2000-CE MATH

PAPER 1

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2000

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)

This paper must be answered in English

1. Write your candidate number, centre number and seat number in the spaces provided on this cover.
2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.
3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your Candidate Number on each sheet and fasten them with string inside this book.
4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
7. The diagrams in this paper are not necessarily drawn to scale.

Candidate Number							
Centre Number							
Seat Number							

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1–2		
3–4		
5–6		
7–8		
9		
10		
11		
12		
13		
14		
Section A Total		

Checker's Use Only	Section A Total		

Section B Question No.*	Marks	Marks
Section B Total		

To be filled in by the candidate.

Checker's Use Only	Section B Total		

Checker No.	

Page total

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3} \times$ base area $\times$ height

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Let $C = \frac{5}{9}(F - 32)$. If $C = 30$, find F . (3 marks)

2. Simplify $\frac{x^{-3}y}{x^2}$ and express your answer with positive indices. (3 marks)

Page total

3. Find the area of the sector in Figure 1. (3 marks)

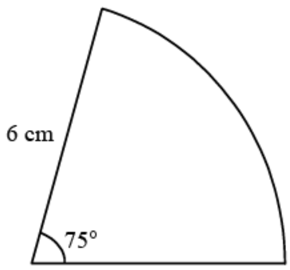

Figure 1

4. In Figure 2, find a and x . (4 marks)

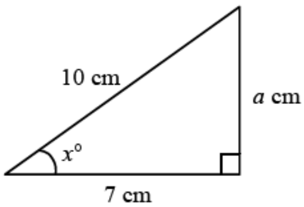

Figure 2

Page total

5. Solve $\frac{11-2x}{5} < 1$ and represent the solution in Figure 3. (4 marks)

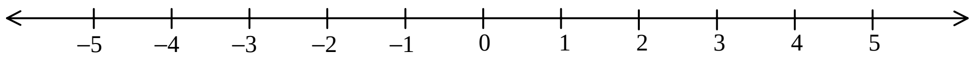

Figure 3

6. Let $f(x) = 2x^3 + 6x^2 - 2x - 7$. Find the remainder when $f(x)$ is divided by $x + 3$. (3 marks)

Page total

7. In Figure 4, AD and BC are two parallel chords of the circle. AC and BD intersect at E. Find x and y. (4 marks)

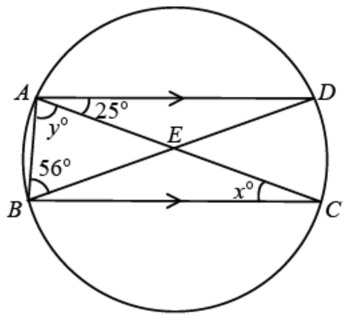

Figure 4

8. On a map of scale $1:5\,000$, the area of the passenger terminal of the Hong Kong International Airport is $220\ \text{cm}^2$. What is the actual area, in m^2, occupied by the terminal on the ground? (4 marks)

Page total

9. Let L be the straight line passing through $(-4, 4)$ and $(6, 0)$. (5 marks)

(a) Find the slope of L.

(b) Find the equation of L.

(c) If L intersects the y-axis at C, find the coordinates of C.

Page total

Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. (a) Solve $10x^2 + 9x - 22 = 0$. (2 marks)

(b) Mr. Tung deposited $\$10\,000$ in a bank on his 25th birthday and $\$9\,000$ on his 26th birthday. The interest was compounded yearly at $r\%$ p.a. , and the total amount he received on his 27th birthday was $\$22\,000$. Find r . (4 marks)

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11. Figure 5 shows the cumulative frequency polygon of the distribution of the lengths of 75 songs.

The cumulative frequency polygon of the distribution of the lengths of 75 songs

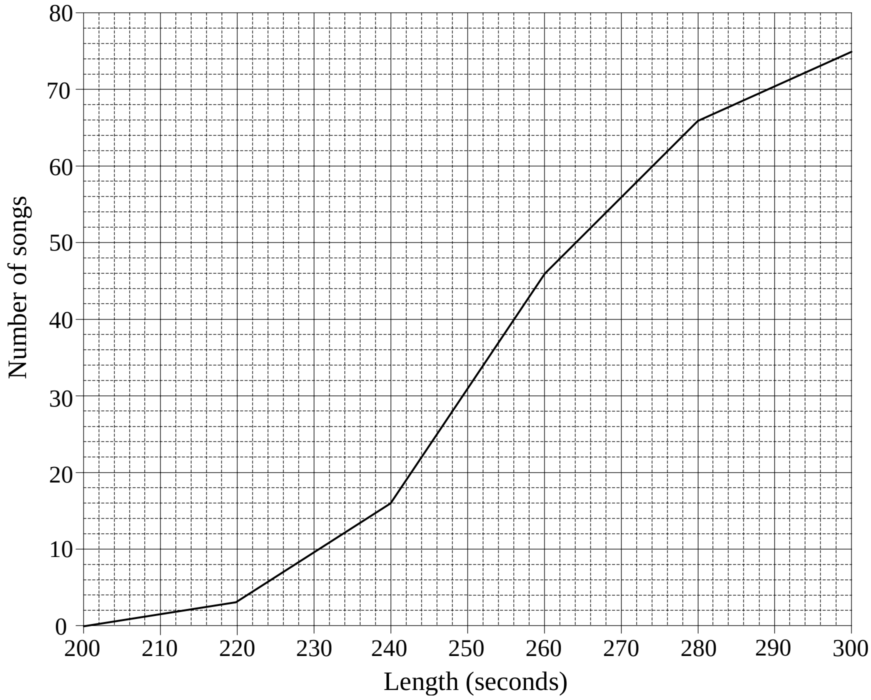

Figure 5

(a) Complete the tables below. (2 marks)

Length (t seconds)	Cumulative frequency
$t \leq 220$	3
$t \leq 240$	16
$t \leq 260$	46
$t \leq 280$	
$t \leq 300$	75

Length (t seconds)	Frequency
$200 < t \leq 220$	3
$220 < t \leq 240$	13
$240 < t \leq 260$	30
$260 < t \leq 280$	
$280 < t \leq 300$	9

(b) Find an estimate of the mean of the distribution. (2 marks)

(c) Estimate from the cumulative frequency polygon the median of the distribution. (1 mark)

(d) What percentage of these songs have lengths greater than 220 seconds but not greater than 260 seconds? (2 marks)

Page total

12. A box contains nine hundred cards, each marked with a different 3-digit number from 100 to 999 . A card is drawn randomly from the box.

(a) Find the probability that two of the digits of the number drawn are zero. (2 marks)

(b) Find the probability that none of the digits of the number drawn is zero. (2 marks)

(c) Find the probability that exactly one of the digits of the number drawn is zero. (2 marks)

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13. In Figure 6, $ABCDE$ is a regular pentagon and $CDFG$ is a square. BG produced meets AE at P.

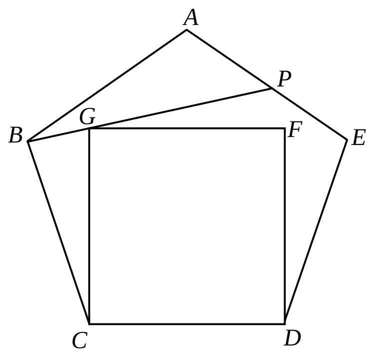

Figure 6

(a) Find $\angle BCG$, $\angle ABP$ and $\angle APB$. (5 marks)

(b) Using the fact that $\dfrac{AP}{\sin \angle ABP} = \dfrac{AB}{\sin \angle APB}$, or otherwise, determine which line segment, AP or PE, is longer. (3 marks)

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14. An auditorium has 50 rows of seats. All seats are numbered in numerical order from the first row to the last row, and from left to right, as shown in Figure 7. The first row has 20 seats. The second row has 22 seats. Each succeeding row has 2 more seats than the previous one.

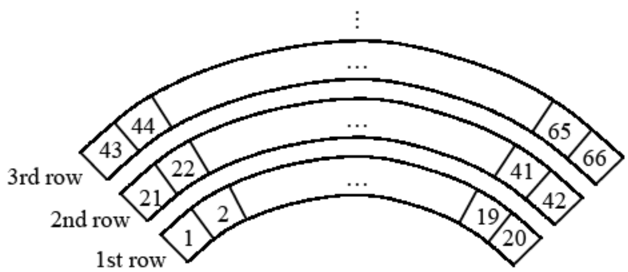

Figure 7

(a) How many seats are there in the last row? (2 marks)

(b) Find the total number of seats in the first n rows.

Hence determine in which row the seat numbered 2000 is located.

(4 marks)

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Page total

SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

15. A company produces two brands, A and B, of mixed nuts by putting peanuts and almonds together. A packet of brand A mixed nuts contains 40 g of peanuts and 10 g of almonds. A packet of brand B mixed nuts contains 30 g of peanuts and 25 g of almonds. The company has 2400 kg of peanuts, 1200 kg of almonds and 70 carton boxes. Each carton box can pack 1000 brand A packets or 800 brand B packets.

The profits generated by a box of brand A mixed nuts and a box of brand B mixed nuts are \$800 and \$1000 respectively. Suppose x boxes of brand A mixed nuts and y boxes of brand B mixed nuts are produced.

(a) Using the graph paper in Figure 8, find x and y so that the profit is the greatest. (8 marks)

(b) If the number of boxes of brand B mixed nuts is to be smaller than the number of boxes of brand A mixed nuts, find the greatest profit. (3 marks)

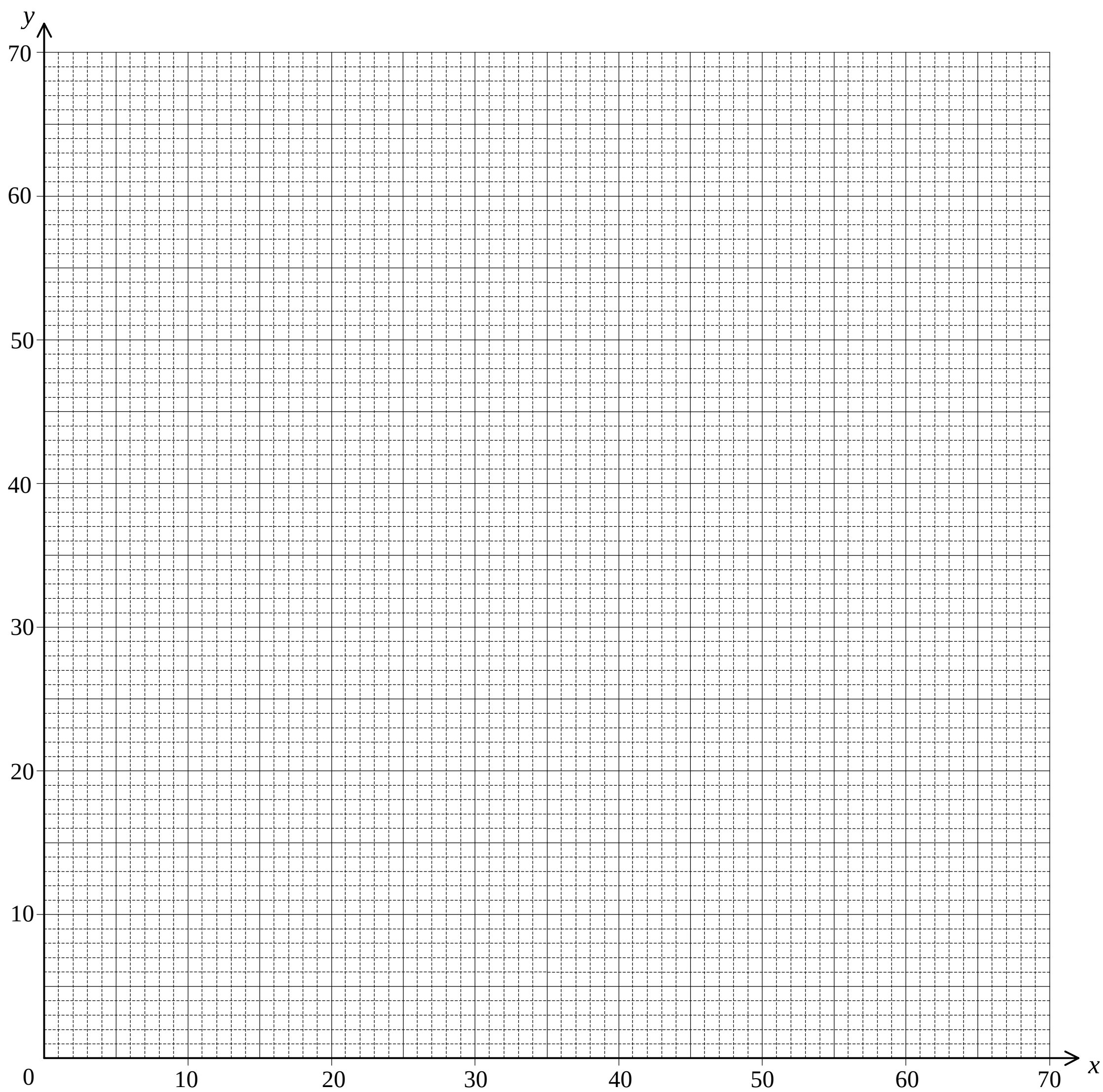

Figure 8

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16. In Figure 9, C is the centre of the circle PQS. OR and OP are tangent to the circle at S and P respectively. OCQ is a straight line and $\angle QOP = 30°$.

(a) Show that $\angle PQO = 30°$. (3 marks)

(b) Suppose $OPQR$ is a cyclic quadrilateral.

(i) Show that RQ is tangent to circle PQS at Q.

(ii) A rectangular coordinate system is introduced in Figure 9 so that the coordinates of O and C are $(0, 0)$ and $(6, 8)$ respectively. Find the equation of QR.

(8 marks)

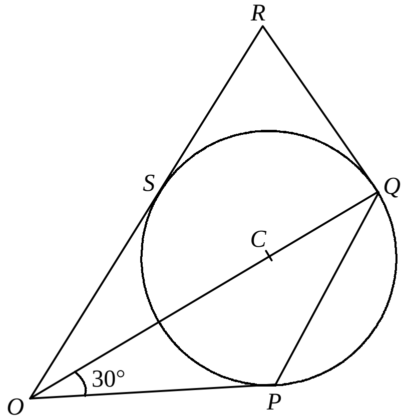

Figure 9

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17. Figure 10 shows a circle with centre O and radius 10 m on a vertical wall which stands on the horizontal ground. A, B and C are three points on the circumference of the circle such that A is vertically below O, $\angle AOB = 90^\circ$ and $\angle AOC = 20^\circ$. A laser emitter D on the ground shoots a laser beam at B. The laser beam then sweeps through an angle of 30° to shoot at A. The angles of elevation of B and A from D are 60° and 30° respectively.

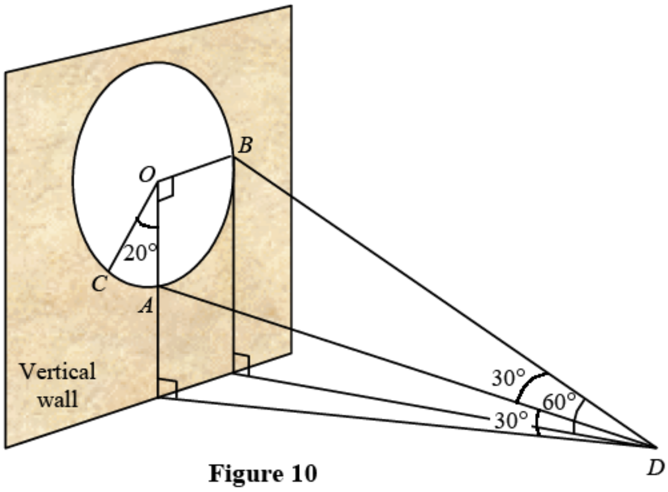

Figure 10

(a) Let A be h m above the ground.

(i) Express AD and BD in terms of h.

(ii) Find h.

(7 marks)

(b) Another laser emitter E on the ground shoots a laser beam at A with angle of elevation 25°. The laser beam then sweeps through an angle of 5° to shoot at C. Find $\angle ACE$.

(4 marks)

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18. Figure 11.1 shows a solid hemisphere of radius 10 cm. It is cut into two portions, P and Q, along a plane parallel to its base. The height and volume of P are h cm and $V\text{ cm}^3$ respectively.

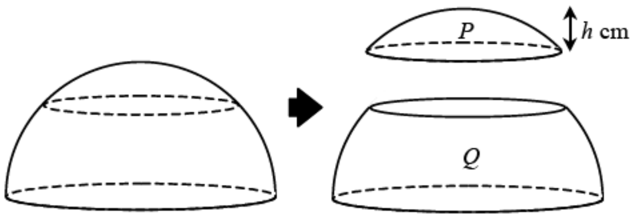

Figure 11.1

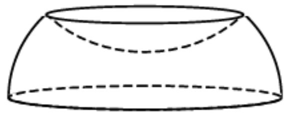

Figure 11.2

It is known that V is the sum of two parts. One part varies directly as h^2 and the other part varies directly as h^3. $V = \frac{29}{3}\pi$ when $h = 1$ and $V = 81\pi$ when $h = 3$.

(a) Find V in terms of h and π. (3 marks)

(b) A solid congruent to P is carved away from the top of Q to form a container as shown in Figure 11.2.

(i) Find the surface area of the container (excluding the base).

(ii) It is known that the volume of the container is $\frac{1400}{3}\pi\text{ cm}^3$. Show that $h^3 - 30h^2 + 300 = 0$.

Using the graph in Figure 11.3 and a suitable method, find the value of h correct to 2 decimal places.

(8 marks)

The graph of $y = x^3 - 30x^2$ for $0 \le x \le 5$

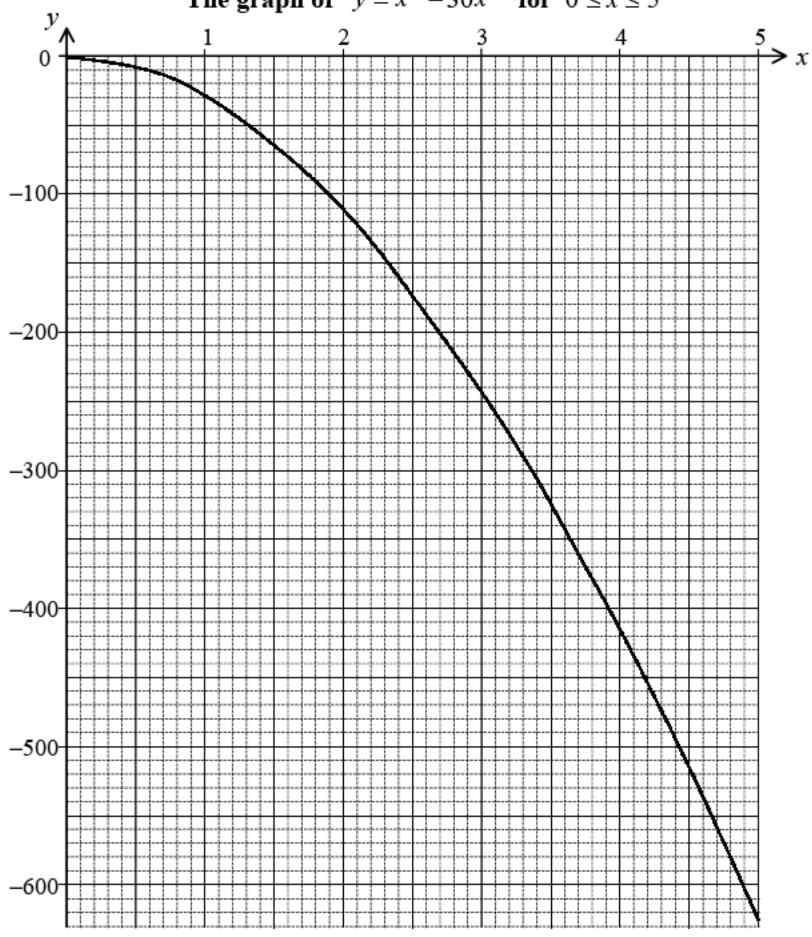

Figure 11.3

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END OF PAPER

2000

Mathematics 1
Section A(1)

1. 86

2. $\frac{y}{x^5}$

3. 23.6 cm^2

4. $a = \sqrt{51}$

   $x \approx 45.6$

5. $x > 3$

6. -1

7. $x = 25$

   $y = 74$

8. 550 000 m^2

9. (a) $-\frac{2}{5}$

   (b) $2x + 5y - 12 = 0$

   (c) $(0, \frac{12}{5})$

Section A(2)

10. (a) $x = -2$ or $\frac{11}{10}$

(b) $10000(1+r\%)^2 + 9000(1+r\%) = 22000$

$10(1+r\%)^2 + 9(1+r\%) - 22 = 0$

From (a), $1 + r\% = 1.1$

$r = 10$

11. (a) Missing value in 1st table = 66
Missing value in 2nd table = 20

(b) An estimate of the mean

$= \frac{210\times 3 + 230\times 13 + 250\times 30 + 270\times 20 + 290\times 9}{75}$ seconds

$\approx$ 255 seconds

(c) Median $\approx$ 254 seconds

(d) Percentage required $= \frac{13+30}{75}\times 100\% \approx 57.3\%$

12. (a) Probability required $= \frac{1}{10}\times\frac{1}{10} = \frac{1}{100}$

(b) Probability required $= \frac{9}{10}\times\frac{9}{10} = \frac{81}{100}$

(c) Probability required $= 1 - \frac{1}{100} - \frac{81}{100} = \frac{9}{50}$

13. (a) Size of each interior angle of the pentagon = $\dfrac{(5-2)\times 180°}{5}$ = 108°

$\angle BCG = 108° - 90° = 18°$

$\angle CBG = \dfrac{180° - 18°}{2} = 81°$

$\angle ABP = 108° - 81° = 27°$

$\angle APB = 180° - 27° - 108° = 45°$

(b) $\because \quad \dfrac{AP}{\sin 27°} = \dfrac{AB}{\sin 45°}$

$\therefore \quad AP = \dfrac{\sin 27°}{\sin 45°} AB = \dfrac{\sin 27°}{\sin 45°} AE \approx 0.642 AE$

$PE \approx (1 - 0.642) AE \approx 0.358\, AE$

$\therefore$ AP is longer than PE .

14. (a) Number of seats in the last row = $20 + 2(50-1) = 118$

(b) Total number of seats in the first n rows = $\dfrac{n}{2}[2\times 20 + 2(n-1)] = n^2 + 19n$

If $n^2 + 19n = 2000$, then

$n^2 + 19n - 2000 = 0$

$n = \dfrac{-19 \pm \sqrt{19^2 - 4(-2000)}}{2}$

$n \approx 36.2$ or -55.2

$\therefore$ The seat numbered 2000 can be found in the 37th row.

Section B

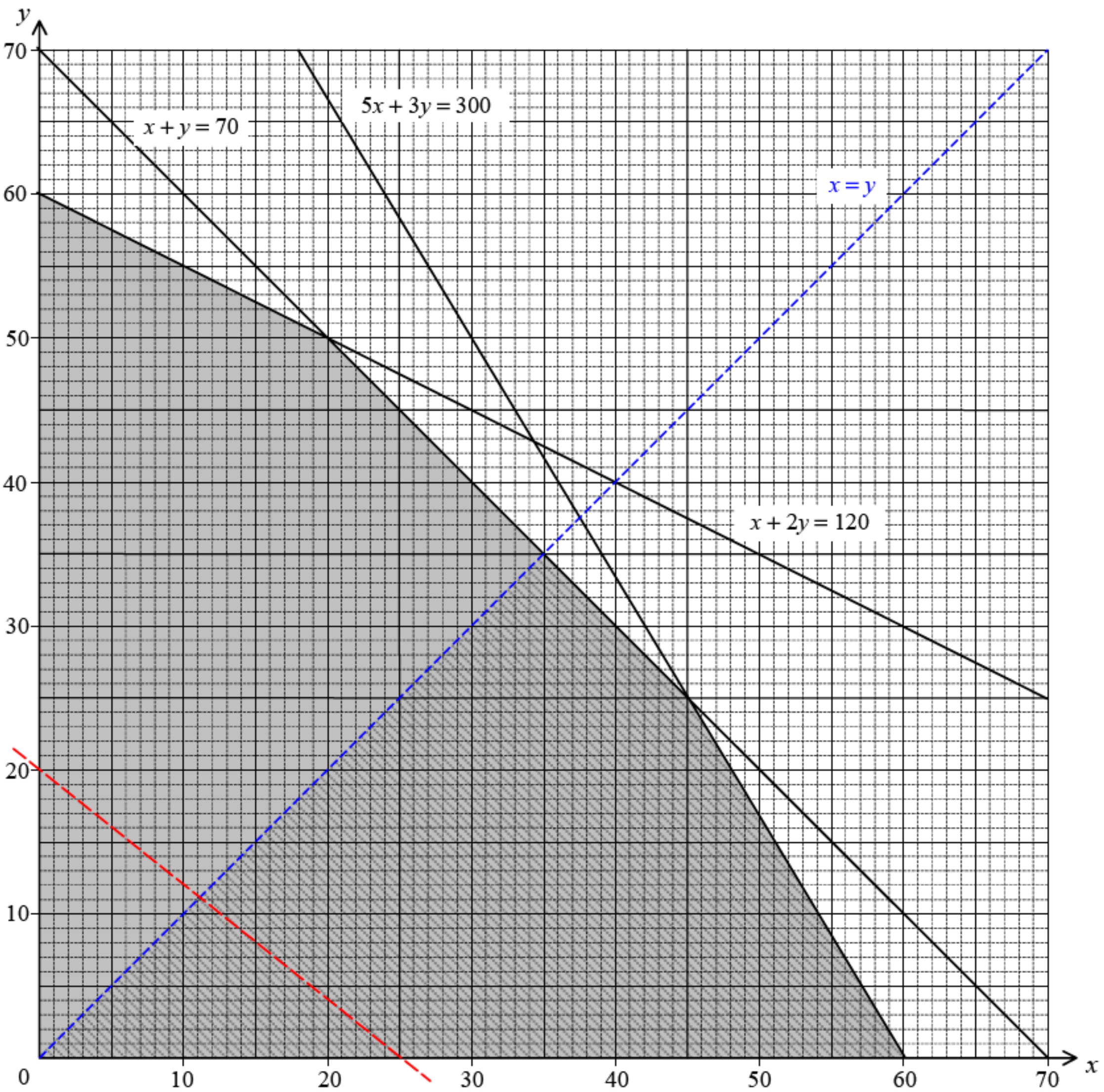
15. (a) x and y satisfy the following conditions:

$1000(40x)+800(30y)\le 2400000$ or $5x+3y\le 300$

$1000(10x)+800(25y)\le 1200000$ or $x+2y\le 120$

$x+y\le 70$

x , y are non-negative integers



Let $\$P(x, y)$ be the profit generated by x boxes of brand A mixed nuts and y boxes of brand B mixed nuts. Then

$$\begin{aligned} P(x, y) &= 800x + 1000y \\ &= 200(4x + 5y) \end{aligned}$$

By drawing parallel lines of $4x + 5y = 0$,
$P(x, y)$ attains its maximum at $(20, 50)$.
$\therefore$ The profit is the greatest when $x = 20$ and $y = 50$.

(b) In addition to the conditions in (a), x , y should also satisfy $y < x$.
By considering lines parallel to $4x + 5y = 0$
$P(x, y)$ attains its maximum at $(36, 34)$.
$\therefore$ The greatest profit is $62800.

16. (a) Join CP.

$\angle OPC = 90°$ (tangent ⊥ radius)

$\angle PCO = 180° - 90° - 30° = 60°$ (∠ sum of Δ)

$\angle PQO = \frac{1}{2}\angle PCO = 30°$ (∠ at centre twice ∠ at circumference)

(b) (i) $\angle ROQ = \angle QOP = 30°$ (tangents from ext. pt.)

$\angle PQO = 30°$ (proved)

∵ $\angle RQP + \angle POR = 180°$ (opp. ∠s of cyclic quad.)

∴ $\angle CQR = 180° - 3\times30° = 90°$

Hence RQ is tangent to circle PQS at Q. (conv. of tangent ⊥ radius)

(b) (ii) ∵ Slope of $OC = \frac{4}{3}$

∴ Slope of $QR = -\frac{3}{4}$

$OC = \sqrt{6^2+8^2} = 10$

$CQ = CP = OC\sin30° = 5$

Let the coordinates of Q be (x, y).

∵ $OC : CQ = 10 : 5 = 2 : 1$

∴ $\frac{2x+1(0)}{3} = 6$ and $\frac{2y+1(0)}{3} = 8$

$x = 9$ and $y = 12$

Hence the equation of QR is

$$\frac{y-12}{x-9} = -\frac{3}{4}$$

$$3x+4y-75=0$$

17. (a) (i) $AD = \dfrac{h}{\sin 30°}\ \text{m} = 2h\ \text{m}$

$BD = \dfrac{h+10}{\sin 60°}\ \text{m} = \dfrac{2}{\sqrt{3}}(h+10)\ \text{m} = \dfrac{2\sqrt{3}}{3}(h+10)\ \text{m}$

(ii) $AB^2 = (10^2 + 10^2)\ \text{m}^2$

By cosine law,

$$AB^2 = AD^2 + DB^2 - 2(AD)(DB)\cos\angle ADB$$

$$200 = \left(\frac{h}{\sin 30°}\right)^2 + \left(\frac{h+10}{\sin 60°}\right)^2 - 2\left(\frac{h}{\sin 30°}\right)\left(\frac{h+10}{\sin 60°}\right)\cos 30°$$

$$200 = 4h^2 + \frac{4}{3}(h+10)^2 - 4h(h+10)$$

$$h^2 - 10h - 50 = 0$$

$h \approx 13.660$ or -3.660

$h \approx 13.7$ or -3.66 (rejected)

(b) $AC = 2(10\sin 10°)\ \text{m} \approx 3.47296\ \text{m}$

$AE = \dfrac{h}{\sin 25°}\ \text{m} \approx 32.3\ \text{m}$

By sine law, $\sin\angle ACE = \dfrac{AE\sin 5°}{AC}$

$\approx \dfrac{h\sin 5°}{20\sin 10°\sin 25°}$

≈ 0.8112

$\therefore\ \angle ACE = 54.2°$ or $126°$

18. (a) Let $V = ah^2 + bh^3$ where a, b are non-zero constants.

$$\begin{cases} \frac{29}{3}\pi = a + b \\ 81\pi = 9a + 27b \end{cases} \quad \text{or} \quad \begin{cases} a + b = \frac{29}{3}\pi & \text{..........(1)} \\ a + 3b = 9\pi & \text{..........(2)} \end{cases}$$

(2) − (1) gives $2b = -\frac{2}{3}\pi$

Hence $b = -\frac{\pi}{3}$ and $a = 10\pi$

$\therefore \quad V = 10\pi h^2 - \frac{\pi}{3}h^3$

(b) (i) Surface area $= 2\pi \times 10^2$ cm^2 ≈ 628 cm^2

(ii) $\because$ Volume of hemisphere $= \frac{2}{3}\pi \times 10^3$ cm^3

$\therefore \quad \frac{2}{3}\pi \times 10^3 - 2V = \frac{1400}{3}\pi$

$$\frac{2}{3}\pi \times 10^3 - 2(10\pi h^2 - \frac{\pi}{3}h^3) = \frac{1400}{3}\pi$$

$$\frac{2}{3}\pi(1000 - 30h^2 + h^3 - 700) = 0$$

$$h^3 - 30h^2 + 300 = 0$$

From the graph in Figure 11.3, $3.3 < h < 3.4$

Let $f(h) = h^3 - 30h^2 + 300$, then $f(3.3) > 0$ and $f(3.4) < 0$.
Using the method of bisection,

Interval	**mid-value (m)**	**f(*m*)**
$3.3 < h < 3.4$	3.35	+ve (0.9204)
$3.35 < h < 3.4$	3.375	−ve (−3.2754)
$3.35 < h < 3.375$	3.363	−ve (−1.2583)
$3.35 < h < 3.363$	3.357	−ve (−0.2519)
$3.35 < h < 3.357$	3.354	+ve (0.2507)
$3.354 < h < 3.357$	3.356	−ve (−0.0843)
$3.354 < h < 3.356$	3.355	+ve (0.0832)

$\therefore \quad 3.355 < h < 3.356$

$h \approx 3.36$ (correct to 2 decimal places)