required probability is

$P(c_1<X \text{ and } X<c_2)$

$= P(10.658<X<10.9038)$ — 1M

$= P(\frac{10.658-10}{0.4}<Z<\frac{10.9038-10}{0.4})$ — 1M

$= P(1.645<Z<2.2595)$

$= 0.4881 - 0.45$ — 1M — For subtraction

$= 0.0381$ — 1A

(d) $P(X<c_3 \text{ where } \sigma=0.4,\ \mu=10) = P(X\ge c_3 \text{ where } \sigma=0.6,\ \mu=12.3)$

i.e. $-(\frac{c_3-10}{0.4}) = \frac{c_3-12.3}{0.6}$ — 1M + 1A

$c_3 = 10.92$ — 1A

(e) The probability would be minimized if μ is in the middle of the 2 limits,

i.e. $\mu = \frac{10.8+9.4}{2}$ — 1M

$= 10.1$ — 1A

11. Point and Interval Estimation

Learning Unit	Learning Objective
Statistics Area	
Point and Interval Estimation	
21. Sampling distribution and point estimates	21.1 recognise the concepts of sample statistics and population parameters 21.2 recognise the sampling distribution of the sample mean from a random sample of size n 21.3 recognise the concept of point estimates including the sample mean, sample variance and sample proportion 21.4 recognise Central Limit Theorem
22. Confidence interval for a population mean	22.1 recognise the concept of confidence interval 22.2 find the confidence interval for a population mean
23. Confidence interval for a population proportion	23.1 find an approximate confidence interval for a population proportion

Summary

A. About population mean

For a normal population, i.e. $X \sim N(\mu, \sigma^2)$, based on a sample size n, $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$. It should be noted that the results are true for samples of any sizes.

For a non-normal population with the population mean μ and a known variance σ^2, if the sample size is large ($n \geq 30$), Central Limit Theorem can be used. $\bar{X}$ is approximately normal and $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.

For a normal or non-normal population with the population mean μ and *unknown* variance σ^2, if the sample size is large ($n \geq 30$), X is approximately normal and $\bar{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ where $\frac{s}{\sqrt{n}}$ is called the standard error of the sample and is denoted by $SE(\bar{x})$. (For normal population and small sample size, t-distribution is used.)

Conditions	95% confidence interval for μ	99% confidence interval for μ
Normal population · with known variance σ^2 · large or small sample size n · sample mean $\bar{x}$	$(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}})$	$(\bar{x} - 2.575\frac{\sigma}{\sqrt{n}}, \bar{x} + 2.575\frac{\sigma}{\sqrt{n}})$
Non-normal population · with known variance σ^2 · large sample size n ($n \geq 30$) · sample mean $\bar{x}$	$(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}})$	$(\bar{x} - 2.575\frac{\sigma}{\sqrt{n}}, \bar{x} + 2.575\frac{\sigma}{\sqrt{n}})$
Non-normal population · with unknown variance σ^2 · large sample size n ($n \geq 30$) · sample mean $\bar{x}$ · sample variance s^2	$(\bar{x} - 1.96\frac{s}{\sqrt{n}}, \bar{x} + 1.96\frac{s}{\sqrt{n}})$	$(\bar{x} - 2.575\frac{s}{\sqrt{n}}, \bar{x} + 2.575\frac{s}{\sqrt{n}})$

B. About population proportion

A number of random samples, each of size n, are drawn from a parent population. If the proportion of successes for each sample is $p_s=\frac{x}{n}$, these proportions form a distribution called the sampling distribution of proportions P_s (P_s denotes a distribution). When n is sufficiently large, the distribution of P_s is approximately normal and $P_s \sim N\left(p, \frac{p(1-p)}{n}\right)$ where $\frac{p(1-p)}{n}$ is the standard error of the proportions. The larger the sample size n is, the better is the approximation. Since p is not known, we use p_s to approximate p.

The approximate confidence interval for the population proportion p:

	Conditions
	large sample size n and sample proportion p_s
95% confidence interval	$\left(p_s-1.96\sqrt{\frac{p_s(1-p_s)}{n}}, p_s+1.96\sqrt{\frac{p_s(1-p_s)}{n}}\right)$
99% confidence interval	$\left(p_s-2.575\sqrt{\frac{p_s(1-p_s)}{n}}, p_s+2.575\sqrt{\frac{p_s(1-p_s)}{n}}\right)$

The sample size, n, should satisfies $\frac{1.96}{\sqrt{n}} \le width$ for 95% C.I. and $\frac{2.575}{\sqrt{n}} \le width$ for 99% C.I..

Section A

1. In an estate, Peter wants to study the proportion p of households who keep pets. He conducts a survey of a random sample of 64 households and finds that an approximate $\beta\%$ confidence interval for p is $(0.0915, 0.3085)$.
   (a) Find
      (i) the sample proportion of households who keep pets,
      (ii) β.
   (b) Using the sample proportion obtained in (a)(i), find the least number of households such that the probability of at least 1 of these households who keeps pets is greater than 0.999.
   (6 marks) (2018 DSE-MATH-M1 Q2)

2. There are many packs of seeds and each pack contains 100 seeds. Let p be the population proportion of seeds that germinate in a pack.
   (a) A pack of seeds is randomly selected, 64 seeds germinate. Find an approximate 95% confidence interval for p.
   (b) It is given that the proportion of seeds that germinate in these packs of seeds follows a normal distribution with a mean of p and a standard deviation of 0.05. Find the least sample size to be taken such that the width of a 90% confidence interval for p is less than 0.04.
   (7 marks) (2016 DSE-MATH-M1 Q4)

3. The manager of a fitness centre wants to promote aerobic classes.
   (a) The manager randomly selected 200 Hong Kong residents and found out that 80 of them had taken part in aerobic classes. Let p be the proportion of Hong Kong residents who had taken part in aerobic classes. Find an approximate 95% confidence interval for p.
   (b) The manager wants to randomly select n Hong Kong residents and invite them to take part in a free aerobic class. The probability that an invited resident will show up is 0.85. Let X be the proportion of the n invited residents who will show up. Assume that X can be modelled by a normal distribution with mean 0.85 and variance $\frac{0.85(1-0.85)}{n}$. Find the maximum number of n such that the probability that more than 100 invited residents will show up is less than 0.05.
   (7 marks) (2014 DSE-MATH-M1 Q9)

4. In a random sample of 120 swimmers in a certain beach, 75 of them are not satisfied with the water quality of the beach. Let p be the population proportion of the swimmers in this beach who are not satisfied with the water quality of the beach. Find an approximate 90% confidence interval for p.

(4 marks) (2013 DSE-MATH-M1 Q6)

5. The lifetime of a randomly selected LED bulb produced by a manufacturer is assumed to be normally distributed with mean μ hours and standard deviation 5000 hours. It is known that 96.41% of the bulbs will have a lifetime shorter than 39000 hours.

(a) Find the value of μ.

(b) Suppose a random sample of 100 bulbs is drawn. Find the probability that the mean lifetime of the sample lies between 30200 hours and 30800 hours.

(c) The manufacturer wants to select another random sample of n bulbs such that the probability that the mean lifetime of the sample exceeding 28500 hours is at least 0.985. Find the least value of n.

(7 marks) (2013 DSE-MATH-M1 Q9)

6. The weights (in kg) of the students in a school can be modelled by the normal distribution with mean 67 and standard deviation 15. A random sample of 36 students is taken.

(a) Find the probability that the mean weight of the 36 students is over 70 kg.

(b) It is found that 9 students in the sample like French fries. Find an approximate 95% confidence interval for the proportion of students in the school who like French fries.

(5 marks) (2012 DSE-MATH-M1 Q6)

7. A random sample of size 10 is drawn from a normal population with mean μ and variance 8. Let $\overline{X}$ be the mean of the sample.

(a) Calculate $\operatorname{Var}(2\overline{X}+7)$.

(b) Suppose the mean of the sample is 50. Construct a 97% confidence interval for μ.

(5 marks) (PP DSE-MATH-M1 Q6)

8. A political party studied the public view on a certain government policy. A random sample of 150 people was taken and 57 of them supported this policy

(a) Estimate the population proportion supporting this policy.

(b) Find an approximate 90% confidence interval for the population proportion.

(4 marks) (SAMPLE DSE-MATH-M1 Q3)

9. A manufacturer produces a large batch of light bulbs, with a mean lifetime of 640 hours and a standard deviation of 40 hours. A random sample of 25 bulbs is taken. Find the probability that the sample mean lifetime of the 25 bulbs is greater than 630 hours.

(5 marks) (SAMPLE DSE-MATH-M1 Q5)

Section B

10. The daily times spent on homework of the students in a school follow a normal distribution with a mean of μ hours and a standard deviation of 0.4 hours.

(a) A survey is conducted in the school to estimate μ.

(i) A sample of 40 students in the school is randomly selected and their daily times spent on homework are recorded below:

Daily time sent (x hours)	Number of student
$0.5 < x \le 1.0$	11
$1.0 < x \le 1.5$	13
$1.5 < x \le 2.0$	8
$2.0 < x \le 2.5$	5
$2.5 < x \le 3.0$	3

Find a 90 % confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 97 % confidence interval for μ is at most 0.3 .

(7 marks)

(b) Suppose that $\mu = 1.48$. If the daily time spent on homework of a student exceeds 2 hours, then the student has to attend homework guidance class.

(i) If a student is randomly selected from the school, find the probability that the student has to attend homework guidance class.

(ii) A sample of 15 students is now randomly drawn from the school and their daily times spent on homework are examined one by one. Given that more than 1 student in the sample have to attend homework guidance class, find the probability that the 10th student is the 2nd student who has to attend homework guidance class.

(6 marks)

(2017 DSE-MATH-M1 Q9)

11. The speeds of cars passing a checkpoint on a highway follow a normal distribution with a mean of μ km/h and a standard deviation of 16 km/h .

(a) A survey on the speeds of cars to estimate μ is conducted.

(i) A random sample of 25 cars is taken and the stem-and-leaf diagram below shows the distribution of their speeds (in km/h) :

Stem (tens)	Leaf (units)
6	0 0 1 1 1 2 2 3 4 4 5 5 6 6 7
7	1 1 2 3 5 5 6
8	3 6 7

Find a 95% confidence interval for μ .

(ii) Find the least sample size to be taken such that the width of a 97.5% confidence interval for μ is less than 9 .

(7 marks)

(b) Suppose that $\mu = 66$. If the speed of a car passing the checkpoint exceeds 90 km/h , a penalty ticket will be issued.

(i) If a car passes the checkpoint, find the probability that a penalty ticket will be issued.

(ii) If 12 cars pass the checkpoint, find the probability that more than 2 penalty tickets will be issued.

(5 marks)

(2015 DSE-MATH-M1 Q9)

12. The delivery time X (in minutes) of an order received by a pizza restaurant follows a normal distribution with mean μ and standard deviation σ . It is known that 27.43% of the delivery times are longer than 25 minutes and 51.60% of the delivery times fall within 3.5 minutes of μ .

(a) Find μ and σ .

(4 marks)

(b) If the delivery time of an order is longer than k minutes, then a coupon will be given as a compensation to the customer who has made the order. Suppose that a total of 200 orders are received in a day. Assuming independence among delivery times of different orders, find the minimum integral value of k such that the expected number of coupons given out is at most 5 in that day.

(3 marks)

(c) The employees of the pizza restaurant recently received training to improve their efficiency. After training, the delivery time Y (in minutes) of an order follows a normal distribution with mean θ and standard deviation 4.7 .

(i) Manager A draws a random sample of 12 orders and the delivery times (in minutes) are recorded as follows:

22	15	18	21	22	31
20	16	21	19	23	24

Construct a 90% confidence interval for θ .

(ii) Manager B is going to draw another random sample of n orders. He requires that the probability that the mean delivery time of the n orders falls within 3 minutes of θ be greater than 0.99. Find the minimum value of n to meet his requirement.

(6 marks)

(2014 DSE-MATH-M1 Q12)

13. The cholesterol levels (in suitable units) of the adults in a city are assumed to be normally distributed with mean μ and variance σ^2. From a random sample of 49 adults, a 95% confidence interval for μ is found to be $(4.596, 5.044)$.

(a) (i) Find the value of σ.

(ii) Find the mean of the sample.

(3 marks)

(b) Another sample of 15 adults is randomly selected and their cholesterol levels are recorded as follows:

3.6	3.8	3.9	4.3	4.3	4.5	4.8	5.0
5.1	5.2	5.3	5.5	5.8	6.0	6.4	

The two samples are then combined. Construct a 99% confidence interval for μ using the combined sample.

(4 marks)

(c) A health organisation classifies the cholesterol level of an adult to be low, medium and high if his/her cholesterol value is respectively at most 5.2, between 5.2 and 6.2, and at least 6.2. Suppose $\mu = 4.8$:

(i) Find the probability that the cholesterol level of a randomly selected adult in the city is low.

(ii) A sample of 20 adults is randomly selected in the city. Find the probability that there are more than 17 adults with low cholesterol level and at least 1 adult with medium cholesterol level in this sample.

(5 marks)

(2013 DSE-MATH-M1 Q12)

14. A company provides cable-car service for tourists. Tourists complain that the waiting time for the cable-car is too long. From past experience, the waiting time (in minutes) of a randomly selected tourist follows a normal distribution with mean μ and standard deviation 9.

(a) The customer service manager of the company conducts a survey on the waiting time to estimate μ.

(i) A random sample of 16 tourists is taken and their waiting times are recorded as below:

56	36	48	63	57	41	50	43
56	55	62	46	55	69	38	50

Construct a 90% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of the 90% confidence interval for μ is less than 6 minutes.

(7 marks)

(b) Suppose that $\mu = 51.5$. The customer service manager of the company interviews tourists and will give a coupon to a tourist whose waiting time is more than 65 minutes.

(i) Find the probability that he gives less than 2 coupons to the first 10 tourists interviewed.

(ii) Find the probability that the 5th coupon is given to the 20th tourist interviewed.

(6 marks)

(2012 DSE-MATH-M1 Q12)

15. A staff of a school studies the school sick room utilization. The number of visits to the sick room per day on 100 randomly selected school days are recorded as follows:

Number of visits per day	0	1	2	3	4	5	6	7
Frequency	6	12	18	21	20	12	7	4

(a) Find an unbiased estimate of the mean number of visits per day.

(1 mark)

(b) (i) Find the sample proportion of school days with less than 4 visits per day.

(ii) Construct an approximate 95% confidence interval for the proportion of school days with less than 4 visits per day.

(3 marks)

(c) Suppose the number of visits per day follows a Poisson distribution with mean λ. Assume that the unbiased estimate obtained in (a) is used for λ. The sick room is said to be *crowded* on a particular day if there are more than 3 visits on that day.

(i) Find the probability that the sick room is *crowded* on a particular day.

(ii) In a certain week of 5 school days, given that the sick room is *crowded* on at least 2 days, find the probability that the sick room is *crowded* on alternate days in the week.

(6 marks)

(PP DSE-MATH-M1 Q12)

16. The Body Mass Index (BMI) value (in kg/m^2) of children aged 12 in a city are assumed to follow a normal distribution with mean μ kg/m^2 and standard deviation 4.5 kg/m^2.

(a) A random sample of nine children aged 12 is drawn and their BMI values (in kg/m^2) are recorded as follows:

16.0, 18.3, 15.2, 17.8, 19.5, 15.9, 18.6, 22.5, 23.6

(i) Find an unbiased estimate for μ.

(ii) Construct a 95% confidence interval for μ.

(3 marks)

(b) Assume $\mu = 18.7$. If a random sample of 25 children aged 12 is drawn and their BMI values are recorded, find the probability that the sample mean is less than 17.8 kg/m^2.

(4 marks)

(c) A child aged 12 having a BMI value greater than 25 kg/m^2 is said to be *overweight*. Children aged 12 are randomly selected one after another and their BMI values are recorded until two *overweight* children are found. Assume that $\mu = 18.7$.

(i) Find the probability that a selected child is *overweight*.

(ii) Find the probability that more than eight children have to be selected in this sampling process.

(iii) Given that more than eight children will be selected in this sampling process, find the probability that exactly ten children are selected.

(8 marks)

(SAMPLE DSE-MATH-M1 Q14)

2021 DSE Q4

Mary conducts a survey to estimate the proportion p of children in a city who learn recorder. In a random sample of 40 children from the city, 28 of them learn recorder.

(a) (i) Find the sample proportion of children who learn recorder.

(ii) Find an approximate 90% confidence interval for p.

(b) Mary now wants to construct an approximate 99% confidence interval for p such that the width of the confidence interval does not exceed 0.1. Using the result of (a)(i), estimate the least number of children that Mary should survey.

(6 marks)

2021 DSE Q10

The number of commercial emails that John receives each hour follows a Poisson distribution with a mean of 1.3 per hour, while the number of non-commercial emails that he receives each hour follows a Poisson distribution with a mean of 0.9 per hour.

(a) Find the probability that the number of non-commercial emails that John receives in a certain hour is fewer than 3. (3 marks)

(b) Find the probability that the number of commercial emails that John receives in 6 hours is 5. (2 marks)

(c) Find the probability that the number of emails that John receives in a certain hour is 2. (3 marks)

(d) Given that the number of emails that John receives in a certain hour is 2, find the probability that both emails are non-commercial emails. (3 marks)

(e) Given that the number of emails that John receives in a certain hour is fewer than 3, find the probability that John does not receive commercial email in that hour. (3 marks)

11. Point and Interval Estimation

1. (2018 DSE-MATH-M1 Q2)

2. (2016 DSE-MATH-M1 Q4)

(a) The point estimate of p is $\frac{64}{100}=0.64$.	1A	
An approximate 95% confidence interval for p		
$=\left(\frac{64}{100}-1.96\sqrt{\frac{(0.64)(0.36)}{100}},\frac{64}{100}+1.96\sqrt{\frac{(0.64)(0.36)}{100}}\right)$	1M+1A	1A for 1.96
$=(0.54592, 0.73408)$	1A	
$\approx(0.5459, 0.7341)$		
(b) Let n be the number of packs in the sample.		
The width of a 90% confidence interval for p is $(2)(1.645)\left(\frac{0.05}{\sqrt{n}}\right)$.		
$(2)(1.645)\left(\frac{0.05}{\sqrt{n}}\right)<0.04$	1M+1A	
$\sqrt{n}>4.1125$		
$n>16.91265625$		
Thus, the least sample size is 17 .	1A	
	----------(7)	

(a)	Very good. More than 60% of the candidates were able to evaluate the approximate 95% confidence interval for the population proportion p. However, some candidates wrongly used $n=64$ instead of $n=100$ in evaluating the approximate confidence interval $\left(\frac{64}{100}-1.96\sqrt{\frac{(0.64)(0.36)}{n}},\frac{64}{100}+1.96\sqrt{\frac{(0.64)(0.36)}{n}}\right)$.
(b)	Good. Some candidates were unable to distinguish the concept between the confidence interval for the population mean and the approximate confidence interval for the population proportion.

3. (2014 DSE-MATH-M1 Q9)

(a) An estimate of $p=\frac{80}{200}$	
$=0.4$	1A
An approximate 95% confidence interval for p	
$=\left(0.4-1.96\sqrt{\frac{0.4\times0.6}{200}},\ 0.4+1.96\sqrt{\frac{0.4\times0.6}{200}}\right)$	1M
$\approx(0.3321, 0.4679)$	1A
(b) $X\sim \mathrm{N}\left(0.85,\frac{0.85\,(1-0.85)}{n}\right)$	
$\mathrm{P}\left(X>\frac{100}{n}\right)<0.05$	1A
$\mathrm{P}\left(Z>\frac{\frac{100}{n}-0.85}{\sqrt{\frac{0.85(0.15)}{n}}}\right)<0.05$	1M
$\frac{100-0.85n}{n}\sqrt{\frac{n}{0.1275}}>1.645$	1M
$0.85n+1.645\sqrt{0.1275n}-100<0$	
$-11.19754391<\sqrt{n}<10.50650569$	
$0<n<110.3866618$	
Hence the maximum number of n is 110 .	1A
	(7)

(a) (b)	Very good. Poor. Some candidates considered $\mathrm{P}(X>100)$ rather than $\mathrm{P}(X>100/n)$. Some candidates used wrong means such as 85 and $0.85n$, or wrong standard deviations such as $\frac{0.85\,(1-0.85)}{n}$, for standardisation. Others got inequalities in $\sqrt{n}$ with incorrect direction of sign.

4. (2013 DSE-MATH-M1 Q6)

An estimate for p is $\frac{75}{120}=0.625$. 1A OR $\frac{5}{8}$

An approximate 90% confidence interval for p

$\approx\left(0.625-1.645\sqrt{\frac{0.625(1-0.625)}{120}}\,,\,0.625+1.645\sqrt{\frac{0.625(1-0.625)}{120}}\right)$ 1M+1M

$\approx(0.5523, 0.6977)$ 1A

(4)

Good. Some candidates treated 75 as the sample size. Some wrote wrong expressions for the approximate standard deviation of the sample proportion.	

5. (2013 DSE-MATH-M1 Q9)

(a) P (lifetime of a bulb <39000) $=0.9641$

$P\left(Z<\frac{39000-\mu}{5000}\right)=0.9641$ 1M

$\frac{39000-\mu}{5000}\approx 1.8$

$\mu\approx 30000$ 1A

(b) P ($30200<$ sample mean <30800) Can use '$\le$' sign

$=P\left(\frac{30200-30000}{\frac{5000}{\sqrt{100}}}<Z<\frac{30800-30000}{\frac{5000}{\sqrt{100}}}\right)$ 1M

$=P(0.4<Z<1.6)$

$\approx 0.4452-0.1554$

$=0.2898$ 1A

(c) P (sample mean >28500) ≥ 0.985

$P\left(Z>\frac{28500-30000}{\frac{5000}{\sqrt{n}}}\right)\ge 0.985$ 1M

$-0.3\sqrt{n}\le -2.17$ 1A

$n\ge 52.32111111$

Thus, the least value of n is 53 . 1A

(7)

(a)	Very good. Some candidates showed poor presentation such as 'P$\left(Z<\frac{39000-\mu}{5000}\right)=0.9641$', 'P$\left(\frac{39000-\mu}{5000}\right)=0.9641$', 'P$\left(Z<\frac{39000-\mu}{5000}\right)=0.4641=0.9641$' or '$0.9641=0.18$'.
(b)	Good. Some candidates used '$0.4452+0.1554$' to find the probability required.
(c)	Fair. Some candidates used same symbols for both random variables before and after standardization. Many did not show enough ability to solve inequalities, such as writing a negative number greater than or equal to a positive number.

6. (2012 DSE-MATH-M1 Q6)

(a) Let X be the weight of a student. The sample mean $\overline{X}\sim N\left(67,\frac{15^2}{36}\right)$.

$P(\overline{X}>70)=P\left(Z>\frac{70-67}{\frac{15}{6}}\right)$ 1M

$=P(Z>1.2)$

≈ 0.1151 1A

(b) The sample proportion is $\frac{9}{36}=0.25$. 1A

An approximate 95% confidence interval for the proportion

$\approx\left(0.25-1.96\times\sqrt{\frac{0.25\times 0.75}{36}}\,,\,0.25+1.96\times\sqrt{\frac{0.25\times 0.75}{36}}\right)$ 1M

$\approx(0.1085, 0.3915)$ 1A

(5)

(a)	Good. Some candidates failed to perform the standardisation related to the distribution of a sample mean correctly.
(b)	Satisfactory. Many candidates found the sample proportion but failed to find the confidence interval required.

7. (PP DSE-MATH-M1 Q6)

(a) $\mathrm{Var}(2\overline{X}+7)=4\mathrm{Var}(\overline{X})$

$=4\left(\frac{8}{10}\right)$ 1M For $\mathrm{Var}(\overline{X})=\frac{\mathrm{Var}(X)}{n}$

$=3.2$ 1A

(b) A 97% confidence interval for μ

$=\left(50-2.17\times\frac{\sqrt{8}}{\sqrt{10}}\,,\,50+2.17\times\frac{\sqrt{8}}{\sqrt{10}}\right)$ 1M+1A 1M for $50\pm d$; 1A for 2.17

$=(48.0591, 51.9409)$ 1A

(5)

(a)	平平。部分學生誤以爲 $\mathrm{Var}(2\overline{X}+7)=2\mathrm{Var}(\overline{X})$ 及 $\mathrm{Var}(\overline{X})=\mathrm{Var}(X)$。
(b)	平平。部分學生誤以爲置信區間是 $\left(50-2.17\times\frac{8}{10},50+2.17\times\frac{8}{10}\right)$。

8. (SAMPLE DSE-MATH-M1 Q3)

(a) The estimate for the population proportion $=\frac{57}{150}=0.38$ 1A

(b) The approximate 90% confidence interval for the population proportion

$=\left(0.38-1.645\sqrt{\frac{0.38(1-0.38)}{150}}\,,\,0.38+1.645\sqrt{\frac{0.38(1-0.38)}{150}}\right)$ 1M+1M 1M for using (a); 1M for $\bar{x}\pm z_c s$

$\approx(0.314805956, 0.445194043)$

$\approx(0.3148, 0.4452)$ 1A

(4)

9. (SAMPLE DSE-MATH-M1 Q5)

Solution	Marks	Remarks
Let X be the lifetime of the bulb, then by Central Limit Theorem, $\overline{X} \sim N\left(640, \frac{40^2}{25}\right)$ approximately.	1A	
The required probability is $P(\overline{X} > 630)$ $= P\left(Z > \frac{630-640}{\frac{40}{\sqrt{25}}}\right)$	1M+1A	1M for standardization
$= P(Z > -1.25)$ $= 0.5 + P(0 < Z < 1.25)$	1M	
$= 0.5 + 0.3944$ $= 0.8944$	1A	
	(5)	

Section B

10. (2017 DSE-MATH-M1 Q9)

Solution	Marks	Remarks
(a) (i) The sample mean $= \frac{(0.75)(11)+(1.25)(13)+(1.75)(8)+(2.25)(5)+(2.75)(3)}{40}$ $= 1.45$ hours	1A	
A 90% confidence interval for μ $= \left(1.45 - 1.645\left(\frac{0.4}{\sqrt{40}}\right), 1.45 + 1.645\left(\frac{0.4}{\sqrt{40}}\right)\right)$	1M+1A	1A for 1.645
$\approx (1.3460, 1.5540)$	1A	r.t. (1.3460 , 1.5540)
(ii) Let n be the sample size. $2(2.17)\left(\frac{0.4}{\sqrt{n}}\right) \le 0.3$	1M+1A	1A for 2.17
$n \ge 33.48551111$ Thus, the least sample size is 34 .	1A	
	(7)	
(b) (i) The required probability $= P\left(Z > \frac{2-1.48}{0.4}\right)$	1M	
$= P(Z > 1.3)$ $= 0.5 - 0.4032$ $= 0.0968$	1A	
(ii) The required probability $= \frac{C_1^9(1-0.0968)^8(0.0968)^2}{1-(1-0.0968)^{15} - C_1^{15}(1-0.0968)^{14}(0.0968)}$	1M+1M+1M	1M for using (b)(i) + 1M for numerator + 1M for denominator
[redacted] ≈ 0.0861	1A	r.t. 0.0861
	(6)	

(a) (i)	Very good. Most candidates were able to find the confidence interval correctly.
(ii)	Very good. A few candidates wrongly used the sample mean obtained in (a)(i) to find the width of the interval concerned.
(b) (i)	Very good. About 80% of the candidates were able to find the required probability.
(ii)	Good. Many candidates were able to find the required conditional probability.

11. (2015 DSE-MATH-M1 Q9)

Solution	Marks	Remarks
(a) (i) The sample mean $= 68.64$ km/h	1A	
A 95% confidence interval for μ $=\left(68.64-1.96\left(\frac{16}{\sqrt{25}}\right), 68.64+1.96\left(\frac{16}{\sqrt{25}}\right)\right)$	1M+1A	1A for 1.96
$=(62.368, 74.912)$	1A	
(ii) Let n be the sample size. $2(2.24)\left(\frac{16}{\sqrt{n}}\right)<9$	1M+1A	1A for 2.24
$n>63.43237531$ Thus, the least sample size is 64 .	1A	
	----------(7)	
(b) (i) The required probability $=\mathrm{P}\left(Z>\frac{90-66}{16}\right)$	1M	
$=\mathrm{P}(Z>1.5)$ $=0.5-0.4332$ $=0.0668$	1A	
(ii) The required probability $=1-(1-0.0668)^{12}-C_1^{12}(1-0.0668)^{11}(0.0668)-C_2^{12}(1-0.0668)^{10}(0.0668)^2$	1M+1M	1M for using (b)(i) + 1M for binomial probability
≈ 0.041574551 ≈ 0.0416	1A	r.t. 0.0416
	----------(5)	

(a) (i)	Very good. Most candidates were able to use the correct formula to find the confidence interval while a few candidates treated 16 as the variance rather than the standard deviation of the given distribution.
(ii)	Very good. A few candidates wrongly used the sample mean in (a)(i) to find the width of the interval concerned.
(b) (i)	Very good. Most candidates were able to perform standardization and find the required probability.
(ii)	Very good. Most candidates were able to formulate the required probability form while a few candidates used wrong probabilities in substitution.

12. (2014 DSE-MATH-M1 Q12)

Solution	Marks	Remarks
(a) $\mathrm{P}(\mu-3.5\le X\le\mu+3.5)=0.5160$	1A	
$\mathrm{P}\left(0\le Z\le\frac{3.5}{\sigma}\right)=0.2580$ $\frac{3.5}{\sigma}=0.7$ $\sigma=5$	1A	
$\mathrm{P}(X>25)=0.2743$	1A	
$\mathrm{P}\left(0<Z<\frac{25-\mu}{\sigma}\right)=0.2257$ $\frac{25-\mu}{5}=0.6$ $\mu=22$	1A	
	(4)	
(b) $\mathrm{P}(X>k)\le\frac{5}{200}$	1A	
$\mathrm{P}\left(0<Z<\frac{k-22}{5}\right)\ge 0.475$ $\frac{k-22}{5}\ge 1.96$	1M	
$k\ge 31.8$ Hence the minimum integral value of k is 32 .	1A	
	(3)	
(c) (i) Sample mean $=\frac{22+15+\cdots+24}{12}$ $=21$	1A	
A 90% confidence interval $\approx\left(21-1.645\times\frac{4.7}{\sqrt{12}}, 21+1.645\times\frac{4.7}{\sqrt{12}}\right)$	1M	
$\approx(18.7681, 23.2319)$	1A	
(ii) Let $\bar{Y}$ be the mean delivery time of the n orders. $\mathrm{P}(\theta-3\le\bar{Y}\le\theta+3)>0.99$ $\mathrm{P}\left(\frac{-3}{\frac{4.7}{\sqrt{n}}}\le Z\le\frac{3}{\frac{4.7}{\sqrt{n}}}\right)>0.99$	1M	Alternative Solution: $\left(\bar{Y}-2.575\times\frac{4.7}{\sqrt{n}}, \bar{Y}+2.575\times\frac{4.7}{\sqrt{n}}\right)$ $\subsetneq(\bar{Y}-3, \bar{Y}-3)$
$\frac{3}{\frac{4.7}{\sqrt{n}}}>2.575$	1M	$\therefore 2.575\times\frac{4.7}{\sqrt{n}}<3$
$n>16.27450069$ Hence the minimum value of n is 17 .	1A	
	(6)	

(a)	Satisfactory. Some candidates found the value of $\mathrm{P}(\mu-1.75\le X\le\mu+1.75)$ instead of $\mathrm{P}(\mu-3.5\le X\le\mu+3.5)$.
(b)	Fair. Some candidates got inequalities in k with incorrect direction of sign.
(c) (i)	Good. Few candidates used the sample standard deviation in the calculation.
(ii)	Poor. Some candidates wrongly treated 21 as the mean of $\bar{Y}$. Some candidates thought that the length of the confidence interval was 3 .

13. (2013 DSE-MATH-M1 Q12)

(a) (i) $2\times 1.96\times\frac{\sigma}{\sqrt{49}}=5.044-4.596$ — 1M

$\sigma=0.8$ — 1A

(ii) The mean of the sample $=\frac{4.596+5.044}{2}$

$=4.82$ — 1A

(3)

(b) The combined sample mean $=\frac{4.82\times 49+3.6+3.8+\cdots+6.4}{49+15}$ — 1M

$=4.83875$ — 1A — OR 4.8388

A 99% confidence interval for μ

$\approx\left(4.83875-2.575\times\frac{0.8}{\sqrt{64}},\ 4.83875+2.575\times\frac{0.8}{\sqrt{64}}\right)$ — 1M

$=(4.58125,\ 5.09625)$ — 1A — OR $(4.5813, 5.0963)$

(4)

(c) Let X be the cholesterol level of a randomly selected adult.

(i) P(low) $=P(X\le 5.2)$

$=P\left(Z\le\frac{5.2-4.8}{0.8}\right)$

$=P(Z\le 0.5)$

≈ 0.6915 — 1A

(ii) P(high) $=P(X\ge 6.2)$

$=P\left(Z\ge\frac{6.2-4.8}{0.8}\right)$

$=P(Z\ge 1.75)$

≈ 0.0401 — 1A

P(medium) $\approx 1-0.6915-0.0401$

$=0.2684$ — 1A

P(more than 17 adults with low level and at least 1 adult with medium level)

$\approx C^{20}_{18}(0.6915)^{18}[C^2_1(0.2684)(0.0401)+(0.2684)^2]+C^{20}_{19}(0.6915)^{19}(0.2684)$ — 1M

≈ 0.0281 — 1A

(5)

(a)	Good. Some wrote $\frac{\sigma}{49}$ or $\frac{\sigma^2}{\sqrt{49}}$ instead of $\frac{\sigma}{\sqrt{49}}$ as the standard deviation of the sample mean.
(b)	Poor. Many candidates tried to use the standard deviation of the combined sample instead of that of the population. Some thought that the mean of the combined sample would be $\frac{4.82+4.9}{2}$.
(c) (i)	Fair. Some candidates assigned wrong values to μ and σ.
(ii)	Poor. Some candidates missed out the factor C^2_1 in calculating the probability required.

14. (2012 DSE-MATH-M1 Q12)

(a) (i) The sample mean $=\frac{56+\cdots+50}{16}$

$=51.5625$ — 1A

A 90% confidence interval

$\approx\left(51.5625-1.645\times\frac{9}{\sqrt{16}},\ 51.5625+1.645\times\frac{9}{\sqrt{16}}\right)$ — 1M+1A

$=(47.86125,\ 55.26375)$ — 1A — OR $(47.8613, 55.2638)$

(ii) Let n be the sample size.

$\therefore\ 2\left(1.645\cdot\frac{9}{\sqrt{n}}\right)<6$ — 1M

$n>24.354225$ — 1A

Hence, the least sample size is 25 . — 1A

(7)

(b) (i) P(a tourist waits for more than 65 minutes)

$=P\left(Z>\frac{65-51.5}{9}\right)$ — 1M

$=P(Z>1.5)$

≈ 0.0668 — 1A

P(less than 2 coupons are sent to the first 10 tourists interviewed)

$\approx(1-0.0668)^{10}+C^{10}_1(1-0.0668)^9(0.0668)$ — 1M

≈ 0.8594 — 1A

(ii) P(the 5th coupon is sent to the 20th tourist interviewed)

$\approx C^{19}_4(1-0.0668)^{15}(0.0668)^4\cdot 0.0668$ — 1M

≈ 0.0018 — 1A

(6)

(a)	(i)	Good. However, some candidates used the standard deviation of the sample instead of the population, used values other than 1.645 , or interchanged the upper and lower confidence limits.
	(ii)	Fair. Besides mistakes similar to (i), many candidates did not write the width of the confidence interval correctly or failed to solve inequalities.
(b)	(i)	Good. Most candidates were able to express the probability of the mentioned event, but some failed in the standardisation of normal distributions.
	(ii)	Satisfactory. Binomial coefficients were omitted or written wrongly by some candidates.

15. (PP DSE-MATH-M1 Q12)

Solution	Marks	Remarks
(a) The estimate of the mean $=\dfrac{0\times 6+\cdots+7\times 4}{100}$ $=3.21$	1A	
	(1)	
(b) (i) The sample proportion of school days with less than 4 visits $=\dfrac{57}{100}$	1A	
(ii) An approximate 95% confidence interval for the proportion $=\left(0.57-1.96\sqrt{\dfrac{0.57\times 0.43}{100}}\ ,\ 0.57+1.96\sqrt{\dfrac{0.57\times 0.43}{100}}\right)$	1M	
$=(0.4730\ ,\ 0.6670)$	1A	
	(3)	
(c) (i) By (a), $\lambda=3.21$. P(*crowded* on a day) $=1-e^{-3.21}\left(1+3.21+\dfrac{3.21^2}{2!}+\dfrac{3.21^3}{3!}\right)$	1M	For Poisson probability
≈ 0.399705729 ≈ 0.3997	1A	
(ii) P(*crowded* on alternate days | *crowded* on at least 2 days) $=\dfrac{(0.399705729)^3(1-0.399705729)^2+(1-0.399705729)^3(0.399705729)^2}{1-(1-0.399705729)^5-5(1-0.399705729)^4(0.399705729)}$	1M+1M+1M	1M for numerator 1M for denominator 1M for binomial probability
≈ 0.0869	1A	
	(6)	

(a)	良好。 少數學生誤以爲估算值是 $\dfrac{321}{99}$ 。
(b) (i)	良好。
(ii)	平平。 部分學生未能運用總體比例的置信區間公式。
(c) (i)	平平。 少部分學生誤以爲所求概率是 $1-P(0)-P(1)-P(2)$ 。
(ii)	甚差。 學生在研究所有的可能性時出現困難。

16. (SAMPLE DSE-MATH-M1 Q14)

Solution	Marks	Remarks
(a) (i) An unbiased estimate for μ is 18.6 .	1A	
(ii) The 95% confidence interval for μ $=\left(18.6-1.96\dfrac{4.5}{\sqrt{9}}\ ,\ 18.6+1.96\dfrac{4.5}{\sqrt{9}}\right)$	1M	1M for $\bar{x}\pm z_c s$
$=(15.66\ ,\ 21.54)$	1A	
	(3)	
(b) Let $\bar{X}$ be the sample mean of the BMI values of 25 children aged 12, then $\bar{X}\sim N\left(18.7\ ,\ \dfrac{4.5^2}{25}\right)$.	1A	
The required probability $=P(\bar{X}<17.8)$ $=P\left(Z<\dfrac{17.8-18.7}{4.5/\sqrt{25}}\right)$	1M+1M	1M for standardization 1M for $\dfrac{\sigma}{\sqrt{n}}$
$=P(Z<-1)$ $=0.5-0.3413$ $=0.1587$	1A	
	(4)	
(c) (i) Let X be the BMI values of the children aged 12 , then $X\sim N(18.7, 4.5^2)$. The required probability $=P(X>25)$ $=P\left(Z>\dfrac{25-18.7}{4.5}\right)$	1M	For standardization
$=P(Z>1.4)$ $=0.5-0.4192$ $=0.0808$	1A	
(ii) The required probability $=(1-0.0808)^8+{}_8C_1(1-0.0808)^7(0.0808)$	1M+1M	1M for 2 cases 1M for binomial prob
≈ 0.868062354 ≈ 0.8681	1A	
(iii) The required probability $=\dfrac{{}_9C_1(1-0.0808)^8(0.0808)\times(0.0808)}{(1-0.0808)^8+{}_8C_1(1-0.0808)^7(0.0808)}$	1M+1M	1M for numerator 1M for denominator using (c)(ii)
≈ 0.034498041 ≈ 0.0345	1A	
<u>Alternative solution</u> The required probability $=\dfrac{(1-0.0808)^8\times(0.0808)^2+{}_8C_1(1-0.0808)^7(0.0808)\times(1-0.0808)(0.0808)}{(1-0.0808)^8+{}_8C_1(1-0.0808)^7(0.0808)}$	1M+1M	1M for numerator 1M for denominator using (c)(ii)
≈ 0.034498041 ≈ 0.0345	1A	
	(8)	