8. Discrete Random Variables

Learning Unit	Learning Objective
Statistics Area	
Binomial, Geometric and Poisson Distributions	
12. Discrete random variables	12.1 recognise the concept of a discrete random variable
13. Probability distribution, expectation and variance	13.1 recognise the concept of discrete probability distribution and its representation in the form of tables, graphs and mathematical formulae 13.2 recognise the concepts of expectation $E(X)$ and variance $\text{Var}(X)$ and use them to solve simple problems 13.3 use the formulae $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$ to solve simple problems

Section A

1. The table below shows the probability distribution of a discrete random variable Y, where m and p are constants:

y	-2	2	m
$P(Y=y)$	p	0.25	0.5

(a) Prove that $\text{Var}(Y)=0.25m^2+2$.

(b) If $\text{Var}(2Y-1)=8E(2Y-1)$, find m.

(7 marks) (2018 DSE-MATH-M1 Q4)

2. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	0	2	4	5	8	9
$P(X=x)$	k^2	0.16	0.18	0.3	k	0.12

Find

(a) k,

(b) $E(X)$,

(c) $\mathrm{Var}(2-3X)$.

(6 marks) (2017 DSE-MATH-M1 Q1)

3. The table below shows the probability distribution of a discrete random variable X, where a and b are constants:

x	2	3	5	7	9
$P(X=x)$	0.08	0.15	a	0.45	b

It is given that $\mathrm{E}(X)=5.64$. Find

(a) a and b ,

(b) $\mathrm{E}((6-5X)^2)$ and $\mathrm{Var}(6-5X)$.

(6 marks) (2015 DSE-MATH-M1 Q1)

4. Let X be a discrete random variable with probability function as shown in the following table.

x	k	0	4	6
$P(X=x)$	0.1	0.2	0.3	0.4

It is given that $\mathrm{E}(X)=3.4$.

(a) Find the value of k .

(b) Find $\mathrm{Var}(3-4X)$.

(c) Let G be the event that $X<4$ and H be the event that $X\geq -1$. Find $P(G\cap H)$.

(5 marks) (2014 DSE-MATH-M1 Q6)

5. Let X and Y be two independent discrete random variables with their respective probability distributions shown as follows:

x	0	1	3	5	7
$\mathrm{P}(X=x)$	0.2	0.3	0.3	0.1	0.1

y	1	2	4	m
$\mathrm{P}(Y=y)$	0.4	0.3	0.2	0.1

Suppose that $\mathrm{E}(Y)=2.4$.

(a) Find the value of m .

(b) Let A be the event that $X+Y\leq 2$ and B be the event that $X=0$.

(i) Find $\mathrm{P}(A)$.

(ii) Are events A and B independent? Justify your answer.

(5 marks) (2013 DSE-MATH-M1 Q7)

6. Let X be a discrete random variable with probability function shown below:

x	1	3	4	6	9	13
$P(X=x)$	0.1	a	0.25	0.15	b	0.05

where a and b are constants. It is known that $\mathrm{E}(X)=5.5$.

(a) Find the values of a and b .

(b) Let F be the event that $X\geq 4$ and G be the event that $X<8$.

(i) Find $P(F\cap G)$.

(ii) Are F and G independent events? Justify your answer.

(6 marks) (2012 DSE-MATH-M1 Q8)

7. The random variable X has probability distribution $P(X=x)$ for $x=1$, 2 and 3 as shown in the following table.

x	1	2	3
$P(X=x)$	0.1	0.6	0.3

Calculate

(a) $\mathrm{E}(X)$,

(b) $\mathrm{Var}(3-2X)$.

(5 marks) (SAMPLE DSE-MATH-M1 Q7)

NEW

Out of syllabus

8. Discrete Random Variable

1. (2018 DSE-MATH-M1 Q4)

2. (2017 DSE-MATH-M1 Q1)

(a) $k^2+0.16+0.18+0.3+k+0.12=1$	1M	
$k^2+k-0.24=0$		
$k=0.2$ or $k=-1.2$ (rejected)	1A	
Thus, we have $k=0.2$.		
(b) $\mathrm{E}(X)$		
$=0(0.04)+2(0.16)+4(0.18)+5(0.3)+8(0.2)+9(0.12)$	1M	
$=5.22$	1A	
(c) $\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9((0-5.22)^2(0.04)+(2-5.22)^2(0.16)+(4-5.22)^2(0.18)$		
$+(5-5.22)^2(0.3)+(8-5.22)^2(0.2)+(9-5.22)^2(0.12))$	1M	
$=56.6244$	1A	
$\mathrm{Var}(2-3X)$		
$=9\mathrm{Var}(X)$		
$=9(\mathrm{E}(X^2)-(\mathrm{E}(X))^2)$		
$=9(33.54-(5.22)^2)$	1M	
$=56.6244$	1A	
	----------(6)	

(a)	Very good. About 98% of the candidates were able to find the value of k by setting up a quadratic equation.
(b)	Very good. Over 90% of the candidates were able to find the value of $\mathrm{E}(X)$.
(c)	Very good. Most candidates were able to find the value of $\mathrm{Var}(2-3X)$.

3. (2015 DSE-MATH-M1 Q1)

(a)	$0.08+0.15+a+0.45+b=1$	1M	either one
	$2(0.08)+3(0.15)+5a+7(0.45)+9b=5.64$		
	Solving, we have $a=0.25$ and $b=0.07$.	1A	for both
(b)	$\mathrm{E}((6-5X)^2)$		
	$=\mathrm{E}(36-60X+25X^2)$		
	$=36-60\mathrm{E}(X)+25\mathrm{E}(X^2)$	1M	
	$=36-60(5.64)+25(35.64)$		
	$=588.6$	1A	
	$\mathrm{Var}(6-5X)$		
	$=\mathrm{E}((6-5X)^2)-(\mathrm{E}(6-5X))^2$		
	$=\mathrm{E}((6-5X)^2)-(6-5\mathrm{E}(X))^2$	1M	accept $(-5)^2\mathrm{Var}(X)$
	$=588.6-(6-5(5.64))^2$		
	$=95.76$	1A	
		----------(6)	

(a)	Very good. Most candidates were able to find the values of a and b by setting up two equations involving them.
(b)	Good. Many candidates were able to find the value of $\mathrm{Var}(6-5X)$ while some candidates wrongly found the value of $(\mathrm{E}(6-5X))^2$ instead of $\mathrm{E}((6-5X)^2)$.

4. (2014 DSE-MATH-M1 Q6)

(a) $0.1k+0.2(0)+0.3(4)+0.4(6)=3.4$ — 1M

$k=-2$ — 1A

(b) $\mathrm{Var}(3-4X)=16\mathrm{Var}(X)$ — 1M

$=16[\mathrm{E}(X^2)-\mathrm{E}(X)^2]$

$=16[0.1(-2)^2+0.2(0)^2+0.3(4)^2+0.4(6)^2-3.4^2]$

Alternative Solution

x	-2	0	4	6
$3-4x$	11	3	-13	-21
$\mathrm{P}(X=x)$	0.1	0.2	0.3	0.4

$\mathrm{E}(3-4X)=0.1(11)+0.2(3)+0.3(-13)+0.4(-21)$ — 1M — OR $3-4(3.4)$

$=-10.6$

$\mathrm{Var}(3-4X)=0.1(11+10.6)^2+0.2(3+10.6)^2+0.3(-13+10.6)^2+0.4(-21+10.6)^2$

$=128.64$ — 1A

(c) $\mathrm{P}(G\cap H)=\mathrm{P}(-1\le X<4)$

$=\mathrm{P}(X=0)$

$=0.2$ — 1A

(5)

(a)	Excellent.
(b)	Very good. Some candidates equated $\mathrm{Var}(3-4X)$ to $3^2\mathrm{Var}(X)$ or $3-4\mathrm{Var}(X)$.
(c)	Good.

5. (2013 DSE-MATH-M1 Q7)

(a) $E(Y)=1\times0.4+2\times0.3+4\times0.2+m\times0.1=2.4$ $\therefore\ m=6$	1A	
(b) (i) $P(A)=P(X=0,Y=1)+P(X=0,Y=2)+P(X=1,Y=1)$ $=0.2\times0.4+0.2\times0.3+0.3\times0.4$	1M	
$=0.26$	1A	
(ii) $P(A\cap B)=P(X=0,Y=1)+P(X=0,Y=2)$ $=0.2\times0.4+0.2\times0.3$ $=0.14$	1A	
$P(A)P(B)=0.26\times0.2$ $=0.052$ $\neq P(A\cap B)$		
Alternative Solution: $P(A\mid B)=P(Y=1)+P(Y=2)$ $=0.4+0.3$ $=0.7$ $\neq P(A)$ by (i)	1A	
Thus, A and B are not independent.	1A	Follow through
	(5)	

(a)		Excellent.
(b)	(i)	Good. Mistakes were occasionally found in computations.
	(ii)	Fair. A lot of candidates thought that the independence of two events A and B could be verified by checking $P(A\cap B)=0$. Among those who found correct values of related probabilities, some did not mention $P(A\cap B)\neq P(A)\cdot P(B)$ as the reason to make conclusion, while some made a wrong conclusion that 'A and B are independent'.

6. (2012 DSE-MATH-M1 Q8)

(a) $P(X=1)+P(X=3)+\cdots+P(X=13)=1$ $0.1+a+0.25+0.15+b+0.05=1$	1M	
$a+b=0.45$ ---------- (1) $E(X)=5.5$		
$1\times0.1+3a+4\times0.25+6\times0.15+9b+13\times0.05=5.5$	1M	
$a+3b=0.95$ ---------- (2)		
Solving (1) and (2), we get $a=0.2$ and $b=0.25$.	1A	For both
(b) (i) $P(F\cap G)=0.25+0.15$ $=0.4$	1A	
(ii) $P(F)\times P(G)=(0.25+0.15+0.25+0.05)(0.1+0.2+0.25+0.15)$ $=0.49$	1A	
$\neq P(F\cap G)$		
Alternative Solution 1: $P(F\mid G)=\dfrac{P(F\cap G)}{P(G)}$ $=\dfrac{0.4}{0.1+0.2+0.25+0.15}$ ≈0.571428571 $P(F)=0.25+0.15+0.25+0.05$ $=0.7$ $\neq P(F\mid G)$	1A	
Alternative Solution 2: $P(G\mid F)=\dfrac{P(F\cap G)}{P(F)}$ $=\dfrac{0.4}{0.25+0.15+0.25+0.05}$ ≈0.571428571 $P(G)=0.1+0.2+0.25+0.15$ $=0.7$ $\neq P(G\mid F)$	1A	
Hence, F and G are not independent.	1	
	(6)	

(a)	Excellent.
(b)	Satisfactory. Quite a number of candidates did not understand the concept of independence — some calculated $P(F\cap G)$ using $P(F)\times P(G)$ and some mixed up independent events with mutually exclusive events.

7. (SAMPLE DSE-MATH-M1 Q7)

(a) $E(X)=1(0.1)+2(0.6)+3(0.3)$	
$=2.2$	1A
(b) $\text{Var}(X)=[1^2(0.1)+2^2(0.6)+3^2(0.3)]-2.2^2$	1M
$=0.36$	1A
$\therefore\ \text{Var}(3-2X)=2^2\,\text{Var}(X)$	1M
$=1.44$	1A
<u>Alternative Solution</u>	
$E(3-2X)=3-2E(X)$	1M
$=-1.4$	1A
$\text{Var}(3-2X)=(3-2\cdot1+1.4)^2(0.1)+(3-2\cdot2+1.4)^2(0.6)+(3-2\cdot3+1.4)^2(0.3)$	1M
$=1.44$	1A
	(5)