## 只限教師參閱 FOR TEACHERS' USE ONLY

## 香港考試局 HONG KONG EXAMINATIONS AUTHORITY

## 一九九七年香港中學會考 HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1997

### 數學 試卷一 MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫,供閱卷員參考之用。閱卷員在完成 閱卷工作後,若將本評卷參考提供其任教會考班的本科同事參閱,本局不表反對, 但須切記,在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取 此等文件,閱卷員/教師應嚴詞拒絕,因學生極可能將評卷參考視爲標準答案,以致 但知硬背死記,活剝生吞。這種落伍的學習態度,既不符現代教育原則,亦有違考 試着重理解能力與運用技巧之旨。因此,本局籲請各閱卷員/教師通力合作,堅守上 述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後,各科評卷參考將存放於教師中心,供教師參閱。
After the examinations, marking schemes will be available for reference at the Teachers' Centres.

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97-CE-MATHS I-1

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## Hong Kong Certificate of Education Examination Mathematics Paper I

#### NOTES FOR MARKERS

- 1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits all the marks allocated to that part, provided that the method used is sound.
- 2. In a question consisting of several parts each depending on the previous parts, marks may be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answers should NOT be awarded. In the marking scheme, marks are classified as:

'M' marks awarded for correct methods being used;
'A' marks awarded for the accuracy of the answers;
Others awarded for correctly completing a proof or arriving at an answer given in a question.

- 3. Use of notation different from those in the marking scheme should not be penalised.
- 4. Each mark deducted for poor presentation (p.p.) should be denoted by [pp-1]:
  - a. At most deduct 1 mark for (p.p.) in each question, up to a maximum of 3 marks for the whole paper.
  - b. For similar (p.p.), deduct 1 mark for the first time that it occurs.
    i.e. do not penalise candidates twice in the paper for the same p.p.
- 5. Each Mark deducted for wrong/no unit (u.) should be denoted by [u-1]:
  - a. No mark can be deducted for (u.) in Section A.
  - b. At most deduct 1 mark for (u.) for the whole paper.
- 6. Marks entered in the Page Total Box should be the NET total scored on that page.

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	Solution	Marks	Remarks
1. <b>(a</b> )	$x^2 - 9 = (x - 3)(x + 3)$	2A	
<b>(b</b> )	ac + bc - ad - bd = (a+b)c - (a+b)d $= (a+b)(c-d)$	1A 1A (4)	
2. (a)	$\sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3}$ $= \sqrt{3}$	1A 1A	For simplifying either term
<b>(b</b> )	$\frac{1}{2\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}-\sqrt{2}}{(2\sqrt{3}+\sqrt{2})(2\sqrt{3}-\sqrt{2})}$	1A	
	$=\frac{2\sqrt{3}-\sqrt{2}}{(2\sqrt{3})^2-(\sqrt{2})^2}$	1A	can be omitted
	$= \frac{2\sqrt{3} - \sqrt{2}}{10} \qquad \text{(or } \frac{\sqrt{3}}{5} - \frac{\sqrt{2}}{10}, \frac{\sqrt{2}(\sqrt{6} - 1)}{10})$	1A (5)	
3. <b>(a</b> )	$\frac{x^3y^2}{x^{-3}y} = x^{3-(-3)}y^{2-1}$	lM	For applying $a^m a^n = a^{m+n}$ ,
	$=x^6y$	1A	$\frac{a^m}{a^n} = a^{m-n} \text{ or } \frac{1}{a^n} = a^{-n}$
(b	$\frac{\log 8 + \log 4}{\log 16} = \frac{\log 2^3 + \log 2^2}{\log 2^4}$	1M	For expressing the numbers as powers of a common number
	$= \frac{3 \log 2 + 2 \log 2}{4 \log 2}$	1M	For applying $\log a^n = n \log a$
	$\frac{\frac{OR}{\log 8 + \log 4}}{\log 16} = \frac{\log 32}{\log 16}$		
-	$= \frac{\log 2^5}{\log 2^4}$ $= \frac{5\log 2}{4\log 2}$	1M	
		1M	
	$=\frac{5}{4}$ (or 1.25)	(5)	
÷			

Solution	Marks	Remarks
<ul> <li>Note: In question 4, accept graphical solutions if no algebraic expressions as answers are provided. Withhold 1 mark for having equal signs in inequalities.</li> <li>4. (i) 2x - 17 &gt; 0         x &gt; 17/2     </li> </ul>	1A	
(ii) $x^2 - 16x + 63 > 0$ (x - 7)(x - 9) > 0 x < 7 or $x > 9$	1A 2A	For factorization, can be omitted
The range of values of $x$ which satisfy both the inequalities in (i) and (ii): x > 9	<u>1A</u> (5)	
5. (a) $AC = 5$ (b) $AD = \sqrt{5^2 + 6^2 - 2(5)(6)(\cos 60^\circ)}$ $= \sqrt{31}$ (or 5.57) (c) Area of $\triangle ACD = \frac{1}{2}(5)(6)\sin 60^\circ$ $= \frac{15}{2}\sqrt{3}$ (or 13.0)	1A 1M 1A 1M	For the cosine rule r.t. 5.57 r.t. 13.0

	Solution		Marks	Remarks
6.	A 140°  N 20 km  L 110°			
(a)	$\angle LAB = 180^{\circ} - 140^{\circ} + 20^{\circ} = 60^{\circ}$ $\therefore \angle ALB = 110^{\circ} - 20^{\circ} = 90^{\circ}$ $\therefore \Delta ALB \text{ is right-angled at } L$	or $\angle LBA = 30^{\circ}$ )	1A	
	$LB = 20 \sin 60^{\circ} \text{ km}$	or 17.3 km)	1M 1A	r.t. 17.3
<b>(b)</b>	∠ABL = 30° Let $\phi$ be the bearing of L from B. Then $\phi = 360^{\circ} - 30^{\circ} - 40^{\circ} = 290^{\circ}$ ∴ The bearing of L from B is 290°. (or		1M 1A (5)	
7. (a)	The height of the smaller cone: the height = 2:3	of the larger cone	1A	
(b)	Total surface area of the smaller cone: total = 4:9	al surface area of the larger cone	1M	
	The cost of painting the larger cone = $$32 \times$	9	1M	
	= \$72	•	1A (4)	
8. (a)	$\alpha + \beta = \frac{7}{2}$		1A	
	$\alpha\beta=2$		1A	
(b)	$(\alpha + 2) + (\beta + 2) = (\alpha + \beta) + 4$ = $\frac{7}{2} + 4$			
	$=\frac{15}{2}$		1A	
	$(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$ = $(2) + 2(\frac{7}{2}) + 4$		1M	
	= 13		IA	
		26 = 0 .	1A	Or equivalent

4	Solution	Marks	Remarks
9.	A 4 cm C		
(a)	$\angle BDC = 30^{\circ}$ $\angle ADC = 90^{\circ}$ $\angle ADB = 90^{\circ} - 30^{\circ}$ $= 60^{\circ}$	1A 1A 1A	can be omitted
(b)	$\stackrel{\frown}{AB}:\stackrel{\frown}{BC} = 60:30$ = 2:1	1A 1M	Accept 2  For finding $AB$ and $BC$
(c)	$AB:BC = 4\cos 30^{\circ}: 4\sin 30^{\circ}$ (or $\tan 60^{\circ}$ ) = $\sqrt{3}:1$ (or $1.73:1, 1:0.577$ )	1A (6)	Accept √3 etc.  Numerical ans. r.t. 1.73, 0.577
10. (a)	Population at the end of $1998 = 300000(1+2\%)^2$ = 312 120	1A 1A	r.t. 312000
	OR Population at the end of $1997 = 300\ 000(1+2\%) = 306\ 000$ Population at the end of $1998 = 306\ 000(1+2\%) = 312\ 120$	1M+1A	r.t. 312000
(b)	If $300000(1+2\%)^n = 330000$ , then $1.02^n = 1.1$ $n \log 1.02 = \log 1.1$ $n \approx 4.81$ $\therefore$ The population will exceed 330 000 at the end of 2001.	1A 1M 1A 1A	Accept n = 5
	OR  Population at the end of $1999 = 300.000(1+2\%)^3 \approx 318362$ Population at the end of $2000 = 300000(1+2\%)^4 \approx 324730$ Population at the end of $2001 = 300000(1+2\%)^5 \approx 331224$ ∴ The population will exceed 330000 at the end of 2001.	1M 1A 1A 1A	1M for calculating the populations of any two years r.t. 325 000 r.t. 331 000
		(6)	·
			en e

		Solution	Marks	Remarks
11 (a)	(i)	Mean = 64.4	1A	r.t. 64.4
( <del>u</del> )		Mode = 95	1A	
		Median = 78	1A	
		Standard deviation = 30.6	1A	r.t. 30.6
(b)		is because the distribution of marks in the Mathematics test ased (to the high end).	1	
(c)	(i)	Let the student scored x marks in the English test.		
		$\frac{x-63}{15} = 0.4$	1A	
		x = 69	1A	
	(ii)	(I) Percentage of classmates scored fewer marks than Lai Wah in the Mathmatics test $= \frac{17}{35} \times 100\%$		
		$\approx 48.6\%$ (or $48\frac{4}{7}\%$ )	1A	r.t. 48.6
		7  (II) The standard score of Lai Wah in the English test  =\frac{78-63}{15}  = 1  \therefore The marks of the English test is normally distributed \therefore More than half (or about 84%) of her classmates scored	1A	Or 84%
		less than her.  Hence Lai Wah performed better in the English test than in the Mathematics test relative to her classmates.	1	
		OR (II) The standard score of Lai Wah in the English test $= \frac{78-63}{15}$ = 1 The standard score of Lai Wah in the Mathematics test $= \frac{78-64.4}{30.6}$ ≈ 0.44 ∴ Lai Wah performed better in the English test than	1A	
		in the Mathematics test relative to her classmates.	1	_
	(iii	The mean of the marks in the English test after the wrong mark has been corrected $= 63 + \frac{10}{35} \qquad (\text{ or } \frac{63 \times 35 + 10}{35})$ $\approx 63.3$	1A 1A	r.t. 63.3

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		Solution	Marks	Remarks
12.		h m		
	(a)	(i) $VN = 3\tan\theta$ m	1A	
		$VM = \frac{3}{\cos \theta}  \text{m} \qquad (\text{ or } 3\sqrt{1 + \tan^2 \theta} \text{m})$	1A	
		(ii) Capacity = $\frac{1}{3} \cdot 6^2 \cdot 3 \tan \theta$ m <sup>3</sup> (1)	1M	For either (1) or (2)
		$= 36 \tan \theta \text{ m}^3$	1A	
-		Total surface area = $4 \cdot \frac{6}{2} \cdot \frac{3}{\cos \theta} \text{ m}^2$ (2) = $\frac{36}{\cos \theta} \text{ m}^2$ (or $\frac{36}{\sqrt{1 + \tan^2 \theta}} \text{ m}^2$ )	1A	
	(b)	(i) $\therefore$ The base areas of the greenhouses are the same $\therefore \pi r^2 = 36$ $r = \frac{6\sqrt{\pi}}{\pi} \qquad (\text{ or } \frac{6}{\sqrt{\pi}})$ (ii) $\therefore$ The capacities of the greenhouses are the same	1A	
		$\therefore 36h = 36\tan\theta  (\text{ or } \pi \left(\frac{6}{\sqrt{\pi}}\right)^2 h = 36\tan\theta )$	1M	
		• **		
		$h = \tan \theta$	1A	
		(iii) If the total surface areas of the greenhouses are equal, then $\pi r^2 + 2\pi r h = \frac{36}{\cos \theta}$	1M	
		$36 + 2\pi \cdot \frac{6}{\sqrt{\pi}} \cdot \tan \theta = \frac{36}{\cos \theta}$ $36 + 12\sqrt{\pi} \tan \theta = \frac{36}{\cos \theta}$	1141	
	•	$3 + \sqrt{\pi} \tan \theta = \frac{3}{\cos \theta}$	1	
		(iv) : $3 + \sqrt{\pi} \tan 61^{\circ} - \frac{3}{\cos 61^{\circ}} (\approx 0.00960) > 0$ $3 + \sqrt{\pi} \tan 62^{\circ} - \frac{3}{\cos 62^{\circ}} (\approx -0.0567) < 0$	} 1M+1A	r.t. 0.01 r.t0.06
		∴ (*) has a root between 61° and 62°.		

Solution	Marks	Remarks
<ul> <li>13. (a) (i) From the graph, y is minimum when x = 10</li> <li>∴ Number of belts in a batch = 10</li> </ul>	1A	
(ii) From the graph, $y < 90$ when $x \ge 2$	1M	Accept $x > 1.6$ , $x \ge 1.6$ or $x = 2, 3, 4,$
i.e. $x = 2, 3,, 11$ $\therefore$ Number of belts in a batch = 2, 3, 4,, 11	1A	Accept $2 \le x \le 11$
(b) (i) $144 = 3^2 - 17(3) + c$ , $c = 186$	1A	
(ii) If $H = 120$ , then $x^2 - 17x + 186 = 120$	1M	
$x^2 - 17x + 66 = 0$		
$x^2 - 20x + 120 = -3x + 54$		
By adding the line $y = -3x + 54$ on the graph,	1A	
110-1		
80		
50	1A	$x = 6 \pm 0.2$ , $11 \pm 0.2$
30		
20		
y = -3x + 5	54	
$0 \frac{2}{2} + \frac{4}{6} + \frac{6}{8} + \frac{10}{12} + \frac{14}{16} \times x$ $x = 6 \text{ or } 11 \text{ (rej.)}$	1A+1A	
The required number of handbags is 6.  (iii) Total cost of 10 belts and 6 handbags $= \$[10 \times (10^2 - 20 \times 10 + 120) + 6(6^2 - 17 \times 6 + 186)]$ $= \$[10 \times 20 + 6 \times 120]$	1A	
= \$920 Total income for selling the belts and handbags = $\{6 \times 100 + 4 \times 300 + 4 \times 10 + 2 \times 60\}$	1A	
61060		I
= \$1960 ∴ She gained \$1040.	1A	

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	Solution	Marks	Remarks
14. (a)	(i) $P(0 < T \le 200) = \frac{40}{50} \cdot \frac{39}{49}$	1A	
	$=\frac{156}{245}$ (or 0.637)	1A	r.t. 0.637 (Ref. A <sub>1</sub> )
	(ii) $P(500 \le T \le 700) = \frac{10}{50} \cdot \frac{40}{49} + \frac{40}{50} \cdot \frac{10}{49}$ = $2 \cdot \frac{10 \times 40}{50 \times 49}$	1M+1A	1M for $p_1p_2 + p_2p_1$ 1A for either term
	$= \frac{16}{49} $ (or 0.327)	1A	r.t. 0.327 (Ref. A <sub>2</sub> )
	(iii) $P(1000 \le T \le 1200) = \frac{10}{50} \cdot \frac{9}{49}$	1A	
	$= \frac{9}{245} $ (or 0.0367)	1A	r.t. 0.0367 (Ref. A <sub>3</sub> )
_	(iv) $P(T>1200) = 0$	1A	Accept 'impossible', 'no chance'
(b)	Let the total weight obtained in the afternoon be $T'$ . (i) $P(T' < 450 \text{ or } T' > 850)$		
	$=\frac{156}{245}+\frac{9}{245}$	1M	For $A_1 + A_3$
	$= \frac{33}{49} $ (or 0.673)	1A	r.t. 0.673
	(ii) $P( T-T'  > 200)$		
	$=1-\left(\frac{156}{245}\right)^2-\left(\frac{16}{49}\right)^2-\left(\frac{9}{245}\right)^2$	1M	For $1 - A_1^2 - A_2^2 - A_3^2$
	$=\frac{29208}{60025}$ (or 0.487)	1A	r.t. 0.487
	$\frac{OR}{=\frac{156}{245}\left(\frac{16}{49} + \frac{9}{245}\right) + \frac{16}{49}\left(\frac{156}{245} + \frac{9}{245}\right) + \frac{9}{245}\left(\frac{156}{245} + \frac{16}{49}\right)}$	1M	
<u> </u>	$=\frac{29208}{60025} \qquad \text{(or } 0.487)$	1A	r.t. 0.487
	,		
		1	1

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	Solution			Marks	Remarks	
15. (a) A <sub>1</sub>	$A_2$ $\frac{\ell}{3}$	; ;	A <sub>3</sub>			,
	Table 1 $A_1 \rightarrow A$ of squares added3of sides of the added $\frac{\ell}{3}$	$ \begin{array}{c c} A_2 \to A_3 \\ \hline 9 \\ \hline \frac{\ell}{9} \end{array} $	$ \begin{array}{c c} A_3 \rightarrow A_4 \\ \hline 27 \\ \hline \frac{\ell}{27} \end{array} $	1A 1A		
	ea of all the squares in $A_4$ $(-1)^2 + 9(\frac{\ell}{9})^2 + 27(\frac{\ell}{27})^2$ $(-1)^2 + \frac{1}{9}(\frac{\ell}{9})^2 + \frac{1}{9}(\frac{\ell}{9})^2$			1A+1M	1A for the first three terms 1M for the 4th term	_
$=\frac{40}{27}\ell^2$	, 2,	or $1.48\ell^2$ )		1A	r.t. 1.48	
(iii) $k = (1 + \frac{1}{3})$ = $\frac{1}{1 - \frac{1}{3}}$ = $\frac{3}{2} \ell^2$		or $15\ell^2$ )	·	1A 1A		
(b) $B_1$	$B_2$		B <sub>3</sub>			, <del>-</del>
(i) Table 2 Perimet			B₄ 10ℓ	1A+1A+1A		
(ii) Perime $= 4\ell + (n - 2\ell)$ $= 2(n + 1)\ell$		$2n\ell + 2\ell$ )		1A		
The perimindefinite	neter of $B_n$ would tend to infinity.	inity if n increa	ises	1A		
					·	

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		Solution	Marks	Remarks
16. (a)		E $C$ $B$		
	(i)		[2] [1]	[同側(旁)內角互補]
	(ii)	Case 3 Any relevant correct argument with correct reason	[1] .	[對頂角] [等腰△底角] [(內)錯角,AB//EF]  Or "base ∠s equal", "converse of 'base ∠s, isos. △'", "equal ∠s, equal sides" [等角對邊相等] 或 [等腰三角形底角等的逆定理] 或 [底角相等] 或 [等邊對等角] 或 [等角對等邊]
<u> </u>	,			[對頂角] [(內)錯角, <i>AB//EF</i> ] Or "AAA" [等角]
		Marking Scheme Case 1 Any correct proof with correct reasons Case 2 Any correct proof without correct reasons In addition, any relevant correct argument with correct reason Case 3 Any relevant correct argument with correct reason	4 [4] [2] [1]	

Solution	Marks	Remarks
(iii) Let $\mathcal{C}$ be the circle passing through $D$ and touching $AE$ at $E$ .	IVIGIRO	AWARMAND
$\mathcal{C}$ touches $AE$ at $E$ and $EF \perp AE$ . $\therefore$ the centre of $\mathcal{C}$ lies on the line $EF$ .	1	Pointing out $EF \perp AE$ for $AE$ touching $\mathcal{C}$
: $ED$ is a chord of $\mathcal{C}$ and $FD = FE$ . : the centre of $\mathcal{C}$ lies on the perpendicular of $DE$ through $F$ .	1	Pointing out $FD = FE$ for $F$ as centre or $FD$ , $FE$ as radii
$F$ is the intersection of the lines which is the centre of $\mathcal C$ .		
ACCEPT Consider the circle with $F$ as centre and $FD$ as radius. $FD = FE$		
$\therefore$ the circle passes through $D$ and $E$ .	1	
∴ $EF \perp AE$ and $EF$ is a radius ∴ the circle touches $AE$ at $E$ .	1	
<sup>, y</sup> ↑		
F B(6,3)		
E(-4,3) (-2,3)		
c		
A(-2,-1) O		
Mid-point of $DE = (-3, 3)$	1M	For any correct method of finding the x-coordinate of I
∴ ED is horizontal ∴ x-coordinate of $F = -3$	1A	mining the x-continue of t
	14	
Slope of $AE = -2$ Equation of $EF: \frac{y-3}{x+4} = \frac{1}{2}$		
Equation of $EF$ : $\frac{x+4}{x+4} = \frac{1}{2}$ x-2y+10=0		
·	1M	For any correct method of
Sub. $x = -3$ into $EF$ ,	1141	finding the y-coordinate of h
-3 - 2y + 10 = 0	1.4	
$y = \frac{7}{2}$	1A	
$\therefore F = (-3, \frac{7}{2})$		
Note: Candidate may use equations of other straight lines for finding the coordinates of $F$ :		
EF   : x - 2y + 10 = 0		For correct mehods of finding
CD   : x + 2y - 4 = 0 Paramenticular from E to DE $ : x = -3$	} 1M	any two of them
Perpendicular from $F$ to $DE : x = -3$ Solving any two of the above equations simultaneously	134	
Solving any two of the above equations simultaneously.  Obtaining $F = (-3, \frac{7}{2})$ .	1M 1A+1A	
( Metaining L' = 1 2 1 1	INTIN	1