10. Normal Distribution

Learning Unit	Learning Objective
Statistics Area	
Normal Distribution	
18. Basic definition and properties	18.1 recognise the concepts of continuous random variables and continuous probability distributions, with reference to the normal distribution 18.2 recognise the concept and properties of the normal distribution
19. Standardisation of a normal variable and use of the standard normal table	19.1 standardise a normal variable and use the standard normal table to find probabilities involving the normal distribution.
20. Applications of the normal distribution	20.1 find the values of $P(X > x_1)$, $P(X < x_2)$, $P(x_1 < X < x_2)$ and related probabilities, given the values of x_1, x_2, μ and σ, where $X \sim N(\mu, \sigma^2)$ 20.2 find the values of x, given the values of $P(X > x)$, $P(X < x)$, $P(a < X < x)$, $P(x < X < b)$ or a related probability, where $X \sim N(\mu, \sigma^2)$ 20.3 use the normal distribution to solve problems

Section A

1. A factory manufactures a batch of marbles. The diameters of the marbles follow a normal distribution with a mean of 9 mm and a standard deviation of 0.125 mm . A marble is classified as *oversized* if its diameter is more than 9.16 mm .
   (a) Find the probability that a randomly selected marble from the batch is *oversized.*
   (b) The diameters of the marbles are measured one by one. Let X be the random variable representing the number of measurements taken when the first *oversized* marble is found. Find
   (i) $P(X \leq 3)$,
   (ii) $E(X)$.

   (6 marks) (2018 DSE-MATH-M1 Q3)

2. In a large farm, the weights of chickens follow a normal distribution with a mean of μ kg and a standard deviation of σ kg . It is given that the percentage of chickens being lighter than 1.83 kg is the same as the percentage of those being heavier than 3.43 kg. Moreover, 89.04% chickens weigh between 1.83 kg and 3.43 kg.
   (a) Find μ and σ.
   (b) If 9 chickens are selected randomly from the farm, find the probability that the mean of their weights lies between 2.5 kg and 3.1 kg.

   (5 marks) (2017 DSE-MATH-M1 Q3)

3. Among the students sitting for a Mathematics test, 73% of them had revised before the test. For those who had revised, their scores are real numbers which can be modelled by $N(59, 10^2)$; and for those who had not revised, their scores are real numbers which can be modelled by $N(35.2, 12^2)$. Students who scored at least 43 passed the test.
   (a) Find the probability that a randomly selected student passed the test.
   (b) Given that a randomly selected student passed the test, find the probability that he had not revised before the test.
   (c) Ten students are randomly selected among those who passed the test. Find the probability that exactly four of them had not revised before the test.

   (7 marks) (2012 DSE-MATH-M1 Q9)

4. The coach of a girls school basketball team recruits new members from the Form One students, of whom 11.7% are taller than 152 cm. Assume that their heights are normally distributed with a mean μ cm and a standard deviation of 5 cm.

(a) Find the value of μ .

(b) It is known that 20% of the Form One students taller than 152 cm do not apply to join the basketball team, while 10% of students shorter than 152 cm apply to join. If a Form One student is selected at random, find the probability that

(i) the student applies to join the basketball team;

(ii) the student is shorter than 152 cm given that she does not apply to join the basketball team.

(7 marks) (2010 ASL-M&S Q6)

5. The amount of money spent by a randomly selected customer of a jewellery shop is assumed to be normally distributed with a mean of \$ μ and a standard deviation of \$6 000. Suppose 24.2% of the customers spend more than \$30 000 in the shop.

(a) Find the value of μ .

(b) It is given that Mrs. Chan spends less than \$30 000 in the shop. Find the probability that she spends more than \$16 500 .

(6 marks) (2008 ASL-M&S Q5)

6. Some statistics from a survey on the monthly incomes (in thousands of dollars) of a group of university graduates are summarized as follows:

Minimum	8
Maximum	52
Lower quartile	10
Median	17
Upper quartile	20
Mean	17.94
Standard deviation	4.7

(a) Using the above information, construct a box-and-whisker diagram to describe the distribution of the monthly incomes.

(b) A student proposes to model the distribution of the monthly incomes of the group of university graduates by a normal distribution with mean and standard deviation given in the above table.

(i) Using the model proposed by the student, find the probability that the monthly income of a randomly selected university graduate from the group is less than \$ 17 000 .

(ii) Is the model proposed by the student appropriate? Explain your answer.

(6 marks) (2004 ASL-M&S Q5)

7. The amount of money involved in a business transaction follows a normal distribution with mean \$215 and standard deviation \$50 . Any transaction with an amount more than \$300 is classified as a Type A transaction.

(a) Find the probability that a transaction will be classified as Type A.

(b) Find the probability that in 7 randomly selected transactions, exactly 2 transactions will be classified as Type A.

(c) Find the probability that the 8th randomly selected transaction is the 3rd transaction which is classified as Type A.

(d) It is known that 64.8% of the transactions each exceeds \$$K$. Find K .

(7 marks) (2003 ASL-M&S Q6)

8. (a) Use the exponential series to find a polynomial of degree 6 which approximates $e^{\frac{-x^2}{2}}$ for x close to 0 .

Hence estimate the integral $\int_0^1 e^{\frac{-x^2}{2}} dx$.

(b) It is known that the area under the standard normal curve between $z=0$ and $z=a$ is $\int_0^a \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz$. Use the result of (a) and the normal distribution table to estimate, to 3 decimal places, the value of π .

(7 marks) (1994 ASL-M&S Q6)

Section B

9. A fruit wholesaler, John, grades a batch of apples according to their weights. The following table shows the classification of the apples, where a is a constant.

Weight of an apple (W g)	$W \le a$	$a < W \le 260$	$W > 260$
Classification	*small*	*medium*	*large*

The weights of the apples follow a normal distribution with a mean of μ g and a standard deviation of 16 g. It is known that 10.56% and 73.57% of the apples are *large* and *medium* respectively. Every 8 of the apples are packed in a box. A box of apples is regarded as *regular* if there are at least 6 *medium* apples in the box.

(a) Find μ and a .

(3 marks)

(b) Find the probability that a randomly chosen box of apples is *regular*.

(2 marks)

(c) John randomly chooses 3 boxes of apples.

(i) Find the probability that these 3 boxes of apples are *regular* and there are totally 21 *medium* apples and 3 *small* apples.

(ii) Given that these 3 boxes of apples are *regular*, find the probability that there are totally 21 *medium* apples and 3 *small* apples.

(iii) Given that there are totally 21 *medium* apples and 3 *small* apples in these 3 boxes of apples, find the probability that these 3 boxes of apples are *regular*.

(7 marks)

(2018 DSE-MATH-M1 Q9)

10. X and Y are two schools with the same number of students. The daily reading times (in minutes) of the students in each school are assumed to be normally distributed. In school X, 0.6% of the students read less than 40 minutes daily while 1.5% read more than 70 minutes. In school Y, 1.5% of the students read less than 48 minutes daily while 1. 7% read more than 72 minutes.

(a) Which school has less students reading more than 60 minutes daily? Explain your answer.

(6 marks)

(b) For the school that has less students reading more than 60 minutes daily, find the probability that the 4th randomly selected student is the 2nd one who reads more than 60 minutes daily.

(2 marks)

(c) Students reading T minutes or more daily will be awarded. What should the least value of T be so that no more than 10% of students are awarded in each school? Give your answer in integral minutes.

(4 marks)

(2016 DSE-MATH-M1 Q9)

11. Let $I=\int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}} dt$.

(a) (i) Use the trapezoidal rule with 6 sub-intervals to estimate I .

(ii) Is the estimate in (a)(i) an over-estimate or under-estimate? Justify your answer.

(7 marks)

(b) Using a suitable substitution, show that $I=2\int_1^2 e^{\frac{-x^2}{2}} dx$.

(3 marks)

(c) Using the above results and the Standard Normal Distribution Table, show that $\pi < 3.25$.

(3 marks)

(2012 DSE-MATH-M1 Q10)

12. In a supermarket, there are two cashier counters: a regular counter and an express counter. It is known that 88% of customers pay at the regular counter. It is found that the waiting time for a customer to pay at the regular counter follows the normal distribution with mean 6.6 minutes and standard deviation 1.2 minutes.

(a) Find the probability that the waiting time for a customer to pay at the regular counter is more than 6 minutes.

(2 marks)

(b) Suppose 12 customers who pay at the regular counter are randomly selected. Assume that their waiting times are independent.

(i) Find the probability that there are more than 10 of the 12 customers each having a waiting time of more than 6 minutes.

(ii) Find the probability that the average waiting time of the 12 customers is more than 6 minutes.

(5 marks)

(b) It is found that the waiting time for a customer to pay at the express counter follows the normal distribution with mean μ minutes and standard deviation 0.8 minutes. It is known that exactly 21.19% of the customers at the regular counter wait less than k minutes, while exactly 3.59% of the customers at the express counter wait more than k minutes.

(i) Find k and μ.

(ii) Two customers are randomly selected. Assume that their waiting times are independent. Given that both of them wait more than μ minutes to pay, find the probability that exactly one of them pays at the regular counter.

(8 marks)

(PP DSE-MATH-M1 Q13)

13. The speeds of the vehicles (X km/h) on a highway follow a normal distribution with mean μ km/h and standard deviation σ km/h . It is known that 12.3% of vehicles have speeds more than 82.64 km/h and 24.2% of vehicles have speeds less than 75.2 km/h . A machine is used to detect the speeds of the vehicles at a spot on the highway. A notice will be issued to the driver if the speed of his/her vehicle is detected to be over 80 km/h .

(a) Find μ and σ .

(3 marks)

(b) (i) A vehicle passes the spot. What is the probability that a notice will be issued?

(ii) Suppose that 10 vehicles pass the spot on the highway. What is the probability that at most 2 notices will be issued?

(4 marks)

(c) On a certain day, the machine does not work properly and there is an error in detecting the speeds of the vehicles. The error (Y km/h) is defined as follows:

Y = speed detected – actual speed ,

and it can be modelled by the following probability distribution:

Error (Y)	2	$2+\theta$
Probability	0.5	0.5

where θ is a non-zero constant. A vehicle passes the spot.

(i) Find the probability that a notice will be issued but the speed of the vehicle is not over 80 km/h for the following two cases:

(1) $\theta=1$,

(2) $\theta=-3$.

(ii) Find the range of values of θ such that the probability that a notice will not be issued but the speed of the vehicle is over 80 km/h is at most 7.125% .

(8 marks)

(2013 ASL-M&S Q10)

14. A manufacturer produces batteries A and B for notebook computers. After fully charged, the operation times (in minutes) of batteries A are normally distributed with mean 168 minutes and standard deviation 32 minutes, and those of batteries B are normally distributed with mean μ minutes and standard deviation σ minutes. Past data revealed that 33% of batteries B have operation times longer than 188 minutes, while 87.7% have operation times shorter than 213.2 minutes.

(a) (i) Find the probability that a randomly chosen battery A has an operation time shorter than 152 minutes or longer than 184 minutes.

(ii) If the probability that a randomly chosen battery A has an operation time longer than k minutes is 5% , find the value of k .

(iii) Find the values of μ and σ .

(iv) Find the probability of a randomly chosen battery B having an operation time shorter than 146 minutes.

(7 marks)

(b) The manufacturer produces 1500 batteries per day. One-third of them are A and the rest are B . A battery is regarded as 'faulty' when the operation time is shorter than 104 minutes. Let λ_A and λ_B be respectively the mean numbers of 'faulty' batteries of A and B produced per day. Assume that the numbers of 'faulty' batteries A and B produced per day can be approximately modelled by Poisson distributions with means λ_A and λ_B.

(i) Find λ_A and λ_B correct to 1 decimal place.

(ii) Find the probability that the number of 'faulty' batteries A produced on a certain day is between 4 and 6 inclusively.

(iii) Given that the total number of 'faulty' batteries A and B produced on a certain day is 10 and the number of 'faulty' batteries A produced is between 4 and 6 inclusively, find the probability that the number of 'faulty' batteries B produced is more than 4 .

(8 marks)

(2012 ASL-M&S Q10)

15. In a scoring game, a player will roll a ball at a starting point along a long horizontal track. When the ball comes to rest, let Y cm be the distance of the ball having travelled. The scoring system is shown in the following table.

Range of Y	$154 \le Y < 160$	$160 \le Y < K$	$K \le Y < 174$	Otherwise
Score	20	50	30	0

It is known that Y can be modelled by a normal distribution with mean 165 and variance 16 . It is also known that 78.88% of the players score 50 in a game. A game in which the player scores 50 is called "Bingo". Assume that the games are independent.

(a) Find the value of K .

(2 marks)

(b) Find the probability that a player will score 30 in a game.

(2 marks)

(c) Find the probability that the 6th game is the 3rd "Bingo".

(2 marks)

(d) If the variance of the number of "Bingo" in n games is at most 2.3 , determine the largest value of n .

(2 marks)

(e) A player will win a prize if his average score in 4 games is at least 40 .

(i) Find the probability that a player will win the prize.

(ii) Find the probability that he wins the prize and his average score in the first 2 games is at least 40 .

(iii) Given that a player wins the prize, find the probability that his average score in the first 2 games is less than 40 .

(7 marks)

(2011 ASL-M&S Q10)

16. A construction company proposes to use the daily rainfall precipitation to determine the effect of rainfall on a construction project. The following table shows the classification system.

Daily Rainfall Precipitation (Y mm)	$Y<100$	$100 \le Y < 150$	$150 \le Y < 200$	$Y \ge 200$
Effect Level of the Day	Low	Medium	High	Severe

Assume that the daily rainfall precipitation recorded follows a normal distribution with mean μ mm and standard deviation σ mm. From past record, 12.10% of the days are classified as Low and 9.18% of the days are classified as Severe.

(a) Find the values of μ and σ.

(3 marks)

(b) Find the probability that a day is classified as High.

(1 mark)

(c) It is given that, in a certain rainy day, the rainfall precipitation exceeds 100mm. Find the probability that the day is classified as High.

(2 marks)

(d) In a construction site, the numbers of days that a project is postponed under the precipitation levels Medium, High and Severe of a rainy day follow Poisson distributions with means 1, 3 and 6 respectively. The project will not be postponed if a day is classified as Low. Given that during the construction of the project, there is exactly 1 rainy day with precipitation exceeding 100 mm.

(i) Find the probability that the project will NOT be postponed.

(ii) Find the probability that the project will be postponed for exactly 1 day.

(iii) Given that the project is postponed for at least 3 days, find the probability that the rainy day is classified as High.

(9 marks)

(2011 ASL-M&S Q12)

17. Suppose the width of the tongues of normal new born babies can be modelled by a normal distribution with mean μ cm and standard deviation 0.4 cm. It is known that 24.2% of the normal babies will have their tongue widths less than 2.22 cm . If babies have inherited a certain genetic disease A , their tongues will be wider. It is known that 5% of new born babies have inherited disease A and the width of their tongues can be modelled by a normal distribution with mean $(\mu + 0.3)$ cm and standard deviation 0.2 cm . A diagnostic test is proposed such that if the width of the tongue of a baby is wider than $(\mu + 0.5)$ cm , he/she is diagnosed to have inherited disease A.

(a) Find the value of μ .

(1 mark)

(b) (i) What is the probability that a normal baby is diagnosed as having inherited disease A ?

(ii) What is the probability of a wrong diagnosis?

(iii) Given that a baby is diagnosed as NOT having inherited disease A , what is the probability that the baby has actually inherited the disease?

(8 marks)

(c) A group of 20 babies are going to take the test one by one.

(i) Given that exactly 4 babies are diagnosed wrongly among the 20 babies, what is the probability that exactly 3 babies are diagnosed wrongly in the first 8 tests?

(ii) Given that at most 4 babies are diagnosed wrongly among the 20 babies, what is the probability that the 8th baby to take the test is the 3rd baby who is diagnosed wrongly?

(6 marks)

(2009 ASL-M&S Q11)

18. A manager of a maintenance centre launches an appraisal system to assess the performance of technicians in terms of the time spent to complete a task. A technician can get 2 points if he takes less than 2 hours to complete a task, 1 point if he takes between 2 and 4.6 hours, and 0 point if he takes longer than 4.6 hours.

Assume the time for a technician to complete a task is normally distributed with a mean of 3 hours and a standard deviation of 0.8 hour, and the number of tasks assigned to a technician follows a Poisson distribution with a mean of 1.8 tasks per day.

(a) Find the probability that a technician is assigned not more than 4 tasks on a certain day.

(3 marks)

(b) Let p_i be the probability of a technician getting i point(s) upon completing a task, where $i = 0, 1, 2$. Find the values of p_0 , p_1 and p_2 .

(3 marks)

(c) Find the probability that a technician gets exactly 4 points on a certain day under each of the following conditions:

(i) 3 tasks are assigned,

(ii) 4 tasks are assigned.

(5 marks)

(d) It is given that a technician is assigned fewer than 5 tasks on a certain day. Find the probability that the technician gets exactly 4 points.

(4 marks)

(2008 ASL-M&S Q11)

19. The manager, Teresa, of a superstore launches a promotion plan to increase the sales volume. The number of customers shopping at the superstore in a minute can be modelled by a Poisson distribution with a mean of 2.4 customers per minute. The expense of customers in the superstore are assumed to be independent and follow a normal distribution with a mean of \$ 375 and a standard deviation of \$ 125. A customer who spends more than \$ 300 but less than \$ 600 in the superstore can enter lucky draw X in which the probability of winning a gift is 0.25. A customer who spends \$ 600 or more in the superstore can enter lucky draw Y in which the probability of winning a gift is 0.8. Assume that each customer enters at most one lucky draw for each visit.

(a) Find the probability that there are more than 2 customers shopping at the superstore in a certain minute.

(3 marks)

(b) Find the probability that a randomly selected customer shopping at the superstore can enter lucky draw X.

(2 marks)

(c) Find the probability that a randomly selected customer shopping at the superstore wins a gift.

(2 marks)

(d) Find the probability that there are exactly 3 customers shopping at the superstore in a certain minute and each of them wins a gift.

(2 marks)

(e) Given that there are more than 2 customers shopping at the superstore in a certain minute, find the probability that there are fewer than 5 customers shopping at the superstore in this minute and each of them wins a gift.

(3 marks)

(f) If Teresa wants to revise that least expense of a customer for entering lucky draw Y so that 33% of the customers shopping at the superstore could enter lucky draw Y, what should be the revised least expense?

(3 marks)

(2007 ASL-M&S Q10)

20. A factory produces brand D coffee beans which are packed into boxes of 30 cans each. The net weight of each can of coffee beans follows a normal distribution with a mean of 300g and a standard deviation of 7.5 g. A can of coffee beans with net weight less than 283.5 g or more than 316.5 g is classified as *exceptional*.

(a) Find the probability that a randomly selected can of brand D coffee beans is *exceptional*.

(2 marks)

(b) The manager of the factory randomly selects a box of brand D coffee beans and inspects every can in the box one by one.

(i) Find the probability that the 12th inspected can is the 1st *exceptional* can of coffee beans in the box.

(ii) Find the probability that there is exactly 1 *exceptional* can of coffee bean in the box.

(iii) Find the probability that there is at most 1 *exceptional* can of coffee beans in the box.

(8 marks)

(c) The shopkeeper of a coffee shop buys one box of brand D coffee beans. The shopkeeper regards a can of coffee beans as *unacceptable* if the net weight of the can is less than 283.5g.

(i) Find the probability that in the box there is exactly 1 *exceptional* can of coffee beans which is *unacceptable*.

(ii) Given that in the box there is at most 1 *exceptional* can of coffee beans, find the probability that there is exactly 1 *unacceptable* can of coffee beans in the box.

(5 marks)

(2007 ASL-M&S Q11)

21. A researcher models the number of cars entering a roundabout in five-second time intervals (FSTIs) by a Poisson distribution with a mean of 4.7 cars per FSTI, and the speed of a car entering the roundabout by a normal distribution with a mean of 42.8km/h and a standard deviation of 12 km/h. A car is *speeding* if the speed of the car is over 50 km/h.

(a) Find the probability that fewer than 6 cars enter the roundabout in a certain FSTI.

(3 marks)

(b) Find the probability that a car entering the roundabout is *speeding*.

(2 marks)

(c) Find the probability that the 6th car entering the roundabout is the 1st *speeding* car.

(3 marks)

(d) The roundabout is *hazardous* in a certain FSTI if at least 4 cars enter the roundabout in that FSTI and more than 2 of them are *speeding*.

(i) If exactly 4 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(ii) Given that fewer than 6 cars enter the roundabout in a certain FSTI, find the probability that the roundabout is *hazardous* in that FSTI.

(7 marks)

(2006 ASL-M&S Q10)

22. In a city, the number of cars entering a filling station for petrol per hour can be modelled by a Poisson distribution with a mean of 6.2 cars per hour.

(a) Find the probability that there are fewer than 5 cars entering the filling station for petrol in a certain hour.

(3 marks)

(b) The manager of the filling station models the amount of petrol for refuelling a car by a normal distribution with a mean of 23.2 litres and a standard deviation of 6 litres.

(i) Find the probability that the amount of petrol for refuelling a car is at least 25 litres.

(ii) Find the probability that the 9th car entering the filling station for petrol is the 3rd car which has been refuelled with at least 25 litres.

(iii) Find the probability that there are exactly 3 cars entering the filling station for petrol in a certain hour and each of them will be refuelled with at least 25 litres.

(iv) If there are exactly 4 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(v) Given that there are fewer than 5 cars entering the filling station for petrol in a certain hour, find the probability that more than 2 of them will each be refuelled with at least 25 litres.

(12 marks)

(2005 ASL-M&S Q10)

23. Every school day, Peter leaves home at 7:00 a.m. to go to the train station to take a train to his school. The time needed for him to go to the train station platform follows a normal distribution with a mean of 17.5 minutes and a standard deviation of 2 minutes.

The following table shows the departure times for trains A , B and C and the probabilities that Peter to be late when taking trains A , B and C respectively:

Train	Departure time	Probability for Peter to be late
A	7:13 a.m.	0.02
B	7:19 a.m.	0.15
C	7:22 a.m.	0.35

Peter takes the earliest departing train when he arrives at the train station platform. Assume that the time needed for him to get on the train from the platform is negligible. It is certain that he will be late if he cannot catch any one of the trains A, B and C .

(a) Find the probability that Peter takes train B to the school on a certain morning.

(2 marks)

(b) Find the probability that Peter is late on a certain morning.

(3 marks)

(c) Given that Peter is late on a certain morning, find the probability that Peter takes train B to the school on this morning.

(2 marks)

(d) Find the probability that Peter is late on exactly 2 mornings in a certain week of 5 school days.

(2 marks)

(e) Given that Peter is late on exactly 2 mornings in a certain week of 5 school days, find the probability that he takes train B to the school only on these 2 mornings.

(3 marks)

(f) If Peter tries to leave home earlier so that the probability of his getting on train A is at least 0.95, what is the latest time that he should leave home? Give your answer correct to the nearest minute.

(3 marks)

(2005 ASL-M&S Q11)

24. A customer who spends \$300 or more in a store during a visit is classified as a 'valuable' customer. The expenses of customers in the store are assumed to be independent and follow a normal distribution with a mean of \$428 and a standard deviation of \$100. The number of customers visiting the store in a minute can be modeled by a Poisson distribution with a mean of 4 customers per minute.

(a) Find the probability that a randomly selected customer of the store is a 'valuable' customer.

(2 marks)

(b) Find the probability that there are at least 2 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day.

(3 marks)

(c) Find the probability that there are exactly 3 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day and exactly 2 of them are 'valuable' customers.

(3 marks)

(d) Given that there are 2 or 3 customers visiting the store between 2:00 p.m. and 2:01 p.m. on a certain day, find the probability that exactly 2 of them are 'valuable' customers.

(3 marks)

(e) A customer who spends \$600 or more in the store during a visit will receive a gift. If the probability of the store giving out gifts is at least 0.99, find the smallest number of customers visiting the store.

(4 marks)

(2004 ASL-M&S Q12)

25. A teacher randomly selected 7 students from a class of 13 boys and 17 girls to form a group to take part in a flag-selling activity.

(a) Find the probability that the group consists of at least 1 boy and 1 girl.

(3 marks)

(b) Given that the group consists of at least 1 boy and 1 girl, find the probability that there are more than 2 girls in the group.

(3 marks)

(c) A group of 3 boys and 4 girls is formed. It is known that the amount of money collected by a boy and a girl in the activity can be modelled respectively by normal distributions with the following means and standard deviation.

Student	Mean	Standard deviation
Boy	\$673	\$100
Girls	\$708	\$100

Any student who collects more than \$800 receives a certificate.

(i) Find the probability that a particular boy in the group will receive a certificate.

(ii) Find the probability that exactly 1 boy and 1 girl in the group will receive certificates.

(iii) Given that the group has received 2 certificates, find the probability that exactly 1 boy and 1 girl received the certificates.

(9 marks)

(2003 ASL-M&S Q12)

26. The weight of each bag of self-raising flour in a batch produced by a factory follows a normal distribution with mean 400 g and standard deviation 10 g . A bag of flour with weight less than 376 g is **underweight**, and more than 424 g is **overweight**.

(a) Find the probability that a randomly selected bag of flour

(i) is **underweight**;

(ii) is **overweight**.

(3 marks)

(b) If a bag of flour is either **underweight** or **overweight**, it will be classified as a **substandard** bag by the director of the factory. The director randomly selects 50 bags as a sample from the batch.

(i) Find the probability that there is no **substandard** bag of flour in the sample.

(ii) Find the probability that there are no more than 2 **substandard** bags of flour in the sample.

(5 marks)

(c) A wholesaler is only concerned about the number of bags of flour which are **underweight** . The wholesaler re-analyses the sample of 50 bags of flour in (b).

(i) Find the probability that in the sample there is only 1 **substandard** bag and it is not **underweight**.

(ii) Find the probability that there are no more than 2 **substandard** bags in the sample and no **underweight** bag of flour in the sample.

(iii) Given that in the sample there are no more than 2 **substandard** bags, find the probability that there is no **underweight** bag in the sample.

(7 marks)

(2002 ASL-M&S Q13)

27. Suppose the number of customers visiting a supermarket per minute follows a Poisson distribution with mean 6 .

(a) Find the probability that the number of customers visiting the supermarket in one minute is more than 2 .

(3 marks)

(b) Suppose the amount $\$X$ spent by a customer in the supermarket follows a normal distribution $N(\mu, \sigma^2)$.

Probability distribution of the amount spent by a customer

Amount spent (\$$X$)	Probability *
$X < 100$	0.063
$100 \le X < 200$	0.364
$200 \le X < 300$	a_1
$300 \le X < 400$	a_2
$X \ge 400$	0.006

* Correct to 3 decimal places.

(i) Using the probabilities provided in the above table, find the values of μ and σ correct to 1 decimal place.
Hence find the values of a_1 and a_2 correct to 3 decimal places.

(ii) What is the median of the normal distribution?

(iii) Given that a customer spends less than \$200, find the probability that the customer spends more than \$50 .

(iv) Find the probability that there are 5 customers visiting the supermarket in a minute and exactly 2 of them each spends less than \$200 .

(12 marks)

(2002 ASL-M&S Q14)

28. The table gives the probability distributions of the lifetimes of two brands of compact fluorescent lamps (CFLs). The lifetime of a Brand X CFL follows a normal distribution with mean μ hours and standard deviation 400 hours. The lifetime of a Brand Y CFL follows another normal distribution with mean 8 800 hours and standard deviation σ hours.

Probability distributions of the lifetimes of brand X and Y CFLs

Lifetime of a CFL (in hours)	Probability *	
	Brand X: $N(\mu, 400^2)$	Brand Y: $N(8, \sigma^2)$
Under 8 200	0.0808	0.1587
8 200 to 8 600	0.2638	b_1
8 600 to 9 000	a_1	b_2
9 000 to 9 400	0.2195	b_3
Over 9 400	a_2	0.1587

* Correct to 4 decimal places.

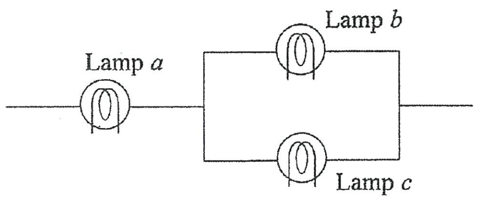
(a) Using the probabilities provided in the table, find μ and σ.
Hence find the values of a_1, a_2, b_1, b_2, b_3 in the table.

(5 marks)

(b) Based on the results of (a), which brand of CFL would you choose to buy? Explain.

(1 mark)

(c) The figure shows a lighting system formed by three lamps. The system will work only if lamp a works and either lamp b or lamp c works.



(i) Suppose all the lamps in the system are brand X CFLs.

(I) Find the probability that the lifetime of the lighting system is more than 8200 hours.

(II) It is known that the lifetime of the lighting system is less than 8 200 hours. Find the probability that only the lifetime of lamp a is less than 8200 hours.

(ii) Suppose the lighting system is formed by 2 brand X and 1 brand Y CFLs. In order for the system to have a better chance of having a lifetime of more than 8 200 hours, where would you put the brand Y CFL in the system? Explain.

(9 marks)

(2001 ASL-M&S Q12)

29. The milk produced by Farm A has been contaminated by dioxin. The amount of dioxin presented in each bottle of milk follows a normal distribution with mean 20 ng $(1ng = 10^{-6} g)$ and standard deviation 5 ng. Bottles which contain more than 12 ng of dioxin are classified as *risky*, and those which contain more than 27 ng are *hazardous*.

(a) Suppose a bottle of milk from Farm A is randomly chosen.

(i) Find the probability that it is *risky* but not *hazardous*.

(ii) If it is *risky*, find the probability that it is *hazardous*.

(6 marks)

(b) A distributor purchases bottles of milk from both Farm A and Farm B and sells them under the same brand name 'Healthy'. It is known that 60% of the milk is from Farm A and the rest from Farm B . A bottle of milk from Farm B has a probability of 0.058 of being *risky* and 0.004 of being *hazardous*.

(i) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is from Farm B .

(ii) If a randomly chosen bottle of 'Healthy' milk is *risky*, find the probability that it is a *hazardous* bottle from Farm B .

(iii) The Health Department inspects 5 randomly chosen bottles of 'Healthy' milk. If 2 or more bottles of milk in the batch are *risky*, the distributor's license will be suspended immediately. Find the probability that the license will be suspended.

(9 marks)

(2000 ASL-M&S Q12)

30. A criminologist has developed a questionnaire for predicting whether a teenager will become a delinquent. Scores on the questionnaire can range from 0 to 100, with higher values indicating a greater criminal tendency. The criminologist sets a critical level at 75, i.e., a teenager scores more than 75 will be classified as a potential delinquent (PD). Extensive studies have shown that the scores of those considered non-PDs follow a normal distribution with a mean of 65 and standard deviation of 5. The scores of those considered PDs follow a normal distribution with a mean of 80 and standard deviation of 5.

(a) Find the probability that

(i) a PD will be misclassified,

(ii) a non-PD will be misclassified.

(4 marks)

(b) What is the probability that out of 10 PDs, not more than 2 will be misclassified?

(3 marks)

(c) If a sociologist wants to ensure that only 1 in 100 PDs should be misclassified, what critical level of score should be used?

(3 marks)

(d) It is known that 10% of all teenagers are PDs. Will the probability of teenagers misclassified by the sociologist in (c) be greater than that misclassified by the criminologist? Explain.

(5 marks)

(1999 ASL-M&S Q10)

31. The weight of each box of washing powder produced by a factory follows a normal distribution with mean 500 g and variance 25 g^2 . The weights of boxes of washing powder are independent of each other. Every thirty minutes, a test consists of one or two parts will be performed as follows:

First part of the test

A randomly selected box of washing powder is weighed. If the weight of this box is greater than 510 g or less than 490 g , a black signal will be generated.

Second part of the test

(*Performed only when the weight of the box in the first part is greater than 508 g or less than 492 g and no black signal has been generated.*)

Another randomly selected box of washing powder is weighed.

(I) A black signal will be generated if the weight of this box is greater than 510 g or less than 490 g .

(II) A red signal will be generated if the weights of the two boxes in the first and second parts are **both between** 508 g and 510 g , or **both between** 490 g and 492 g.

(a) Find the probability that a black signal will be generated in the first part of a test.

(2 marks)

(b) Find the probability that the second part has to be performed in a test.

(3 marks)

(c) Find the probability that a black signal will be generated in a test.

(3 marks)

(d) Given that the second part has to be performed in a test, find the probability that the weights of the two boxes selected are both between 508 g and 510 g.

(3 marks)

(e) Given that the second part has to be performed in a test, find the probability that a red signal is generated.

(2 marks)

(f) Find the probability that a red signal will be generated in a test.

(2 marks)

(1998 ASL-M&S Q13)

32. The number of fire insurance claims (FICs) received by an insurance company is modelled by a Poisson distribution with mean 4 claims per day. The company found that 60% of the FICs are related to house fires.

(a) Find the probability that no FICs are received on a particular day.

(2 marks)

(b) If 5 FICs are received on a certain day, find the probability that at least 2 of them are related to house fires.

(3 marks)

(c) It is known that the amounts of FICs related and not related to house fires can be modelled respectively by normal distributions with the following means and standard deviations:

FICs	Mean	Standard deviation
Related to house fires	\$100 000	\$50 000
Not related to house fires	\$150 000	\$20 000

If the amount of a FIC is greater than \$ 200 000, the FIC is said to be *large*.

(i) Find the probability that a certain FIC is *large*.

(ii) Given that a FIC is *large*. Find the probability that the FIC is related to a house fire.

(iii) Find the probability that on a particular day, the company receives 5 FICs and at least 2 of them are *large*.

(10 marks)

(1997 ASL-M&S Q11)

33. Every morning, Mr. Wong wears a necktie to work. If the length of the front portion of his necktie is between 44 cm and 45 cm, he regards it to be a *perfect tying*. Otherwise, he has to tie it again until he gets the *perfect tying*. Suppose that the length of the front portion of his necktie can be modelled by a normal distribution with mean 44.6 cm and standard deviation 1.2 cm.

(a) Find the probability that Mr. Wong gets a *perfect tying* in one trial.

(3 marks)

(b) Find the mean number of trials to be taken by Mr. Wong to get the first *perfect tying*.

(2 marks)

(c) Find the probability that Mr. Wong gets the *perfect tying* in not more than 3 trials.

(2 marks)

(d) Mr. Wong will have to go to work by taxi only if he doesn't get the *perfect .tying* in the first 3 trials in any morning.

(i) Find the probability that Mr. Wong will have to go to work by taxi in less than 2 out of 6 days.

(ii) Given that Mr. Wong has to go to work by taxi on a certain morning, find the probability that he could not get the *perfect tying* until the 5th trial.

(iii) Find the probability that in a certain week of 6 working days (Monday to Saturday), Mr. Wong will have to go to work by taxi on 2 consecutive mornings and he will not have to take a taxi on the other 4 mornings.

(8 marks)

(1997 ASL-M&S Q13)

34. A machine discharges soda water once for each cup of soda water purchased. The amount of soda water in each discharge is independently normally distributed with mean 210 ml and standard deviation 15 ml.

(a) Find the probability that the amount of a cup of soda water is between 200 ml and 220 ml.

(2 marks)

(b) Suppose cups of capacity 240 ml each are used.

(i) Find the probability that a discharge will overflow.

(ii) What is the probability that there will be exactly 1 overflow out of 30 discharges?

(iii) If Sam buys a cup of soda water from the machine every day starting on 1st July, find the probability that he will get the second overflow on 31st July.

(5 marks)

(c) The vendor has decided to use cups of capacity 220 ml each and to repair the machine so that, on the average, 80 in 100 cups contain more than 205 ml of soda water in each and only 1 in 100 discharges overflows. The amount of soda water in each discharge is still independently normally distributed.

(i) What will the new mean and standard deviation of the amount of soda water in each discharge be? Give the answers correct to 1 decimal place.

(ii) If a discharge from the repaired machine overflows, find the probability that the amount of soda water in this discharge exceeds 225 ml. Give the answer correct to 2 decimal places.

(8 marks)

(1996 ASL-M&S Q11)

35. A test is used to diagnose a disease. For people *with* the disease, it is known that the test scores follow a normal distribution with mean 70 and standard deviation 5. For people *without* the disease, the test scores follow another normal distribution with mean μ and the same standard deviation 5. It is known that 33 % of those people *without* the disease will achieve a test score over 63.2.

(a) Find μ.

(3 marks)

(b) It is estimated that 15 % of the population of a city has the disease. A doctor has proposed that a person be classified as having the disease if the person's test score exceeds 66, otherwise the person will be classified as not having the disease.
If a person is randomly selected from the population to take the test,

(i) what is the probability that this person will be classified as having the disease?

(ii) find the probability that this person will be misclassified.

(12 marks)

(1995 ASL-M&S Q12)

36. Batches of screws are produced by a manufacturer under two different sets of conditions, favourable and unfavourable. If screws are produced under favourable conditions, the diameters of the screws will follow a normal distribution with mean 10 mm and standard deviation 0.4 mm. If screws are produced under unfavourable conditions, the diameters of the screws will follow a normal distribution with mean 12.3 mm and standard deviation 0.6 mm. A batch of screws is examined by measuring the diameter X mm of a screw randomly selected from the batch.

(a) The batch is classified as acceptable by the manufacturer if $X < c_1$ and as unacceptable if otherwise. The value c_1 satisfies $P(X < c_1) = 0.95$ under favourable conditions. Determine the value of c_1 .

(3 marks)

(b) The buyer uses a different criterion instead. He classifies the batch as acceptable if $X < c_2$ and as unacceptable if otherwise. The value c_2 satisfies $P(X < c_2) = 0.01$ under unfavourable conditions. Determine the value of c_2 .

(3 marks)

(c) For a batch of screws produced under favourable conditions and based on the same measurement of a screw, find the probability that the batch will be classified as unacceptable by the manufacturer but acceptable by the buyer.

(4 marks)

(d) After some negotiation, the manufacturer and the buyer agree to use a common cut-off point c_3 such that $P(X < c_3)$ under favourable conditions is equal to $P(X \geq c_3)$ under unfavourable conditions. Determine the value of c_3 .

(3 marks)

(e) The manufacturer and the buyer later agree that a batch will be rejected in the future if $X \geq 10.8$ (too thick) or $X < 9.4$ (too thin). If the population mean μ mm of the diameters of the screws produced can be modified by adjusting the machine, find μ so that the probability of rejection, $P(X < 9.4 \text{ or } X > 10.8)$, is minimized.

(2 marks)

(1994 ASL-M&S Q13)

2021 DSE Q9

The weight of each potato in a large farm follows a normal distribution with a mean of 200 grams and a standard deviation of σ grams. The classification of the potatoes is as follows:

Weight of a potato (W grams)	$W < 180$	$180 \leq W < 230$	$W \geq 230$
Classification	*small*	*medium*	*big*

It is given that 21.19% of the potatoes in the farm are *small*.

(a) Find the percentage of *medium* potatoes in the farm. (3 marks)

(b) The potatoes in the farm are now inspected one by one. Find the probability that the 4th potato inspected is the 2nd *big* potato inspected. (3 marks)

(c) From the farm, 5 potatoes are randomly selected.

(i) Find the probability that there are exactly 1 *big* potato and 2 *small* potatoes.

(ii) Given that there is exactly 1 *big* potato, find the probability that there are at least 2 *small* potatoes.

(5 marks)

10. Normal Distribution

Section A

1. (2018 DSE-MATH-M1 Q3)

2. (2017 DSE-MATH-M1 Q3)

(a) μ

$= \dfrac{1.83+3.43}{2}$

$= 2.63$ 1A

$P\left(\dfrac{1.83-2.63}{\sigma} < Z < \dfrac{3.43-2.63}{\sigma}\right) = 0.8904$ 1M

$P\left(\dfrac{-0.8}{\sigma} < Z < \dfrac{0.8}{\sigma}\right) = 0.8904$

$P\left(0 < Z < \dfrac{0.8}{\sigma}\right) = 0.4452$

$\dfrac{0.8}{\sigma} = 1.6$

$\sigma = 0.5$ 1A

(b) The required probability

$= P\left(\dfrac{2.5-2.63}{\frac{0.5}{\sqrt{9}}} < Z < \dfrac{3.1-2.63}{\frac{0.5}{\sqrt{9}}}\right)$ 1M

$= P(-0.78 < Z < 2.82)$

$= 0.2823 + 0.4976$

$= 0.7799$ 1A

----------(5)

(a)	Very good. Most candidates were able to find the required mean μ and standard deviation σ .
(b)	Good. Some candidates mistook σ as the standard deviation of the sample mean.

3. (2012 DSE-MATH-M1 Q9)

(a) Let X be the score of a student who had revised.

$P(X \geq 43) = P\left(Z \geq \dfrac{43-59}{10}\right)$

$= P(Z \geq -1.6)$

≈ 0.9452 1A

Let Y be the score of a student who had not revised.

$P(Y \geq 43) = P\left(Z \geq \dfrac{43-35.2}{12}\right)$

$= P(Z \geq 0.65)$

≈ 0.2578

(Either one)

$\therefore$ P(pass the test) $\approx 0.73 \times 0.9452 + 0.27 \times 0.2578$ 1M

$= 0.759602$ 1A OR 0.7596

(b) P(a student had not revised for the test | he passed the test)

$= \dfrac{0.27 \times 0.2578}{0.759602}$ 1M

≈ 0.091634829

≈ 0.0916 1A

(c) P(4 students had not revised for the test among 10 passed students)

$\approx C_6^{10}(0.091634829)^4(1-0.091634829)^6$ 1M

≈ 0.0083 1A

(7)

(a)	Good. Nevertheless, some candidates did not figure out that the required probability was $0.73\,P(X \geq 43) + 0.27\,P(Y \geq 43)$, and some failed to use the standard normal distribution table.
(b)	Satisfactory. Many candidates were able to apply the correct method, although some got wrong numerical answers.
(c)	Fair. Some candidates wrote a binomial probability but did not use the result of (b).

4. (2010 ASL-M&S Q6)

Solution	Marks	Remarks
(a) $P\left(Z>\frac{152-\mu}{5}\right)=0.117$	1A	
$P\left(0<Z<\frac{152-\mu}{5}\right)=0.383$	1M	
$\frac{152-\mu}{5}\approx 1.19$ $\mu\approx 146.05$	1A	
(b) (i) The required probability $=0.117\times(1-0.2)+(1-0.117)\times 0.1$ $=0.1819$	1A	
(ii) The required probability $=\frac{(1-0.117)\times(1-0.1)}{1-0.1819}$	1M+1M	
$=\frac{883}{909}$	1A	OR 0.9714
	(7)	

Very good. Candidates performed well in simple application of normal and conditional probabilities.

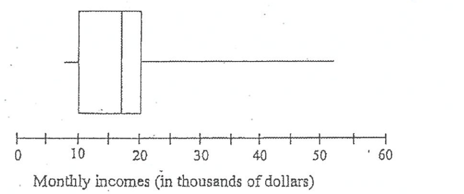
5. (2008 ASL-M&S Q5)

Solution	Marks	Remarks
(a) Let $\$X$ be the amount of money spent by a randomly selected customer. $P(X>30000)=0.242$ $P\left(Z>\frac{30000-\mu}{6000}\right)=0.242$	1M	For standardization
$\therefore P\left(0<Z\le\frac{30000-\mu}{6000}\right)=0.258$ $\therefore \frac{30000-\mu}{6000}=0.7$	1A	
i.e. $\mu=25800$	1A	
(b) The required probability $=\frac{P(16500<X<30000)}{P(X<30000)}$	1A	For $P(16500<X<30000)$
$=\frac{P\left(\frac{16500-25800}{6000}<Z<\frac{30000-25800}{6000}\right)}{1-P(X\ge 30000)}$ $=\frac{P(-1.55<Z<0.7)}{1-0.242}$	1A	For denominator
$=\frac{0.4394+0.258}{0.758}$ ≈ 0.9201	1A	
	(6)	

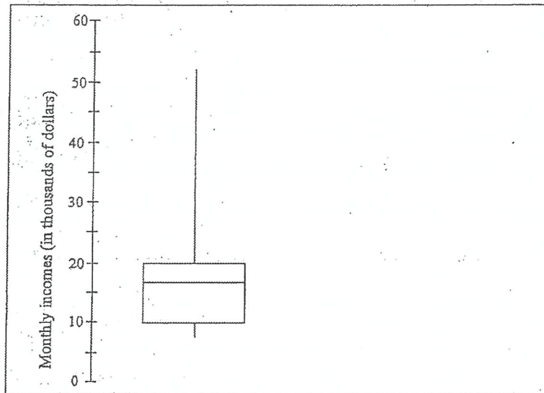
Fair. Some candidates were not aware that a conditional probability is required.

6. (2004 ASL-M&S Q5)

(a)



1A for any correct box-and-whisker diagram
1A for correct scale
pp−1 for omitting the title



1A for any correct box-and-whisker diagram
1A for correct scale
pp−1 for omitting the title

Solution	Marks	Remarks
(b) (i) Let $\$X$ be the monthly income of a randomly selected university graduate from the group. Then, we have $X\sim N(17940, 4700^2)$. The required probability $=P(X<17000)$ $=P(Z<\frac{17000-17940}{4700})$ $=P(Z<-0.2)$	1A	
$=0.4207$	1A	$a-1$ for r.t. 0.421
(ii) Since the distribution is skewed to the right side,	1M	accept skewed to one side or not symmetrical
the model proposed by the student is not appropriate.	1M	
	(6)	

Good. Many candidates did not construct the box-and-whisker diagram to the scale which is necessary for correctly describing the distribution of data.

7. (2003 ASL-M&S Q6)

Let \$$X$ be the amount of a business transaction. Then, $X \sim N(215, 50^2)$.

(a) The required probability

$= P(X > 300)$

$= P\left(Z > \frac{300-215}{50}\right)$ — 1A accept $P\left(Z \geq \frac{300-215}{50}\right)$

$= P(Z > 1.7)$

$= 0.0446$ — 1A $a-1$ for r.t. 0.045

(b) The required probability

$= C_2^7 (0.0446)^2 (1-0.0446)^5$ — 1M for Binomial probability

≈ 0.033251802

≈ 0.0333 — 1A $a-1$ for r.t. 0.033

(c) The required probability

$\approx (0.033251802)(0.0446)$ (by (b))

≈ 0.001483030369

≈ 0.0015 — 1A $a-1$ for r.t. 0.001

(d) $P(X > K) = 64.8\%$

$P\left(Z > \frac{K-215}{50}\right) = 0.648$

$\frac{K-215}{50} = -0.38$ — 1M accept $\frac{K-215}{50} = 0.38$

$K = 196$ — 1A

---------(7)

Very good. Most candidates were able to make use of the normal distribution table.

8. (1994 ASL-M&S Q6)

(a) Since $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ for $x \approx 0$,

$e^{-\frac{x^2}{2}} \approx 1 + \left(-\frac{x^2}{2}\right) + \frac{1}{2}\left(-\frac{x^2}{2}\right)^2 + \frac{1}{6}\left(-\frac{x^2}{2}\right)^3$ — 1M

$\approx 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}$ for $x \approx 0$. — 1A

$\int_0^1 e^{-\frac{x^2}{2}} dx \approx \int_0^1 \left(1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}\right) dx$

$= \left[x - \frac{x^3}{6} + \frac{x^5}{40} - \frac{x^7}{336}\right]_0^1$ — 1M

$= 1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336}$

$= 0.8554$ — 1A

(b) From the normal distribution table,

$\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{x^2}{2}} dx = 0.3413$ — 1A

Hence $\frac{1}{\sqrt{2\pi}} \times 0.8554 = 0.3413$ — 1M

$\therefore \pi = \frac{0.8554^2}{2 \times 0.3413^2} = 3.141$ — 1A 3.140 for using exact value of (a)

7

Section B

9. (2016 DSE-MATH-M1 Q9)

Let J minutes and K minutes be the random variables representing the daily reading times of the students in schools X and Y respectively.

(a) Let μ_1 minutes and σ_1 minutes be the mean and the standard deviation of the daily reading times of the students in school X respectively, while μ_2 minutes and σ_2 minutes be the mean and the standard deviation of the daily reading times of the students in schools Y respectively.

$\begin{cases} \frac{40-\mu_1}{\sigma_1} = -2.51 \\ \frac{70-\mu_1}{\sigma_1} = 2.17 \end{cases}$

$\begin{cases} \frac{48-\mu_2}{\sigma_2} = -2.17 \\ \frac{72-\mu_2}{\sigma_2} = 2.12 \end{cases}$ — 1M+1A either one

Solving, we have

$\mu_1 = \frac{4375}{78}, \sigma_1 = \frac{250}{39}$ — 1A for both

$\mu_1 \approx 56.08974359, \sigma_1 \approx 6.41025641$

$\mu_1 \approx 56.0897, \sigma_1 \approx 6.4103$ — r.t. $\mu_1 \approx 56.0897, \sigma_1 \approx 6.4103$

$\mu_2 = \frac{8600}{143}, \sigma_2 = \frac{800}{143}$ — 1A for both

$\mu_2 \approx 60.13986014, \sigma_2 \approx 5.594405594$

$\mu_2 \approx 60.1399, \sigma_2 \approx 5.5944$ — r.t. $\mu_2 \approx 60.1399, \sigma_2 \approx 5.5944$

P(students reading more than 60 minutes daily in school X)

$= P(J > 60)$

$= P\left(Z > \frac{60 - \frac{4375}{78}}{\frac{250}{39}}\right)$

$= P(Z > 0.61)$

$= 0.2709$

P(students reading more than 60 minutes daily in school Y)

$= P(K > 60)$

$= P\left(Z > \frac{60 - \frac{8600}{143}}{\frac{800}{143}}\right)$

$= P(Z > \frac{-1}{40})$

$> P(Z > 0)$

$= 0.5$

> 0.2709 — 1M either one

Thus, there are less students reading more than 60 minutes daily in school X. — 1A f.t.

---------(6)

Solution	Marks	Remarks
(b) The required probability		
$= C_1^3(0.2709)(1-0.2709)^2(0.2709)$	1M	
≈ 0.11703438		
≈ 0.1170	1A	r.t. 0.1170
	----------(2)	
(c) For school X,		
$P(J \geq T) \leq 0.1$		
$\dfrac{T-\frac{4375}{78}}{\frac{250}{39}} \geq 1.29$	1M+1A	1A for 1.29 (either one)
$T \geq \frac{2510}{39}$		
$T \geq 64.35897436$		
$T \geq 65$	1A	accept $T = 65$ (either one)
For school Y,		
$P(K \geq T) \leq 0.1$		
$\dfrac{T-\frac{8600}{143}}{\frac{800}{143}} \geq 1.29$		
$T \geq \frac{9632}{143}$		
$T \geq 67.35664336$		
$T \geq 68$		
Thus, the least value of T should be 68 .	1A	f.t.
	----------(4)	

(a)	Good. Many candidates were able to formulate the corresponding equations in means and standard deviations, but some candidates were unable to give the numerical answers either in an exact fraction or correct to 4 decimal places.
(b)	Good. Many candidates were able to apply the result of (a) .
(c)	Fair. About half of the candidates were unable to use inequality to formulate the problem. Besides, many candidates used 1.28 instead of 1.29 in the inequality.

10. (2012 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) (i) $I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}}\, dt$		
$= \frac{1}{2} \cdot \frac{4-1}{6}\left[\frac{1}{\sqrt{1}}e^{\frac{-1}{2}} + \frac{1}{\sqrt{4}}e^{\frac{-4}{2}} + 2\left(\frac{1}{\sqrt{1.5}}e^{\frac{-1.5}{2}} + \frac{1}{\sqrt{2}}e^{\frac{-2}{2}} + \frac{1}{\sqrt{2.5}}e^{\frac{-2.5}{2}} + \frac{1}{\sqrt{3}}e^{\frac{-3}{2}} + \frac{1}{\sqrt{3.5}}e^{\frac{-3.5}{2}}\right)\right]$	1M	
≈ 0.692913377		
≈ 0.6929	1A	
(ii) $\frac{d}{dt}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right) = \frac{-1}{2} t^{\frac{-3}{2}} e^{\frac{-t}{2}} + t^{\frac{-1}{2}} \cdot \frac{-1}{2} e^{\frac{-t}{2}}$	1M+1A	
$= \frac{-1}{2} e^{\frac{-t}{2}}\left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)$		
$\frac{d^2}{dt^2}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right) = \frac{-1}{2}\left[e^{\frac{-t}{2}}\left(\frac{-3}{2}t^{\frac{-5}{2}} + \frac{-1}{2}t^{\frac{-3}{2}}\right) + \frac{-1}{2}e^{\frac{-t}{2}}\left(t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)\right]$	1M+1A	
$= \frac{1}{4} e^{\frac{-t}{2}}\left(3t^{\frac{-5}{2}} + 2t^{\frac{-3}{2}} + t^{\frac{-1}{2}}\right)$		
> 0 for $1 \leq t \leq 4$.		
Hence the estimation in (i) is an over-estimate.	1	
	(7)	
(b) Let $t = x^2$.	1M	
$dt = 2x dx$	1A	
When $t = 1$, $x = 1$; when $t = 4$, $x = 2$.	(1A)	
$I = \int_1^4 \frac{1}{\sqrt{t}} e^{\frac{-t}{2}}\, dt$		
$= \int_1^2 \frac{1}{x} e^{\frac{-x^2}{2}}\, 2x dx$		
$= 2\int_1^2 e^{\frac{-x^2}{2}}\, dx$	1	
	(3)	
(c) $2\int_1^2 e^{\frac{-x^2}{2}}\, dx < 0.692913377$	1M	
$2\sqrt{2\pi}\int_1^2 \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx < 0.692913377$		
$2\sqrt{2\pi}(0.4772 - 0.3413) < 0.692913377$	1A	For 0.4772 and 0.3413
$\pi < 3.249593152$		
$\therefore \pi < 3.25$	1	
	(3)	

(a) (i)	Good. Many candidates applied the trapezoidal rule correctly.
(ii)	Poor. Many candidates used $\frac{d}{dt}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right)$ instead of $\frac{d^2}{dt^2}\left(t^{\frac{-1}{2}} e^{\frac{-t}{2}}\right)$ to determine whether the estimate in (i) is an over-estimate or under-estimate.
(b)	Fair. Many candidates used wrong substitutions.
(c)	Very poor. Only a few candidates attempted this part. Among them, some wrote $I \approx 0.692913377$ instead of $I < 0.692913377$.

11. (PP DSE-MATH-M1 Q13)

Solution	Marks	Remarks
Let X_r minutes and X_e minutes be the waiting times for a customer in the regular and express counter respectively.		
(a) $P(X_r > 6) = P\left(Z > \frac{6-6.6}{1.2}\right)$	1M	
$= P(Z > -0.5)$		
≈ 0.6915	1A	
	(2)	
(b) (i) P(more than 10 from 12 customers with $X_r > 6$)		
$= C_{11}^{12}(0.6915)^{11}(1-0.6915)+(0.6915)^{12}$	1M+1M	
≈ 0.0759	1A	
(ii) Let Y minutes be the average waiting time of the 12 customers		
$Y \sim N\left(6.6, \frac{1.2^2}{12}\right) = N(6.6, 0.12)$	1A	
$P(Y > 6) = P\left(Z > \frac{6-6.6}{\sqrt{0.12}}\right)$		
$\approx P(Z > -1.73)$		OR $P(Z > -1.732)$
≈ 0.9582	1A	OR 0.9584
	(5)	
(c) (i) $P(X_r < k) = 0.2119$		
$P\left(Z < \frac{k-6.6}{1.2}\right) = 0.2119$	1M	
$\frac{k-6.6}{1.2} = -0.8$		
$k = 5.64$	1A	
$P(X_e > k) = 0.0359$		
$P\left(Z > \frac{5.64-\mu}{0.8}\right) = 0.0359$	1M	
$\frac{5.64-\mu}{0.8} = 1.8$		
$\mu = 4.2$	1A	
(ii) $P(X_r > \mu) = P\left(Z > \frac{4.2-6.6}{1.2}\right)$		
≈ 0.9772	1A	
P(1 customer pays at regular counter | 2 customers wait more than μ min)		
$\approx \frac{2(0.88)(0.9772)(0.12)(0.5)}{[(0.88)(0.9772)+(0.12)(0.5)]^2}$	1M+1M	1M for numerator 1M for denominator
≈ 0.1219	1A	
	(8)	

(a)	良好。
(b) (i)	良好。
(ii)	平平。 部分學生不知道平均等候時間服從的正態分佈的標準差。
(c) (i)	甚差。 很多學生沒有完全明白題目所描述的特快櫃台付款與普通櫃台付款的分別。
(ii)	甚差。 很少學生嘗試答這部分。

12. (2013 ASL-M&S Q10)

Solution	Marks	Remarks
Let X be the speed of a randomly selected vehicle.		
(a) $P(X > 82.64) = 0.123$ and $P(X < 75.2) = 0.242$		
$\frac{82.64-\mu}{\sigma} = 1.16$ and $\frac{75.2-\mu}{\sigma} = -0.7$	1M	
Dividing the equations, we have $\frac{82.64-\mu}{75.2-\mu} = \frac{1.16}{-0.7}$.		
i.e. $\mu = 78$	1A	
$\therefore \sigma = 4$	1A	
	(3)	
(b) (i) P(a notice will be issued) $= P(X > 80)$	1M	
$= P\left(Z > \frac{80-78}{4}\right)$		
$= P(Z > 0.5)$		
≈ 0.3085	1A	
(ii) P(at most 2 notices will be issued for the 10 vehicles)		
$\approx (1-0.3085)^{10} + C_1^{10}(0.3085)(1-0.3085)^9 + C_2^{10}(0.3085)^2(1-0.3085)^8$	1M	
≈ 0.3604	1A	
	(4)	
(c) (i) (I) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = 1$)		
= P(speed of vehicle $\le$ 80 and (speed of vehicle + error) > 80)		
$= P(78 < X \le 80 \mid Y = 2)P(Y = 2) + P(77 < X \le 80 \mid Y = 3)P(Y = 3)$	1M+1A	1A for either term
$= P(0 < Z \le 0.5)(0.5) + P(-0.25 < Z \le 0.5)(0.5)$		
$\approx (0.1915)(0.5) + (0.0987 + 0.1915)(0.5)$		
≈ 0.2409	1A	Accept 0.24085
(II) P(a notice will be issued but the speed of vehicle is not over 80 if $\theta = -3$)		
= P(speed of vehicle $\le$ 80 and (speed of vehicle + error) > 80)		
$= P(78 < X \le 80 \mid Y = 2)P(Y = 2) + 0 \cdot P(Y = -1)$	1A	
$= P(0 < Z \le 0.5)(0.5)$		
$\approx (0.1915)(0.5)$		
≈ 0.0958	1A	Accept 0.09575
(ii) We need $2 + \theta < 0$ for the scenario happens.		
P(a notice will not be issued but the speed of vehicle is over 80) ≤ 0.07125		
P(speed of vehicle > 80 and (speed of vehicle + error) $\le$ 80) ≤ 0.07125	1M	
$P(80 < X \le 80-(2+\theta) \mid Y = 2+\theta)\,P(Y = 2+\theta) \le 0.07125$		
$P\left(0.5 < Z \le \frac{78-\theta-78}{4}\right)(0.5) \le 0.07125$		
$P\left(0 < Z \le \frac{-\theta}{4}\right) \le 0.1425 + 0.1915$		
$\frac{-\theta}{4} \le 0.97$	1A	For 0.97
$\theta \ge -3.88$		
Hence the range is $-3.88 \le \theta < -2$.	1A	
	(8)	

(a)			Good. When making use of the normal distribution table, some candidates equated $\frac{75.2-\mu}{\sigma}$ to 0.7 rather than to -0.7 .
(b)			Good.
(c)	(i)		Fair. Some candidates wrongly assumed that the variable was discrete.
	(ii)		Poor.

13. (2012 ASL-M&S Q10)

Let A and B be the operation time of a randomly chosen battery A and B respectively.

(a) (i) $P(A<152 \text{ or } A>184)$

$=P\left(Z<\frac{152-168}{32} \text{ or } Z>\frac{184-168}{32}\right)$

$=P(Z<-0.5 \text{ or } Z>0.5)$

$=0.617$ — 1A

(ii) $P(A>k)=0.05$

$\frac{k-168}{32}=1.645$ — 1M (Accept 1.64 or 1.65)

$k=220.64$ — 1A (Accept 220.48 or 220.8)

(iii) $P(B>188)=0.33$ and $P(B<213.2)=0.877$

$P\left(Z>\frac{188-\mu}{\sigma}\right)=0.33$ and $P\left(Z<\frac{213.2-\mu}{\sigma}\right)=0.877$

$\frac{188-\mu}{\sigma}=0.44$ and $\frac{213.2-\mu}{\sigma}=1.16$ — 1M

Solving, $\mu=172.6$ and $\sigma=35$. — 1A+1A

(iv) $P(B<146)=P\left(Z<\frac{146-172.6}{35}\right)$

$=P(Z<-0.76)$

$=0.2236$ — 1A

(7)

(b) (i) $\lambda_A=1500\times\frac{1}{3}\times P(A<104)$ — 1M (Either one)

$=500\times P\left(Z<\frac{104-168}{32}\right)$

$=500\times P(Z<-2)$

$=11.4$ (correct to 1 d.p.) — 1A

$\lambda_B=1500\times\frac{2}{3}\times P(B<104)$ — (Either one)

$=1000\times P\left(Z<\frac{104-172.6}{35}\right)$

$=1000\times P(Z<-1.96)$

$=25.0$ (correct to 1 d.p.) — 1A (Accept 25)

(ii) $P(4\le$ number of 'faulty' batteries A produced $\le 6)$

$=\frac{e^{-11.4}11.4^4}{4!}+\frac{e^{-11.4}11.4^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}$ — 1M

≈ 0.0600 — 1A

(iii) The required probability

$$=\frac{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}}{\frac{e^{-11.4}11.4^4}{4!}\times\frac{e^{-25}25^6}{6!}+\frac{e^{-11.4}11.4^5}{5!}\times\frac{e^{-25}25^5}{5!}+\frac{e^{-11.4}11.4^6}{6!}\times\frac{e^{-25}25^4}{4!}}$$ — 1M+1M (1M for numerator, 1M for denominator)

≈ 0.8815 — 1A

(8)

(a) (i)(ii)		Very good.
(iii)(iv)		Good.
(b) (i)		Fair. Some candidates were not able to make use of the given information of 1500 batteries.
(ii)		Very good.
(iii)		Poor. Many candidates had difficulty in counting the number of outcomes and considering all the relevant ones. Some candidates failed to recognise that a conditional probability should be considered.

14. (2011 ASL-M&S Q10)

(a) $P(160\le Y<K)=78.88\%$

$P\left(\frac{160-165}{4}\le Z<\frac{K-165}{4}\right)=0.7888$

$0.3944+P\left(0\le Z<\frac{K-165}{4}\right)=0.7888$ — 1M

$\frac{K-165}{4}=1.25$

$K=170$ — 1A

(2)

(b) $P(\text{score }30)=P(170\le Y<174)$

$=P(1.25\le Z<2.25)$ — 1M

$=0.4878-0.3944$

$=0.0934$ — 1A

(2)

(c) $P(\text{6th game is the 3rd Bingo})=C_2^5(0.2112)^3(0.7888)^3$ — 1M

≈ 0.0462 — 1A

(2)

(d) The number of "Bingo" in n games $\sim B(n, 0.7888)$.

$\therefore\ n(0.7888)(0.2112)\le 2.3$ — 1M

$n\le 13.80597302$

Thus the largest value of n is 13 . — 1A

(2)

(e) (i) $P(\text{score }20)=P(-2.75\le Z<-1.25)=0.1026$ — 1A

$\therefore P(\text{win a prize})$

$=P(\text{total score in 4 games}\ge 160)$

$=(0.7888)^4+C_1^4(0.7888)^3(0.0934+0.1026)+C_2^4(0.7888)^2(0.0934)^2$ — 1M

≈ 0.804490478

≈ 0.8045 — 1A

(ii) $P(\text{win a prize and average score in the first 2 games}\ge 40)$

$=P(\text{total score in 4 games}\ge 160 \text{ and total score in first 2 games}\ge 80)$

$=(0.7888)^4+C_1^4(0.7888)^3(0.0934)+C_1^2(0.7888)^3(0.1026)$

$+(C_2^4-1)(0.7888)^2(0.0934)^2$ — 1M

<u>Alternative Solution</u>

$=P(\text{total score in 4 games}\ge 160)-P(\text{total score in 4 games}\ge 160$ and total score in first 2 games $<80)$

$\approx 0.804490478-(0.7888)^2(0.0934)^2-C_1^2(0.7888)^3(0.1026)$ — 1M

≈ 0.698351364

≈ 0.6984 — 1A

(iii) $P(\text{average score in the first 2 games}<40 \mid \text{win a prize})$

$\approx\frac{0.804490478-0.698351364}{0.804490478}$ — 1M

≈ 0.1319 — 1A

(7)

(a)(b)(c)		Very good.
(d)		Good. Some candidates were not familiar with the variance of a binomial distribution.
(e) (i) (ii)		Fair. Many candidates were unable to exhaust all relevant outcomes.
(iii)		Fair. Many candidates were unable to fully understand the rules of the game described in the question.

15. (2011 ASL-M&S Q12)

(a) $P(Y<100)=0.121$

$P\left(\frac{100-\mu}{\sigma}\le Z<0\right)=0.379$

$\frac{100-\mu}{\sigma}=-1.17$ ---------- (1) 1A

$P(Y\ge 200)=0.0918$

$P\left(0<Z<\frac{200-\mu}{\sigma}\right)=0.4082$

$\frac{200-\mu}{\sigma}=1.33$ ---------- (2) 1A

Solving (1) and (2), we get $\mu=146.8$ and $\sigma=40$ 1A

(3)

(b) P(High level)

$=P(150\le Y<200)$

$=P(0.08\le Z<1.33)$

$\approx 0.4082-0.0319$

$=0.3763$ 1A

(1)

(c) P(High | rainfall exceeds 100 mm)

$=\frac{0.3763}{1-0.121}$ 1M For conditional probability

≈ 0.428100113

≈ 0.4281 1A

(2)

(d) (i) P(Severe | rainfall exceeds 100 mm)

$=\frac{0.0918}{1-0.121}$

≈ 0.10443686 1A

P(Medium | rainfall exceeds 100 mm)

$=\frac{1-0.121-0.0918-0.3763}{1-0.121}$

≈ 0.467463026 1A

P(job will NOT be postponed | rainfall exceeds 100 mm)

$=(0.467463026)e^{-1}+(0.428100113)e^{-3}+(0.10443686)e^{-6}$ 1M

≈ 0.193542759

≈ 0.1935 1A

(ii) P(job will be postponed for 1 day | rainfall exceeds 100 mm)

$=(0.467463026)\cdot e^{-1}1+(0.428100113)\cdot e^{-3}3+(0.10443686)\cdot e^{-6}6$ 1M

≈ 0.237464824

≈ 0.2375 1A

(iii) P(job will be postponed for 2 days | rainfall exceeds 100 mm)

$=(0.467463026)\cdot\frac{e^{-1}1^2}{2!}+(0.428100113)\cdot\frac{e^{-3}3^2}{2!}+(0.10443686)\cdot\frac{e^{-6}6^2}{2!}$

≈ 0.186557057 1A

P(High level | job will be postponed for at least 3 days)

$=\frac{0.428100113\left(1-e^{-3}-e^{-3}3-\frac{e^{-3}3^2}{2!}\right)}{1-0.193542759-0.237464824-0.186557057}$ 1M

≈ 0.6457 1A

(9)

(a) (b) (c)		Good.
(d) (i) (ii)		Satisfactory. The given condition in the stem of (d) was overlooked by some candidates.
(iii)		Fair. Many candidates were unable to analyse the situation and exhaust all relevant cases.

16. (2009 ASL-M&S Q11)

Solution	Marks	Remarks
Let X_N cm and X_D cm be the widths of the tongue of a normal baby and a baby having inherited disease A respectively.		
(a) $\text{P}(X_N < 2.22) = 0.242$		
$\frac{2.22-\mu}{0.4} = -0.7$		
$\mu = 2.5$	1A	
	(1)	
(b) (i) The required probability		
$= \text{P}(X_N > 2.5 + 0.5)$		
$= \text{P}\left(Z > \frac{3-2.5}{0.4}\right)$	1M	
$= 0.5 - 0.3944$		
$= 0.1056$	1A	
(ii) The required probability		
$= 0.05 \times \text{P}(X_D < 2.5 + 0.5) + 0.95 \times \text{P}(X_N > 2.5 + 0.5)$	1M+1A	
$= 0.05 \times \text{P}\left(Z < \frac{0.2}{0.2}\right) + 0.95 \times 0.1056$		
$= 0.05 \times 0.8413 + 0.95 \times 0.1056$		
$= 0.142385$	1A	OR 0.1424
(iii) The required probability		
$= \frac{0.05(0.8413)}{0.05(0.8413) + 0.95(1-0.1056)}$	1M+1A	
≈ 0.0472	1A	
	(8)	
(c) (i) The required probability		
$= \frac{C_3^8 C_1^{12}(0.142385)^4(1-0.142385)^{16}}{C_4^{20}(0.142385)^4(1-0.142385)^{16}}$	1M	
$= \frac{224}{1615}$	1A	OR 0.1387
(ii) The required probability		
$= \frac{C_2^7(0.142385)^3(1-0.142385)^{17} + C_2^7 C_1^{12}(0.142385)^4(1-0.142385)^{16}}{(1-0.142385)^{20} + C_1^{20}(0.142385)(1-0.142385)^{19} + C_2^{20}(0.142385)^2(1-0.142385)^{18} + C_3^{20}(0.142385)^3(1-0.142385)^{17} + C_4^{20}(0.142385)^4(1-0.142385)^{16}}$	1M+1M+1A	1M for numerator 1M for denominator
≈ 0.0156	1A	
	(6)	

(a)		Fair. Candidates were not familiar with the use of normal tables especially in determining whether the z-value is positive or negative.
(b) (i)		Good. Except for the wrong answer carried forward from part (a).
(ii)		Fair. Many candidates could not formulate the problem.
(iii)		Fair. Many candidates did not fully understand the question and hence could not work out the conditional probability.
(c) (i)		Fair. Many candidates were affected by the wrong answers obtained in the previous parts.
(ii)		Very poor. Very few candidates got to attempt this part.

17. (2008 ASL-M&S Q11)

Solution	Marks	Remarks
(a) The required probability		
$= \frac{1.8^0 e^{-1.8}}{0!} + \frac{1.8^1 e^{-1.8}}{1!} + \frac{1.8^2 e^{-1.8}}{2!} + \frac{1.8^3 e^{-1.8}}{3!} + \frac{1.8^4 e^{-1.8}}{4!}$	1M+1M	1M for $\text{P}(X \le 4)$ 1M for Poisson probability with correct λ
≈ 0.963593339		
≈ 0.9636	1A	
	(3)	
(b) $p_0 = \text{P}\left(Z > \frac{4.6-3}{0.8}\right) = \text{P}(Z > 2) = 0.5 - 0.4772 = 0.0228$	1M	For standardization
$p_2 = \text{P}\left(Z < \frac{2-3}{0.8}\right) = \text{P}(Z < -1.25) = 0.5 - 0.3944 = 0.1056$	1A	For any one correct
$p_1 = 1 - p_0 - p_2 = 1 - 0.0228 - 0.1056 = 0.8716$	1A	For all correct
	(3)	
(c) (i) The required probability		
$= C_1^3 p_2 {p_1}^2 + C_1^3 {p_2}^2 p_0$	1M	1M for form correct
$= 3(0.1056)(0.8716)^2 + 3(0.1056)^2(0.0228)$		
≈ 0.241431455		
≈ 0.2414	1A	
(ii) The required probability		
$= C_2^4 {p_2}^2 {p_0}^2 + \frac{4!}{1!2!1!} p_2 {p_1}^2 p_0 + {p_1}^4$	1M	1M for form correct
$= 6(0.1056)^2(0.0228)^2 + 12(0.1056)(0.8716)^2(0.0228) + (0.8716)^4$	1A	
≈ 0.599107436		
≈ 0.5991	1A	
	(5)	
(d) The required probability		
$\approx \frac{\frac{1.8^2 e^{-1.8}}{2!}(0.1056)^2 + \frac{1.8^3 e^{-1.8}}{3!}(0.241431455) + \frac{1.8^4 e^{-1.8}}{4!}(0.599107436)}{0.963593339}$	1M+1M+1A	1M for any 2 cases 1M for denominator using (a) 1A for all correct
≈ 0.0883	1A	
	(4)	

(a)		Very good.
(b)		Good.
(c) (i) (ii)		Fair. Some candidates did not do the counting right and missed some of the eligible events.
(d)		Poor. Many candidates had difficulty in identifying the joint probabilities required for the numerator of the conditional probability.

18. (2007 ASL-M&S Q10)

(a) The required probability

$=1-\left(\frac{2.4^0e^{-2.4}}{0!}+\frac{2.4^1e^{-2.4}}{1!}+\frac{2.4^2e^{-2.4}}{2!}\right)$ — 1M for complemeantary events + 1M for Poisson probability

≈ 0.430291254

≈ 0.4303 — 1A $a-1$ for r.t. 0.430 ----------(3)

(b) Let $\$X$ be the expense of a customer.

Then, $X \sim N(375, 125^2)$.

The required probability

$= P(300 < X < 600)$

$= P(\frac{300-375}{125} < Z < \frac{600-375}{125})$ — 1M (accept $P(\frac{300-375}{125} \le Z \le \frac{600-375}{125})$)

$= P(-0.6 < Z < 1.8)$

$= 0.2257 + 0.4641$

$= 0.6898$ — 1A $a-1$ for r.t. 0.690 ----------(2)

(c) The required probability

$= (0.25)(0.6898) + (0.8)(0.5 - 0.4641)$ — 1M for $0.25(b) + 0.8p$, $0 < p < 0.5$

≈ 0.20117

≈ 0.2012 — 1A $a-1$ for r.t. 0.201 ----------(2)

(d) The required probability

$\approx \frac{2.4^3e^{-2.4}}{3!}(0.20117)^3$ — 1M for $\frac{2.4^3e^{-2.4}}{3!}(c)^3$

≈ 0.00170163

≈ 0.0017 — 1A $a-1$ for r.t. 0.002 ----------(2)

(e) The required probability

$\approx \frac{0.00170163 + (0.20117)^4\left(\frac{2.4^4e^{-2.4}}{4!}\right)}{0.430291254}$ — 1M for numerator using (c) and (d) + 1M for denominator using (a)

≈ 0.00445193

≈ 0.0044 — 1A $a-1$ for r.t. 0.004 ----------(3)

(f) Suppose that the revised least expense is $\$x$.

Then, we have $P(X \ge x) = 0.33$. — 1M

So, we have $P(Z \ge \frac{x-375}{125}) = 0.33$.

Therefore, we have $\frac{x-375}{125} = 0.44$. — 1A

Hence, we have $x = 430$. — 1A

Thus, the revised least expense is $\$430$. ----------(3)

(a)		Very good.
(b)		Very good.
(c)		Good.
(d)		Good. Some candidates overlooked the given condition that *each* of the three customers wins a gift.
(e)		Fair. Many candidates were able to handle conditional probabilities but some were not able to identify the compound events and get the numerator right.
(f)		Fair. A number of candidates could not establish the inequality and some could not solve for the required value.

19. (2007 ASL-M&S Q11)

Let X g be the net weight of a can of brand D coffee beans.

Then, $X \sim N(300, 7.5^2)$.

(a) The required probability

$= P(X < 283.5$ or $X > 316.5)$

$= P(Z < \frac{283.5-300}{7.5}$ or $Z > \frac{316.5-300}{7.5})$ — 1M (accept $P(Z \le \frac{283.5-300}{7.5}$ or $Z \ge \frac{316.5-300}{7.5})$)

$= P(Z < -2.2$ or $Z > 2.2)$

$= 2(0.0139)$

$= 0.0278$ — 1A $a-1$ for r.t. 0.028 ----------(2)

(b) (i) The required probability

$= (1-0.0278)^{11}(0.0278)$ — 1M for $(1-p)^{11}p$

≈ 0.020387152

≈ 0.0204 — 1A $a-1$ for r.t. 0.020

(ii) The required probability

$= C_1^{30}(1-0.0278)^{29}(0.0278)$ — 1M for $C_1^{30}(1-p)^{29}p$; 1M for $p=$((either one

≈ 0.368195889

≈ 0.3682 — 1A $a-1$ for r.t. 0.368

(iii) The required probability

$\approx (1-0.0278)^{30} + 0.368195889$ — 1M for $\left((1-p)^{30}+q\right)$ + 1M for $q = $(b)(ii)

≈ 0.797404576

≈ 0.7974 — 1A $a-1$ for r.t. 0.797 ----------(8)

(c) (i) The required probability

$\approx \frac{1}{2}(0.368195889)$ — 1M for $\frac{1}{2}((b)(ii))$

≈ 0.184097944

≈ 0.1841 — 1A $a-1$ for r.t. 0.184

(ii) The required probability

$\approx \frac{0.184097944}{0.797404575}$ — 1M for numerator using (c)(i) + 1M for denominator using (b)(iii)

≈ 0.230871443

≈ 0.2309 — 1A $a-1$ for r.t. 0.231 ----------(5)

(a)		Very good.
(b)		Good.
(c) (i)		Poor. Many candidates could not apply the well known multiplication rule: $P(A \cap B) = P(A)P(B \mid A)$. In this particular case, $P(A)$ is from b(ii) and $P(B \mid A) = \frac{1}{2}$.
(ii)		Satisfactory.

20. (2006 ASL-M&S Q10)

(a) The required probability

$= \frac{4.7^0 e^{-4.7}}{0!} + \frac{4.7^1 e^{-4.7}}{1!} + \frac{4.7^2 e^{-4.7}}{2!} + \frac{4.7^3 e^{-4.7}}{3!} + \frac{4.7^4 e^{-4.7}}{4!} + \frac{4.7^5 e^{-4.7}}{5!}$ — 1M for the 6 cases + 1M for Poisson probability

≈ 0.668438485

≈ 0.6684 — 1A a–1 for r.t. 0.668

----------(3)

(b) Let X km/h be the speed of a car entering the roundabout.

Then, $X \sim \mathrm{N}(42.8, 12^2)$.

The required probability

$= \mathrm{P}(X > 50)$

$= \mathrm{P}(Z > \frac{50-42.8}{12})$ — 1M (accept $\mathrm{P}(Z \ge \frac{50-42.8}{12})$)

$= \mathrm{P}(Z > 0.6)$

$= 0.2743$ — 1A a–1 for r.t. 0.274

----------(2)

(c) The required probability

$= (1-0.2743)^5(0.2743)$ — 1M for $(1-p)^5 p$ + 1M for $p = $ (b)

≈ 0.055209196

≈ 0.0552 — 1A a–1 for r.t. 0.055

----------(3)

(d) (i) The required probability

$= C_1^4(0.2743)^3(1-0.2743)+(0.2743)^4$ — 1M for the 2 cases + 1M for binomial probability

≈ 0.065570471

≈ 0.0656 — 1A a–1 for r.t. 0.066

(ii) The required probability

$$\approx \frac{0.065570471\left(\frac{(4.7)^4 e^{-4.7}}{4!}\right) + \left((0.2743)^5 + C_1^5(0.2743)^4(1-0.2743) + C_2^5(0.2743)^3(1-0.2743)^2\right)\left(\frac{(4.7)^5 e^{-4.7}}{5!}\right)}{0.668438485}$$

1M + 1M for numerator + 1M for denominator using (a)

≈ 0.052151265

≈ 0.0522 — 1A a–1 for r.t. 0.052

----------(7)

(a)		Very good.
(b)		Good. However, some candidates did not define the notation X when they used it to denote a random variable.
(c)		Good. A number of candidates could not adopt the geometric distribution.
(d) (i)		Fair. Some candidates mistook the required probability to be a conditional probability.
(ii)		Not satisfactory. Many candidates were not able to count the number of relevant events and clearly formulate the required probability.

21. (2005 ASL-M&S Q10)

(a) The required probability

$= \frac{6.2^0 e^{-6.2}}{0!} + \frac{6.2^1 e^{-6.2}}{1!} + \frac{6.2^2 e^{-6.2}}{2!} + \frac{6.2^3 e^{-6.2}}{3!} + \frac{6.2^4 e^{-6.2}}{4!}$ — 1M for the 5 cases + 1M for Poisson probability

≈ 0.259177368

≈ 0.2592 — 1A a–1 for r.t. 0.259

----------(3)

(b) (i) Let X litres be the amount of the petrol for refuelling a car.

Then, $X \sim \mathrm{N}(23.2, 6^2)$.

The required probability

$= \mathrm{P}(X \ge 25)$

$= \mathrm{P}(Z \ge \frac{25-23.2}{6})$ — 1M (accept $\mathrm{P}(Z > \frac{25-23.2}{6})$)

$= \mathrm{P}(Z \ge 0.3)$

$= 0.3821$ — 1A a–1 for r.t. 0.382

(ii) The required probability

$= C_2^8(0.3821)^2(1-0.3821)^6(0.3821)$ — 1M for $C_2^8 p^2(1-p)^6 p$ + 1M for $p = $ (b)(i)

≈ 0.086935732

≈ 0.0869 — 1A a–1 for r.t. 0.087

(iii) The required probability

$= \frac{6.2^3 e^{-6.2}}{3!}(0.3821)^3$ — 1M for $\frac{6.2^3 e^{-6.2}}{3!} p^3$ --------either one

≈ 0.004497064778

≈ 0.0045 — 1A a–1 for r.t. 0.004

(iv) The required probability

$= C_3^4(0.3821)^3(1-0.3821)+(0.3821)^4$ — 1M for $C_3^4 p^3(1-p)+p^4$

≈ 0.159198667

≈ 0.1592 — 1A a–1 for r.t. 0.159

(v) The required probability

$$\approx \frac{0.004497064 + 0.159198667\left(\frac{6.2^4 e^{-6.2}}{4!}\right)}{0.259177368}$$

1M for numerator using (b)(iii) and (b)(iv) + 1M for denominator using (a)

≈ 0.094100184

≈ 0.0941 — 1A a–1 for r.t. 0.094

----------(12)

(a)		Very good.
(b)(i)		Very good.
(ii)		Very good.
(iii)		Fair. Some candidates mistook the required probability to be a conditional probability.
(iv)		Not satisfactory. Many candidates mistook the required probability to be a conditional probability.
(v)		Fair. Many candidates were unable to correctly work out the numerator.

22. (2005 ASL-M&S Q11)

Let X minutes be the time needed for Peter to go to the train station platform. Then, $X \sim N(17.5, 2^2)$.	
(a) The required probability $= P(13 < X \le 19)$	
$= P(\frac{13-17.5}{2} < Z \le \frac{19-17.5}{2})$	1M (accept $P(\frac{13-17.5}{2} \le Z < \frac{19-17.5}{2})$)
$= P(-2.25 < Z \le 0.75)$ $= 0.4878 + 0.2734$ $= 0.7612$	1A a–1 for r.t. 0.761 ----------(2)
(b) The required probability $= (0.02)(0.0122) + (0.15)(0.7612) + (0.35)(0.2144) + (1)(0.0122)$	1M for $(0.02)p_1 + (0.15)p_2 + (0.35)p_3$ + 1M for $(1)(1-p_1-p_2-p_3)$
$= 0.201664$	1A
≈ 0.2017	a–1 for r.t. 0.202 ----------(3)
(c) The required probability $= \frac{(0.15)(0.7612)}{0.201664}$	1M for $\frac{(0.15)(a)}{(b)}$
≈ 0.566189305 ≈ 0.5662	1A (accept 0.5661) a–1 for r.t. 0.566 ----------(2)
(d) The required probability $= C_2^5 (0.201664)^2 (1-0.201664)^3$	1M for $C_2^5 (b)^2 (1-(b))^3$
≈ 0.206925443 ≈ 0.2069	1A (accept 0.2070) a–1 for r.t. 0.207 ----------(2)
(e) The required probability $\approx \frac{C_2^5((0.15)(0.7612))^2((0.0122)(1-0.02)+(0.2144)(1-0.35))^3}{0.206925443}$	1M for $\frac{C_2^5 p^2 q^3}{(d)}$ + 1A
≈ 0.002182834 ≈ 0.0022	1A a–1 for r.t. 0.002
The required probability $\approx (0.566189305)^2 \left(\frac{(0.0122)(1-0.02)+(0.2144)(1-0.35)}{1-0.201664}\right)^3$	1M for $(c)^2 r^3$ + 1A
≈ 0.002182834 ≈ 0.0022	1A a–1 for r.t. 0.002 ----------(3)
(f) Suppose Peter leaves home t minutes before 7:00 a.m. Then, we have $P(X \le 13+t) \ge 0.95$.	1M withhold 1M for equality or strict inequality
So, we have $P(Z \le \frac{13+t-17.5}{2}) \ge 0.95$.	
Therefore, we have $\frac{t-4.5}{2} \ge 1.645$.	1A (accept $\frac{t-4.5}{2} \ge z$, $1.64 \le z \le 1.65$)
Hence, we have $t \ge 7.79$. Thus, the required time is 6:52 a.m.	1A ----------(3)

(a)		Fair. Some candidates were not able to express the required probability.
(b)		Fair. Many candidates overlooked the case that Peter cannot catch any one of the three trains.
(c)		Fair. Some candidates got the numerator wrong and forgot that the required probability should be a joint probability.
(d)		Very good.
(e)		Poor. Many candidates were not able to count the number of relevant events and hence they were unable to correctly work out the numerator.
(f)		Poor. Many candidates could not formulate the problem using the correct inequality.

23. (2004 ASL-M&S Q12)

Let \$$X$ be the amount of money spent by a customer. Then, $X \sim N(428, 100^2)$. Also let Y be the number of customers visiting the store in a minute. Then, $X \sim P_0(4)$.	
(a) The required probability $= P(X \ge 300)$ $= P(Z \ge \frac{300-428}{100})$ $= P(Z \ge -1.28)$	1A accept $P(Z > -1.28)$
$= 0.8997$	1A a–1 for r.t. 0.900 ----------(2)
(b) The required probability $= 1 - P(Y=0) - P(Y=1)$	1M for complementary probability
$= 1 - \frac{4^0 e^{-4}}{0!} - \frac{4^1 e^{-4}}{1!}$	1A
$= 1 - 5e^{-4}$	1A
≈ 0.9084218056 ≈ 0.9084	a–1 for r.t. 0.908 ----------(3)
(c) The required probability $= P(Y=3)(C_2^3 (0.8997)^2 (1-0.8997))$ $= (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1-0.8997))$	1M for $C_2^3 (a)^2 (1-(a))$ + 1M for multiplication rule
≈ 0.0475848193 ≈ 0.0476	1A a–1 for r.t. 0.048 ----------(3)
(d) The required probability $= \frac{(\frac{4^2 e^{-4}}{2!})(C_2^2 (0.8997)^2) + (\frac{4^3 e^{-4}}{3!})(C_2^3 (0.8997)^2 (1-0.8997))}{\frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!}}$	1A for denominator + 1M for numerator
$\approx \frac{0.16619104895}{0.34189192592}$ ≈ 0.48609234776 ≈ 0.4861	1A a–1 for r.t. 0.486 ----------(3)
(e) $P(X \ge 600)$ $= P(Z \ge \frac{600-428}{100})$ $= P(Z \ge 1.72)$ $= 0.0427$	1A
Let n be the number of customers visiting the store. Then, we have $1 - (1-0.0427)^n \ge 0.99$ $(0.9573)^n \le 0.01$	1M accept $(1-0.0427)^n \le 0.01$ withhold 1M for using equality or strict inequality
$n \ln 0.9573 \le \ln 0.01$ $n \ge \frac{\ln 0.01}{\ln 0.9573}$ $n \ge 105.5300874$	1M for using ln or trial and error
Thus, the smallest number of customers visiting the store is 106.	1A ----------(4)

(a)		Good. The skill is straightforward but some candidates did not understand the question and were unable to correctly find the probability.
(b)		Good.
(c)		Good. Some candidates were not capable of applying the multiplication rule.
(d)		Fair.
(e)		Poor. Very few candidates managed to establish the inequality that the stated probability ≥ 0.99.

24. (2003 ASL-M&S Q12)

(a) The required probability	
$= 1 - \dfrac{C_7^{17} + C_7^{13}}{C_7^{30}}$	1M for counting cases + 1A for correctness of probability
$= \dfrac{38743}{39150}$	1A
≈ 0.989604086	
≈ 0.9896	$a-1$ for r.t. 0.990
	---------(3)
(b) The required probability	
$= \dfrac{\dfrac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30}}}{\dfrac{38743}{39150}}$	1M for denominator using (a) + 1A for numerator
$= \dfrac{1498}{2279}$	1A
≈ 0.657305835	
≈ 0.6573	$a-1$ for r.t. 0.657
The required probability	
$= \dfrac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30} - C_7^{17} - C_7^{13}}$	1M for denominator using (a) + 1A for numerator
$= \dfrac{1498}{2279}$	1A
≈ 0.657305835	
≈ 0.6573	$a-1$ for r.t. 0.657
	---------(3)
(c) Let $\$X$ be the amount of money collected by a boy and $\$Y$ be the amount of money collected by a girl. Then, $X \sim N(673, 100^2)$ and $Y \sim N(708, 100^2)$.	
(i) The required probability	
$= P(X > 800)$	
$= P\left(Z > \dfrac{800-673}{100}\right)$	1M accept $Z \geq \dfrac{800-708}{100}$
$= P(Z > 1.27)$	
$= 0.102$	1A accept 0.1020

(ii) $P(Y > 800)$	
$= P\left(Z > \dfrac{800-708}{100}\right)$	
$= P(Z > 0.92)$	
$= 0.1788$	1A
The required probability	
$= \left(C_1^3(0.102)(0.898)^2\right)\left(C_1^4(0.1788)(0.8212)^3\right)$	1M for Binomial probability + 1M for Binomial × Binomial
≈ 0.097734619	
≈ 0.0977	1A $a-1$ for r.t. 0.098
(iii) The required probability	
$\approx \dfrac{0.097734619}{0.097734619 + C_2^3(0.102)^2(0.898)(0.8212)^4 + (0.898)^3 C_2^4 (0.1788)^2 (0.8212)^2}$	1M for numerator +1M for denominator
≈ 0.478730045	
≈ 0.4787	1A (accept 0.4786) $a-1$ for r.t. 0.479
	---------(9)

(a/b)		Good. Most candidates successfully managed to count the number of combinations.
(c)		Parts (i) and (ii) were well attempted. Part (iii) was more demanding and most candidates were unable to obtain the probability of getting two certificates.

25. (2002 ASL-M&S Q13)

Let Xg be the weight of a bag of self raising flour in the batch.

(a) (i) P(a bag of flour is underweight) $= P(X < 376)$

$= P(\frac{X-400}{10} < \frac{376-400}{10})$ — 1M (either one)

$= P(Z < -2.4)$

≈ 0.0082 — 1A (either one)

(ii) P(a bag of flour is overweight) $= P(X > 424)$

$= P(\frac{X-400}{10} > \frac{424-400}{10})$

$= P(Z > 2.4)$

≈ 0.0082 — 1A ----------(3)

(b) (i) P(a bag of flour is substandard)

$= P(X < 376) + P(X > 424)$

$\approx 0.0082 + 0.0082 = 0.0164$ — 1A

Let Y be the number of substandard bags in the sample.

P(there is no substandard bags in the sample) $= P(Y = 0)$

$= C_0^{50}\ 0.0164^0 \times (1-0.0164)^{50}$ — 1M

$= 0.9836^{50} \approx 0.4374$ — 1A (0.43745) a–1 for r.t. 0.437

(ii) $P(Y \le 2)$

$= P(Y=0) + P(Y=1) + P(Y=2)$

$= C_0^{50} 0.0164^0 \times 0.9836^{50} + C_1^{50}\ 0.0164 \times 0.9836^{49} + C_2^{50}\ 0.0164^2 \times 0.9836^{48}$ — 1M

$\approx 0.43745 + 0.36469 + 0.14897$

≈ 0.9511 — 1A (0.95111) a–1 for r.t. 0.951 ----------(5)

(c) Let W be the number of underweight bags in the sample.

(i) $P(W = 0, Y = 1)$

$= P(W = 0 \mid Y = 1) \cdot P(Y = 1)$

$= \frac{1}{2} \times C_1^{50} (0.0164)(0.9836)^{49}$ — 1M + 1M for $\frac{1}{2}$ and cond. prob.

≈ 0.1823 — 1A a–1 for r.t. 0.182

(ii) The required probability is $P(W = 0, Y \le 2)$

$= P(W = 0, Y = 0) + P(W = 0, Y = 1) + P(W = 0, Y = 2)$ — 1M

$= P(Y = 0) + P(W = 0, Y = 1) + P(W = 0 \mid Y = 2) \cdot P(Y = 2)$

$\approx 0.43745 + 0.18235 + \left(\frac{1}{2}\right)^2 \cdot C_2^{50} (0.0164)^2 (0.9836)^{48}$ — 1M for the last term

≈ 0.6570 — 1A (0.65704) (Accept 0.6569) a–1 for r.t. 0.657

(iii) The required probability is $P(W = 0 \mid Y \le 2)$

$= \frac{P(W = 0, Y \le 2)}{P(Y \le 2)}$

$\approx \frac{0.65704}{0.95111}$ — 1M

≈ 0.6908 — (Accept 0.6907) ----------(7)

26. (2002 ASL-M&S Q14)

(a) Let N be the number of customers visiting the supermarket in one minute.

$P(N \le 2) = \sum_{k=0}^{2} \frac{6^k}{k!} e^{-6}$ — 1A

$= e^{-6} + \frac{6}{1!}e^{-6} + \frac{6^2}{2!}e^{-6}$

$\approx 0.002479 + 0.01487 + 0.04462$

≈ 0.0620

$\therefore\ P(N > 2) = 1 - P(N \le 2) \approx 0.9380$ — 1M+1A a–1 if 0.938 ----------(3)

(b) (i) $X \sim N(\mu, \sigma^2)$

$P(X < 100) = 0.063$

$P(Z < \frac{100-\mu}{\sigma}) = 0.063$

$\frac{100-\mu}{\sigma} \approx -1.53$ .... (1) — 1A

$P(X \ge 400) = 0.006$

$p(Z \ge \frac{400-\mu}{\sigma}) = 0.006$

$\frac{400-\mu}{\sigma} \approx 2.51$ .... (2) — 1A (Accept $\frac{200-\mu}{\sigma} \in [-0.185, -0.18]$)

Solving (1) and (2), we get

$\mu \approx 213.6$ — 1A a–1 for more than 1 d.p. (Accept $\mu \in [213.3, 213.8]$)

$\sigma \approx 74.26 \approx 74.3$ — 1A a–1 for more than 1 d.p. (Accept $\sigma \in [74.1, 74.3]$)

$a_1 = P(200 \le X < 300)$

$= p(Z < \frac{300-213.6}{74.3}) - P(Z < \frac{200-213.6}{74.3})$

≈ 0.4484

≈ 0.448 — 1A a–1 for more than 3 d.p. (Accept $a_1 \in [0.448, 0.453]$)

$a_2 = P(300 \le X < 400)$

≈ 0.117 — 1A a–1 for more than 3 d.p. (Accept $a_2 \in [0.115, 0.119]$)

(ii) For normal distribution,
median = mean = 213.6 — 1M

(iii) $P(X > 50 \mid X \le 200)$

$= \frac{P(50 \le X < 200)}{P(X < 100) + P(100 \le X < 200)}$ — 1M

$= \frac{P(-2.20 \le Z < -0.18)}{0.063 + 0.364}$

$\approx \frac{0.4861 - 0.0714}{0.427}$

≈ 0.9712 — 1A a–1 for more than 4 d.p. (Accept probability $\in [0.9620, 0.9749]$)

$P(X > 50 \mid X \le 200)$

$= \dfrac{P(50 \le X < 200)}{P(X < 100) + P(100 \le X < 200)}$ — 1M

$= \dfrac{P(X < 200) - P(X \le 50)}{P(X \le 200)}$

$= \dfrac{P(X < 200) - P(Z \le -2.20)}{P(X \le 200)}$

$= \dfrac{0.427 - 0.0139}{0.427}$

≈ 0.9674 — 1A $a-1$ for more than 4 d.p. (Accept probability $\in [0.9620, 0.9749]$)

(iv) The required probability

$= C_2^5 P(X < 200)^2 (1 - P(X < 200))^3 \cdot P(N = 5)$

$= 10(0.063 + 0.364)^2 (1 - (0.063 + 0.364))^3 \cdot \dfrac{6^5}{5!} e^{-6}$ — 1M for Binomial/Poisson probability; 1M for the multiplication rule (Binomial × Poisson)

$\approx 10(0.1823)(0.1881) \cdot (0.1606)$

≈ 0.0551 — 1A $a-1$ for r.t. 0.055 (Accept probability $\in [0.0550, 0.0552]$)

----------(12)

27. (2001 ASL-M&S Q12)

Let E_X and E_Y be the lifetimes of brand X and brand Y CFLs respectively.

(a) $P(E_X < 8200) = 0.1151 \Rightarrow P\left(\dfrac{E_X - \mu}{400} < \dfrac{8200 - \mu}{400}\right) = 0.0808$

$\Rightarrow \dfrac{8200 - \mu}{400} = -1.4$ — 1A for either

$\Rightarrow \mu = 8760$ — 1A

$P(E_Y < 8200) = 0.1587 \Rightarrow P\left(\dfrac{E_Y - 8800}{\sigma} < \dfrac{8200 - 8800}{\sigma}\right) = 0.1587$

$\Rightarrow \dfrac{8200 - 8800}{\sigma} = -1.00$

$\Rightarrow \sigma = 600$ — 1A

$a_1 = 0.3811$, $a_2 = 0.0548$ — 1A

$b_1 = 0.2120$, $b_2 = 0.2586$, $b_3 = 0.2120$ — 1A $b_1 = b_3 \in [0.2101, 0.2120]$, $b_2 \in [0.2586, 0.2624]$

[$b_1 = 0.2109$, $b_2 = 0.2608$, $b_3 = 0.2109$]

----------(5)

(b) The mean of the lifetimes of the 2 brands only differ a little but the standard deviation of the lifetimes of brand X CFLs is significantly smaller than that of brand Y.

I shall choose brand X because the lifetimes of its CFLs are more reliable. — 1M

[I shall choose brand Y because there will be a bigger chance of getting a long life CFL. — 1M]

[I shall choose brand Y because the mean lifetime is larger. — 1M]

----------(1)

(c) (i) Let X_a, X_b and X_c be the lifetimes of lamps a, b and c resp.

(I) The required probability

$= P(X_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$ — 1M

$= [1 - P(E_X < 8200)]\{1 - [P(E_X < 8200)]^2\}$ — 1M

$\approx (1 - 0.0808)(1 - 0.0808^2)$ — 1M

[$\approx 2(0.9192)^2(1 - 0.9192) + (0.9192)^3$ — 1M+1M+1M]

≈ 0.9132 — 1A

(II) The required probability

$= \dfrac{P(X_a < 8200)P(X_b > 8200)P(X_c > 8200)}{1 - 0.9132}$ — 1M for numerator; 1M for denominator

$= \dfrac{0.0808(1 - 0.0808)^2}{1 - 0.9132}$

$= 0.7865$ — 1A

(ii) Note that $P(E_X < 8200) \approx 0.0808$

and $P(E_Y < 8200) \approx 0.1578$.

Since a brand X CFL is less likely than a brand Y CFL to have a lifetime less than 8200 hours, and lamp a is the most critical lamp for the lighting system to work (according to the result of (c)(i)(II)), — 1

$\therefore$ Lamp a should be a brand X CFL.

Hence I will put the brand Y CFL as lamp b or c. — 1A with explanation

[Let X_a and Y_a be the lifetimes of lamp a when using brand X CFL and brand Y CFL respectively. Similar notations are used for the other two lamps.

$P(Y_a > 8200)[P(X_b > 8200 \text{ or } X_c > 8200)]$

$= (1 - 0.1587)(1 - 0.0808^2)$

$= 0.8358$

$P(X_a > 8200)[P(Y_b > 8200 \text{ or } X_c > 8200)]$

$= (1 - 0.0808)[(1 - 0.0808) + (1 - 0.1587) - (1 - 0.0808)(1 - 0.1587)]$

$= 0.9074$ — [1]

Hence putting the brand Y CFL as lamp b or c will yeild a better system. — [1A] with explanation]

----------(9)

28. (2000 ASL-M&S Q12)

(a) Let $X \sim N(20, 5^2)$ and $Z \sim N(0, 1)$.

(i) P(risky but not hazardous | A)

$= P(12 < X < 27)$ — 1A

$= P(\frac{12-20}{5} < Z < \frac{27-20}{5})$

$= P(-1.6 < Z < 1.4)$ — 1M

$\approx 0.4452 + 0.4192$

≈ 0.8644 — 1A a–1 for r.t. 0.864

(ii) P(risky | A) $= P(X > 12)$

$= P(Z > -1.6)$

$\approx 0.4452 + 0.5$ — 1A

≈ 0.9452

P(hazardous | A) $= P(X > 27)$

$= P(Z > 1.4)$

$\approx 0.5 - 0.4192$ — 1A

≈ 0.0808

$\therefore$ P (a risky bottle is hazardous | A) $\approx \frac{0.0808}{0.9452} \approx 0.0855$ — 1M

(b) (i) P(risky) $= 0.6$ P(risky | A) $+ 0.4$ P(risky | B)

$\approx 0.6(0.9452) + 0.4(0.058)$ — 1M

≈ 0.59032

≈ 0.5903 (p) — 1A a–1 for r.t. 0.590

P(B and risky | risky) $= \frac{\text{P(risky | B)P(B)}}{\text{P(risky)}}$

$\approx \frac{(0.058)(0.4)}{0.59032}$ — 1A numerator; 1M Bayes' theorem

≈ 0.0393

(ii) P(B and hazardous | risky) $= \frac{\text{P(hazardous | B)P(B)}}{\text{P(risky)}}$

$\approx \frac{(0.004)(0.4)}{0.59032}$ — 1A numerator; 1M Bayes' theorem

≈ 0.00271

≈ 0.0027

(iii) P(license suspended) $= 1-(1-p)^5 - 5p(1-p)^4$ — 1M binomial; 1M complement of cases 0 & 1; 1M p from b(i)

$\approx 1-(1-0.59032)^5 - 5(0.59032)(1-0.59032)^4$

≈ 0.9053

29. (1999 ASL-M&S Q10)

Let X be the score on the questionnaire.

(a) (i) P(classify as non-PD | PD)

$= P(X < 75 \mid X \sim N(80, 5^2))$

$= P(Z < \frac{75-80}{5})$ — 1A

$= P(Z < -1)$

$\approx 0.5 - 0.3413$

$= 0.1587$ — 1A

(ii) P(classify as PD | non-PD)

$= P(X > 75 \mid X \sim N(65, 5^2))$

$= P(Z > \frac{75-65}{5})$ — 1A

$= P(Z > 2)$

$\approx 0.5 - 0.4772$

$= 0.0228$ — 1A

(b) The probability that out of 10 PDs, not more than 2 will be misclassified

$\approx (1-0.1587)^{10} + C_1^{10}(0.1587)(1-0.1587)^9 + C_2^{10}(0.1587)^2(1-0.1587)^8$ — 1M+1M   1M for 2nd or 3rd term; 1M for all

≈ 0.7971 — 1A

(c) Let x_0 be the required critical level of score.

$P(X < x_0 \mid X \sim N(80, 5^2)) = 0.01$

$P(Z < \frac{x_0 - 80}{5}) = 0.01$ — 1A

$\frac{x_0 - 80}{5} \approx -2.3267$ — 1M   accept −2.325 to −2.33, for the case '$Z <$ ...' only

$x_0 \approx 68.3665$ — 1A   accept 68.35 to 68.375

(d) If a teenager is classified by the sociologist, then

P(classify as PD | non-PD)

$= P(X > 68.3665 \mid X \sim N(65, 5^2))$ — 1M   accept 68.35 to 68.375

$= P(Z > 0.6733)$ accept 0.67 to 0.675

$\approx 0.5 - 0.2496$

$= 0.2504$ accept 0.2498 to 0.2514

$\therefore$ P(misclassified) $\approx (0.01)(0.1) + (0.2504)(0.9)$ — 1M for either

≈ 0.2264 — 1A   accept 0.2258 to 0.2273

If a teenager is classified by the criminologist, then

P(misclassified) $\approx (0.1587)(0.1) + (0.0228)(0.9)$

≈ 0.0364 — 1A

$\because$ $0.2264 > 0.0364$

$\therefore$ The probability of teenagers miscalssified by the sociologist is greater than that by the criminologist. — 1

30. (1998 ASL-M&S Q13)

Solution	Marks	Remarks
Let X, Y be the weights of the randomly selected boxes in parts 1 and 2 of a test respectively.		
(a) $P(X<490 \text{ or } X>510)$		
$=1-P(\frac{490-500}{5}\le Z\le\frac{510-500}{5})$	1A	deduct 1 mark once for the whole question for any wrong inequality sign
$=1-P(-2\le Z\le 2)$		
$\approx 1-2\times 0.4772$		
≈ 0.0456	1A	
(b) $P(490\le X<492)+P(508<X\le 510)$	1A	
$=P(\frac{490-500}{5}\le Z<\frac{492-500}{5})+P(\frac{508-500}{5}<Z\le\frac{510-500}{5})$	1A	
$=P(-2\le Z<-1.6)+P(1.6<Z\le 2)$		
$\approx(0.4772-0.4452)\times 2$		
≈ 0.0640	1A	
Alternatively,		
$P(X<492)+P(X>508)-$ P(a black signal is generated in the first part)	1A	
$=P(Z<\frac{492-500}{5})+P(Z>\frac{508-500}{5})-0.0456$	1A	
$\approx 0.0548+0.0548-0.0456$		
≈ 0.0640	1A	
(c) P(black)		
= P(black in part 1) + P(black in part 2)		
$\approx 0.0456+0.0640\times 0.0456$	1M+1M	
≈ 0.0485	1A	
(d) $P(508<X\le 510 \text{ and } 508<Y\le 510 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$=\frac{P(508<X\le 510)P(508<Y\le 510)}{P(490\le X<492)+P(508<X\le 510)}$		
$\approx\frac{0.0320\times 0.0320}{0.0320+0.0320}$	1M+1M	
≈ 0.0160	1A	
(e) P(red | part 2)		
$=P(508<X\le 510 \text{ and } 508<Y\le 510 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$+P(490\le X<492 \text{ and } 490\le X<492 \mid 490\le X<492 \text{ or } 508<X\le 510)$		
$\approx 2\times 0.0160$	1M	
≈ 0.0320	1A	
(f) P(red) = P(red | part 2) P(part 2)		
$\approx 0.0320\times 0.0640$	1M	
≈ 0.0020	1A	
Alternatively,		
$P(\text{red})=P(508<X\le 510 \text{ and } 508<Y\le 510)$		
$+P(490\le X<492 \text{ and } 490\le Y<492)$		
$=0.0320^2\times 2$	1M	
$=0.0020$	1A	

31. (1997 ASL-M&S Q11)

Solution	Marks
(a) Let X be the number of FICs per day, then $X\sim Po(4)$.	
$P(X=0)=\frac{4^0e^{-4}}{0!}$	1M
≈ 0.0183	1A
(b) Let Y be the number of FICs which are related to house fires in 5 FICs, then $Y\sim B(5,0.6)$.	
$P(Y\ge 2)=1-P(Y=0)-P(Y=1)$	
$=1-C_0^5(0.4)^5-C_1^5(0.6)(0.4)^4$	1M+1A
≈ 0.9130	1A
(c) Let H and L be the events of "a FIC is related to a house fire" and "a FIC is *large*". Let A be the amount of a FIC.	
(i) $P(L\mid H)=P(A>20\,000)$	
$=P(Z>\frac{200\,000-100\,000}{50\,000})$	1M
$=P(Z>2)$	
≈ 0.0228	1A
$P(L\mid\overline{H})=P(A>20\,000)$	
$=P(Z>\frac{200\,000-150\,000}{20\,000})$	
$=P(Z>2.5)$	
≈ 0.0062	1A
$P(L)=P(L\mid H)P(H)+P(L\mid\overline{H})P(\overline{H})$	
$\approx 0.0228(0.6)+0.0062(0.4)$	1M
≈ 0.0162	1A
(ii) $P(H\mid L)=\frac{P(L\mid H)P(H)}{P(L)}$	
$\approx\frac{0.0228\times 0.6}{0.0162}$	1M
≈ 0.8444	1A
(iii) P(5 FICs and at least 2 of them are *large*)	
= P(2 or more out of 5 FICs are *large*)P(X=5)	
$\approx[1-(1-0.0162)^5-5(0.0162)(1-0.0162)^4]\frac{e^{-4}4^5}{5!}$	1M+1A
≈ 0.0004	1A

32. (1997 ASL-M&S Q13)

Let L cm be the length of the front portion of Mr. Wong's necktie.		
(a) $P(44 < L < 45)$		
$= P(\frac{44-44.6}{1.2} < Z < \frac{45-44.6}{1.2})$	1M	
$\approx P(-0.5 < Z < 0.3333)$		
$\approx 0.1915 + 0.1293$ (or $0.1915 + 0.1306$)	1A	For either
≈ 0.3208 (or 0.3221)	1A	
(b) Let Y be the number of trials that Mr. Wong gets the first *perfect tying*, then $Y \sim$ Geometric(p), where $p \approx 0.3208$ (or 0.3221)		
$E(Y) = \frac{1}{p}$	1M	
≈ 3.1172 (or 3.1046)	1A	
(c) P(not more than 3 trials) = P(1 trial) + P(2 trials) + P(3 trials)		
$= p + p(1-p) + p(1-p)^2$	1M	or $1-(1-p)^3$
≈ 0.6867 (or 0.6885)	1A	
(d) Let T be the event that Mr. Wong has to go to work by taxi.		
(i) $P(T) \approx 1 - 0.6867$ (or $1 - 0.6885$) $= 0.3133$ (or 0.3115)		
P(less than $2T$ out of 6 days) $\approx C_0^6(0.6867)^6 + C_1^6(0.6867)^5(0.3133)$	1M+1A	
(or $C_0^6(0.6885)^6 + C_1^6(0.6885)^5(0.3115)$)		
≈ 0.3919 (or 0.3957)	1A	
(ii) $P(Y=5 \mid T) \approx \frac{(1-p)^4 p}{P(T)}$	1M	
≈ 0.2179 (or 0.2184)	1A	
(iii) Probability required $\approx 5(0.3133)^2(0.6867)^4$ (or $5(0.3115)^2(0.6885)^4$)	1M+1A	
≈ 0.1091 (or 0.1090)	1A	

33. (1996 ASL-M&S Q11)

Let X ml be the amount of soda water in each discharge. $X \sim N(210, 15^2)$.		
(a) $P(200 < X < 220)$		
$= P(\frac{200-210}{15} < Z < \frac{220-210}{15})$	1M	
$\approx P(-0.6667 < Z < 0.6667)$		
≈ 0.4972	1A	Accept value in [0.494, 0.4972]
(b) (i) $P(X > 240)$		
$= P(Z > \frac{240-210}{15})$	1M	
$= P(Z > 2)$		
≈ 0.0228	1A	
(ii) The probability that there is exactly 1 overflow out of 30 discharges is $C_1^{30}(0.0228)(0.9772)^{29}$	1M	
≈ 0.3504	1A	
(iii) The probability that Sam will get the second overflow on 31st July is 0.3504×0.0228		
≈ 0.0080	1M	
(c) (i) $\because P(X > 205) = 0.8$		
$\therefore P(Z > \frac{205-\mu}{\sigma}) = 0.8$		
$\frac{205-\mu}{\sigma} = -0.84$ ....(1)	1M+1A	Accept value in [−0.845, −0.84]
$\because P(X > 220) = 0.01$		
$\therefore P(Z > \frac{220-\mu}{\sigma}) = 0.01$		
$\frac{220-\mu}{\sigma} = 2.33$ ....(2)	1A	Accept value in [2.32, 2.33]
Solving (1) & (2): $\begin{cases}\sigma = 4.7 \\ \mu = 209.0\end{cases}$	1A+1A	
(ii) $P(X > 225)$		
$= P(Z > \frac{225-209}{4.7})$		
$\approx P(Z > 3.4042)$		
≈ 0.0003	1A	
Probability required $= \frac{0.0003}{0.01}$	1M	
$= 0.03$	1A	

34. (1995 ASL-M&S Q12)

Let X denote the test score and D the event that a person has the disease.

(a) $P(X>63.2\,|\,D') = 0.33$

$P(Z>\frac{63.2-\mu}{5}) = 0.33$ [1A]

From the normal distribution table,

$\frac{63.2-\mu}{5} = 0.44$ [1A]

$\therefore \mu = 61$ [1A]

(b) (i) $P(X>66\,|\,D) = P(Z>\frac{66-70}{5})$ [1A]

$= P(Z>-0.8)$

$= 0.7881$ [1A]

and $P(X>66\,|\,D') = P(Z>\frac{66-61}{5})$

$= P(Z>1)$

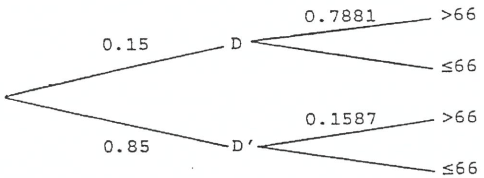
$= 0.1587$ [1A]

$\therefore$ P(the person will be classified as having the disease)

$= 0.15\times0.7881+(1-0.15)\times0.1587$ [1M + 1A]

$= 0.2531$ [1A]

(ii)



[1M]

$P(X\le66\,|\,D) = 1 - 0.7881$ [1M]

$= 0.2119$ [1A]

$\therefore$ P(the person will be misclassified)

$= 0.15\times0.2119+(1-0.15)\times0.1587$ [1M + 1A]

$= 0.1667$ [1A]

35. (1994 ASL-M&S Q13)

(a) $\because P(Z<\frac{c_1-10}{0.4}) = 0.95$ [1M]

$\therefore \frac{c_1-10}{0.4} = 1.645$ [1A] Accept 1.64-1.65

$c_1 = 10.658$ [1A] Accept 10.656-10.66

(b) $\because P(Z<\frac{c_2-12.3}{0.6}) = 0.01$ [1M]

$\therefore \frac{c_2-12.3}{0.6} = -2.327$ [1A] -2.33 to -2.32

$c_2 = 10.9038$ [1A] 10.902 to 10.908

(c) Given the batch is produced under the favourable condition, the required probability is

$P(c_1<X \text{ and } X<c_2)$

$= P(10.658<X<10.9038)$ [1M]

$= P(\frac{10.658-10}{0.4}<Z<\frac{10.9038-10}{0.4})$ [1M]

$= P(1.645<Z<2.2595)$

$= 0.4881 - 0.45$ [1M] For subtraction

$= 0.0381$ [1A]

(d) $P(X<c_3 \text{ where } \sigma=0.4,\ \mu=10) = P(X\ge c_3 \text{ where } \sigma=0.6,\ \mu=12.3)$

i.e. $-(\frac{c_3-10}{0.4}) = \frac{c_3-12.3}{0.6}$ [1M + 1A]

$c_3 = 10.92$ [1A]

(e) The probability would be minimized if μ is in the middle of the 2 limits,

i.e. $\mu = \frac{10.8+9.4}{2}$ [1M]

$= 10.1$ [1A]