香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 一 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

數學(課程一) 試卷一

MATHEMATICS (SYLLABUS 1) PAPER I

二小時完卷

上午八時三十分至上午十時三十分

本試卷必須用英文作答

Two hours

8.30 a.m.—10.30 a.m.

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.
Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3} \times$ base area $\times$ height

SECTION A **Answer ALL questions in this section.**
There is no need to start each question in this section on a fresh page.

1. The capacities of two spherical tanks are in the ratio $27:64$. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?

(5 marks)

2. Find a quadratic equation whose roots are

$$2+3i \text{ and } 2-3i \text{ where } i=\sqrt{-1}.$$

Express your answer in the form $x^2+bx+c=0$ where b and c are real numbers.

(5 marks)

3. There are 40 students in a class, including students A and B. If two students are to be chosen at random as class representatives, find the probability that both A and B are chosen.

(5 marks)

4. Solve $\cos(200^\circ+\theta)=\sin 120^\circ$ where $0^\circ < \theta < 180^\circ$.

(6 marks)

5. Solve $4^x = 10 - 4^{x+1}$.

(6 marks)

6. Figure 1 shows the cumulative frequency polygon of the marks obtained by 100 students taking a mathematics test.

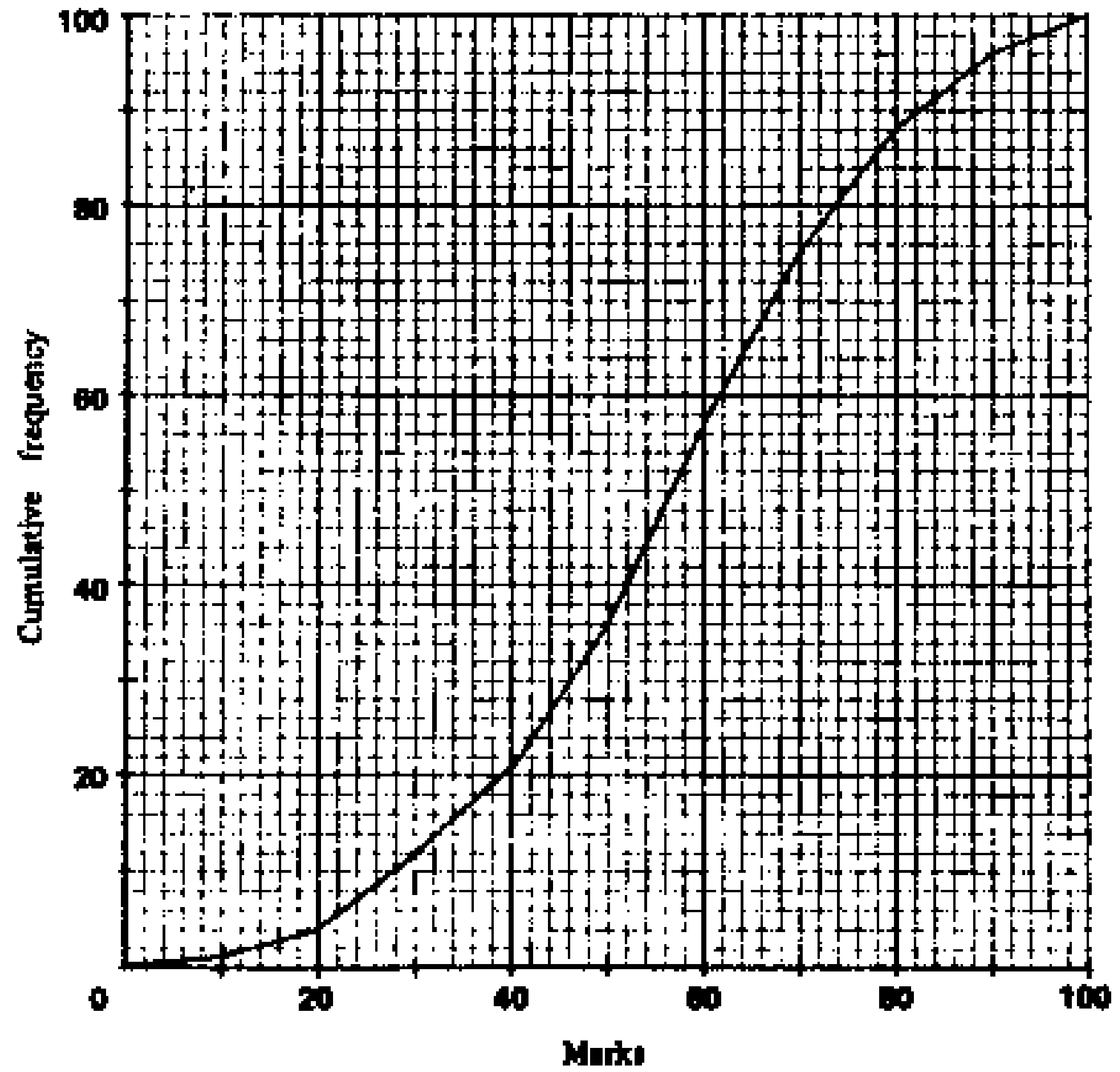


Figure 1

(a) If 75% of the students pass the test, what is the pass mark, correct to the nearest integer ?

(b) If the pass mark were 40 , how many students would pass the test ?

(c) Find the inter-quartile range.

(6 marks)

7. In a class of 42 students, 28 have been to Ocean Park and 34 have been to the Space Museum.

(a) Find the least number of students who have been to both Ocean Park and the Space Museum.

(b) If 7 of the 42 students have never been to Ocean Park or the Space Museum, find the number of students who have been to both places.

(6 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

SECTION B Answer FIVE questions in this section. Each question carries 12 marks.

8. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.

An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions :

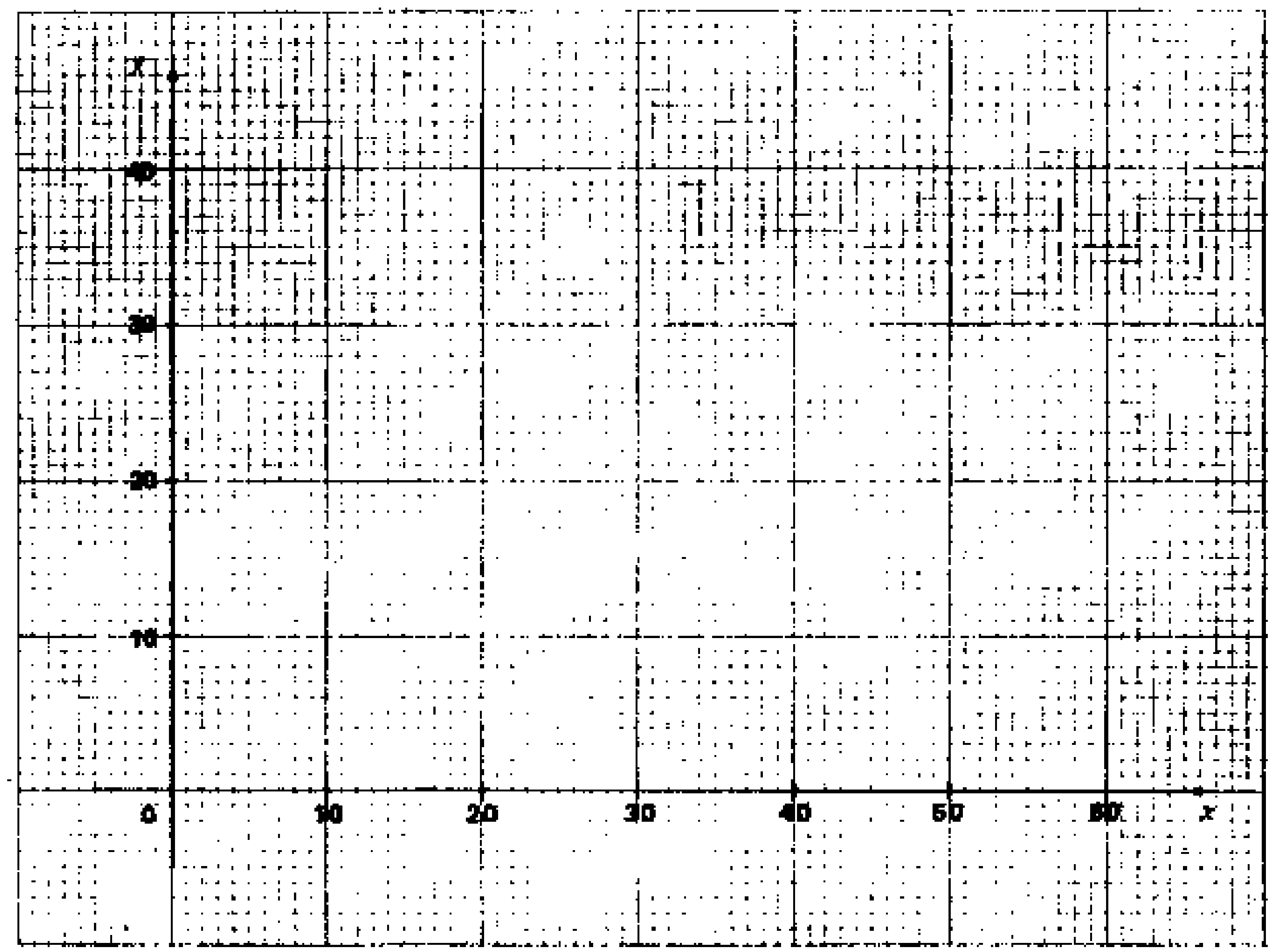
(1) The hostel will accommodate at least 48 persons.

(2) Each single room will occupy an area of 10 m^2 , each double room will occupy an area of 15 m^2 and the total available floor area for the rooms is 450 m^2

(3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)



9. Normally, a factory produces 400 radios in x days. If the factory were to produce 20 more radios each day, then it would take 10 days less to produce 400 radios. Calculate x.

(12 marks)

10.

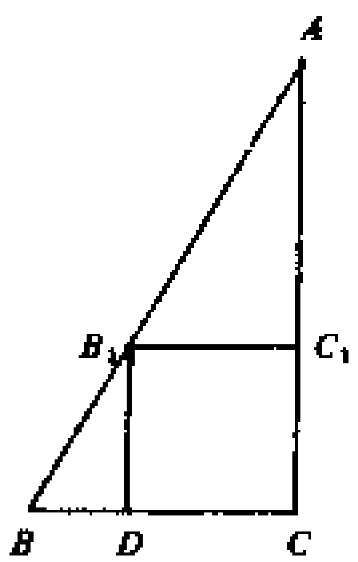


Figure 2(a)

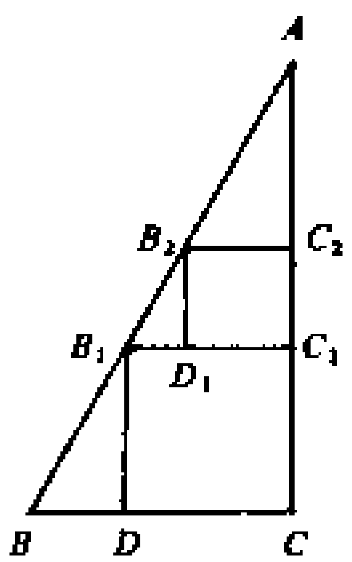


Figure 2(b)

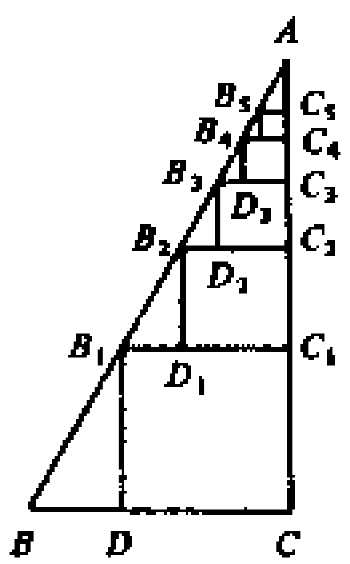


Figure 2(c)

In Figure 2(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $\angle C = 90°$, $BC = a$, $AC = 2a$, $B_1C_1 = b$.

(a) Express b in terms of a.

(3 marks)

(b) $B_2C_2C_1D_1$ is a square inscribed in $\triangle AB_1C_1$ (see Figure 2(b)).

(i) Express B_2C_2 in terms of b.

(ii) Hence express B_2C_2 in terms of a.

(2 marks)

(c) If squares $B_3C_3C_2D_2$, $B_4C_4C_3D_3$, $B_5C_5C_4D_4$, ... are drawn successively as indicated in Figure 2(c),

(i) write down the length of B_5C_5 in terms of a,

(ii) find, in terms of a, the sum of the areas of the infinitely many squares drawn in this way.

(7 marks)

Go on to the next page

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11. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.**

A piece of wire 20 cm long is bent into a rectangle. Let one side of the rectangle be x cm long and the area be y cm^2.

(a) Show that $y = 10x - x^2$.

(2 marks)

(b) Figure 3 shows the graph of $y = 10x - x^2$ for $0 \leq x \leq 10$.
Using the graph, find
(i) the value of y correct to 1 decimal place, when $x = 3.4$,
(ii) the values of x correct to 1 decimal place, when the area of the rectangle is 12 cm^2,
(iii) the greatest area of the rectangle,
(iv) the value of x for which y is three times x, by drawing a suitable line on the graph.

(10 marks)

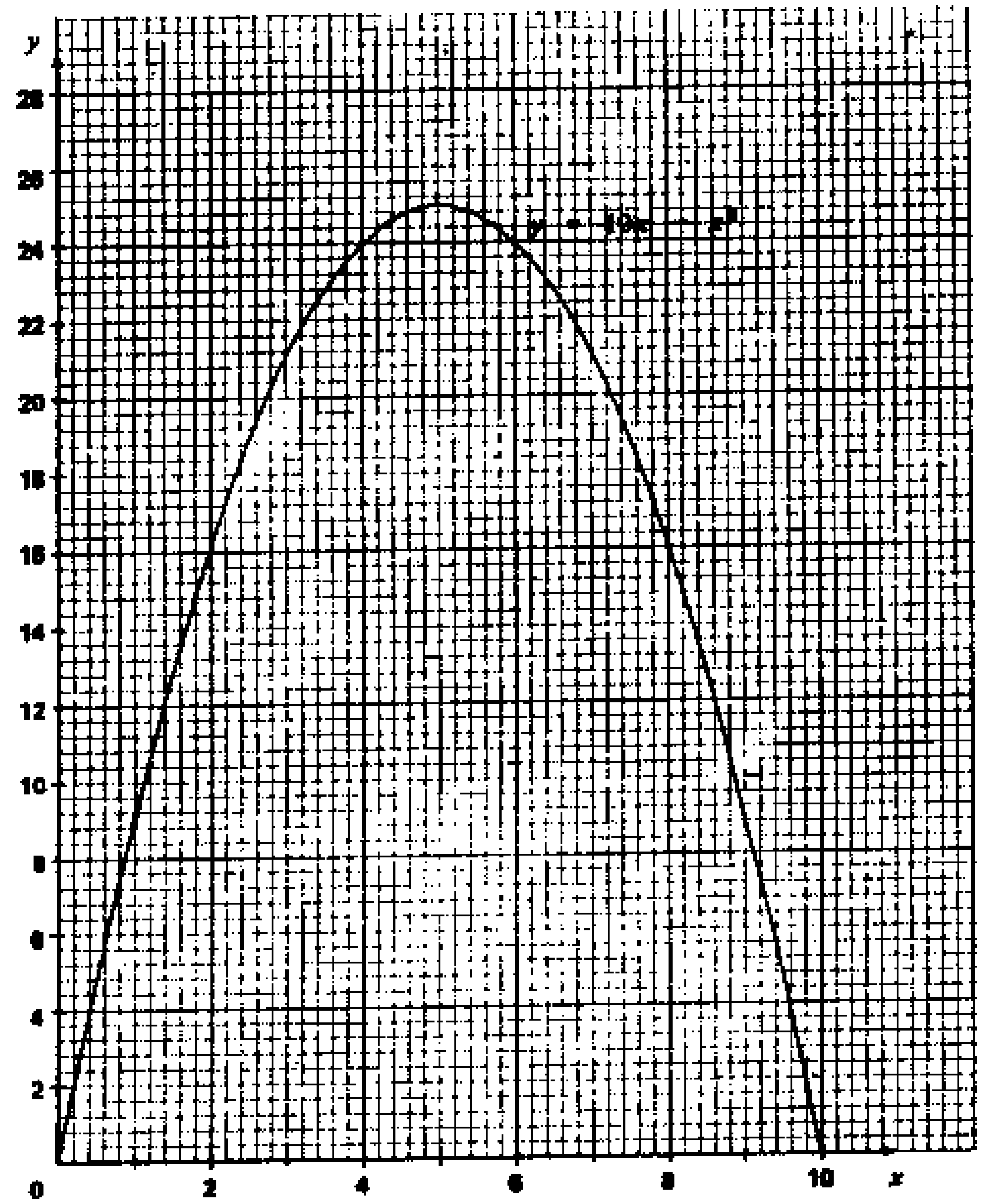

Figure 3

12. Figure 4 shows a cylinder 10 metres high and 10 metres in radius used for storing coal-gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 2.4 radians at the point O, which is the centre of the top of the cylinder.

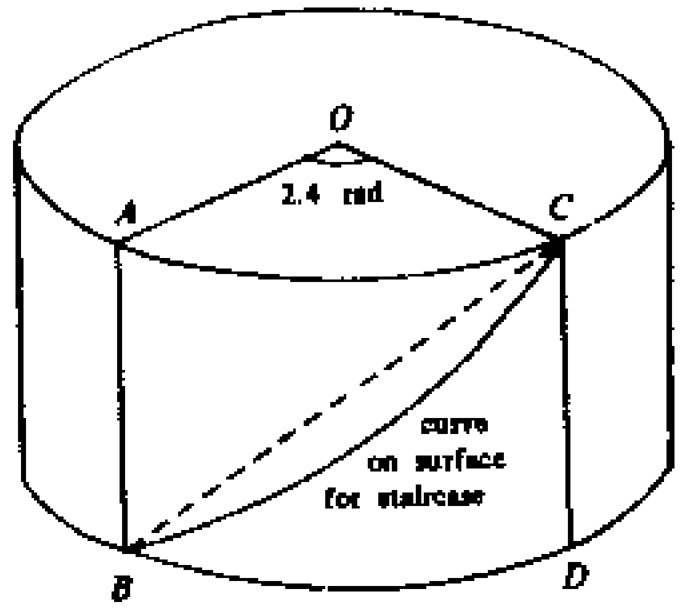

Figure 4

(a) Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures. (5 marks)

(b) Calculate the area of the curved surface $ABDC$ bounded by the minor arcs AC, BD and the lines AB, CD. (3 marks)

(c) A staircase from B to C is built along the shortest curve on the curved surface $ABDC$. Find the length of the curve. (4 marks)

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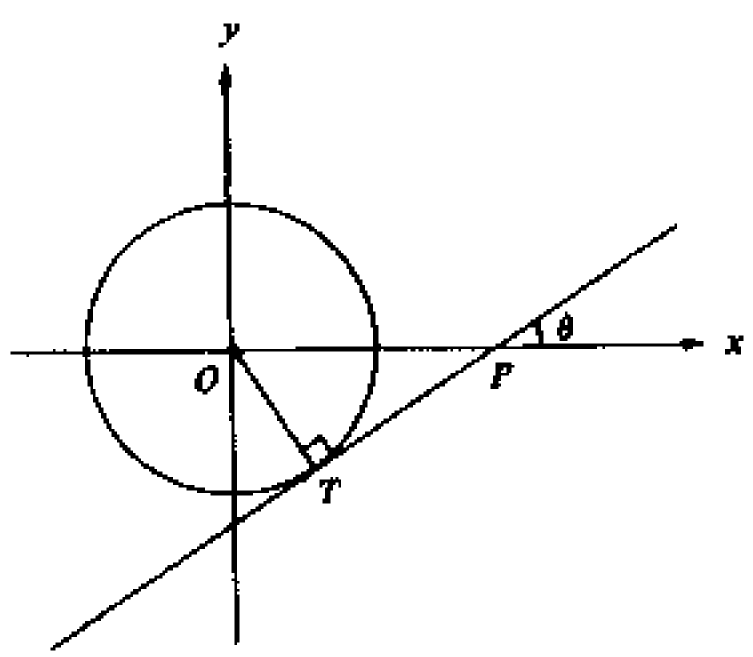

Figure 5(a)

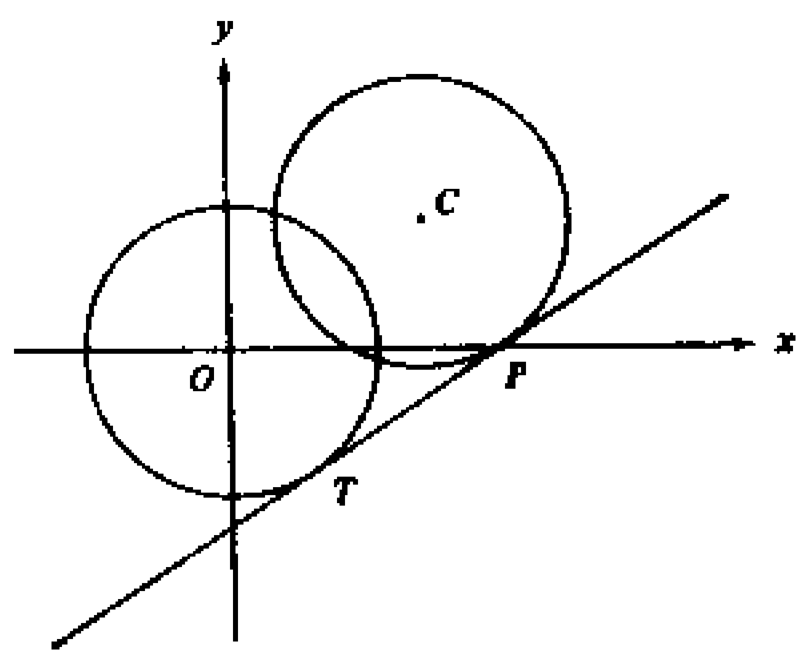

Figure 5(b)

Figure 5(a) shows a circle of radius 15 with centre at the origin O. The line TP, of slope $\frac{3}{4}$ ($= \tan\theta$), touches the circle at T and cuts the x-axis at P.

(a) Find the equation of the circle. (1 mark)

(b) Calculate the length of OP. (3 marks)

(c) Find the equation of the line TP. (2 marks)

Another circle, with centre C and radius 15, is drawn to touch TP at P (see Figure 5(b)).

(d) Find the equation of the line OC. (1 mark)

(e) Find the equation of the circle with centre C. (5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

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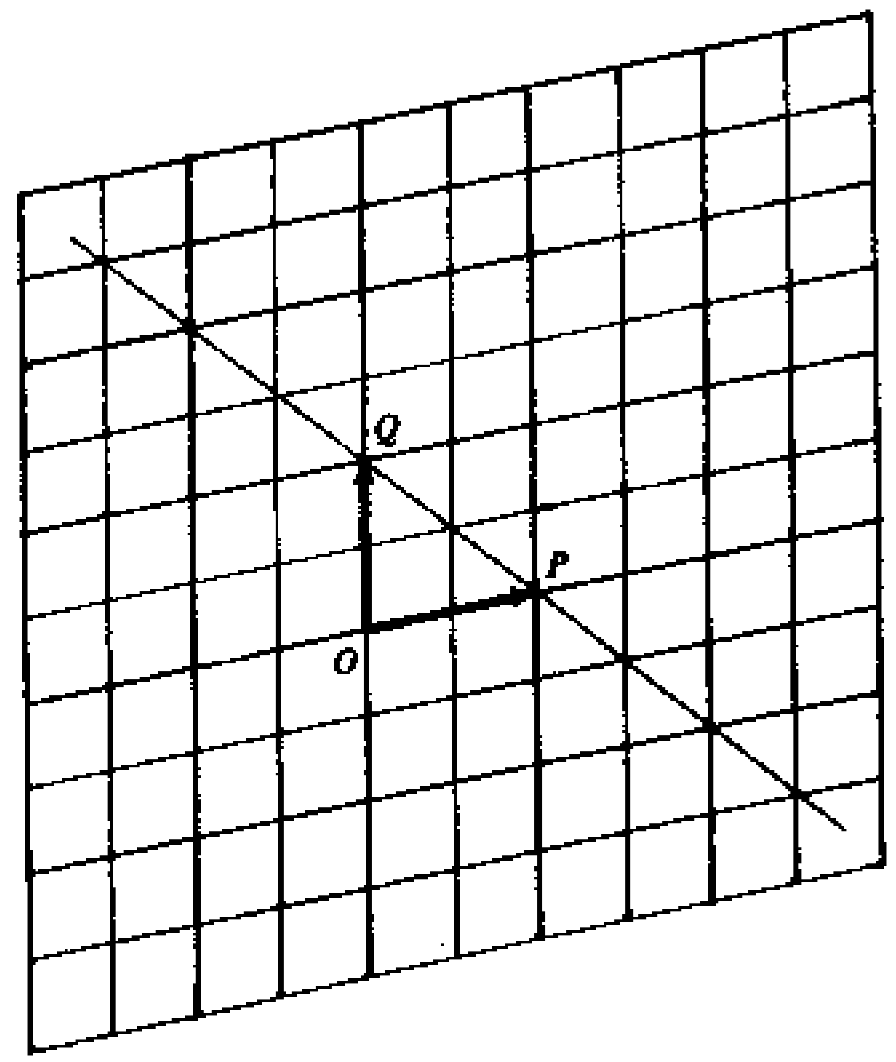

Figure 6

In Figure 6, $\vec{OP} = \vec{p}$ and $\vec{OQ} = \vec{q}$.

(a) If $\vec{OA} = \frac{1}{2}\vec{p} + \frac{1}{2}\vec{q}$ and $\vec{OB} = \frac{3}{2}\vec{p} - \frac{1}{2}\vec{q}$, draw the vectors $\vec{OA}$ and $\vec{OB}$ onto Figure 6. (2 marks)

(b) T is a point on PQ produced such that $\vec{QT} = \frac{1}{2}\vec{PQ}$. Express $\vec{OT}$ in terms of $\vec{p}$ and $\vec{q}$. (3 marks)

(c) Let $\vec{p} = 6\vec{i} + 2\vec{j}$ and $\vec{q} = 5\vec{j}$ where $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors. R is a point such that

$$\vec{OR} = r\vec{p} + (1-r)\vec{q}.$$

(i) Express the dot product $\vec{PQ}\cdot\vec{OR}$ in terms of r.

(ii) If $\vec{PQ} \perp \vec{OR}$, find the value of r. (7 marks)

END OF PAPER

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一九八一年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

數學（課程二） 試卷一

MATHEMATICS (SYLLABUS 2) PAPER I

二小時完卷

上午八時三十分至上午十時三十分

本試卷必須用英文作答

Two hours

8.30 a.m.—10.30 a.m.

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PYRAMID	Volume	=	$\frac{1}{3}\times$ base area $\times$ height

SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be referred to when used.

1. The capacities of two spherical tanks are in the ratio 27 : 64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank? (5 marks)

2. If $x = (a + by^2)^{\frac{1}{3}}$, express y in terms of a, b and x. (5 marks)

3. Let $f(x) = (x+2)(x-3) + 3$.
When $f(x)$ is divided by $(x-k)$, the remainder is k. Find k. (5 marks)

4. Solve $\cos(200^\circ + \theta) = \sin 120^\circ$ where $0^\circ < \theta < 180^\circ$. (6 marks)

5. Factorize $(1+x)^4 - (1-x^2)^2$. (6 marks)

6. Solve $4^x = 10 - 4^{x+1}$. (6 marks)

7. In Figure 1, O is the centre of circle ABC. $\angle OAB = 40°$.
Calculate $\angle BCA$. (6 marks)

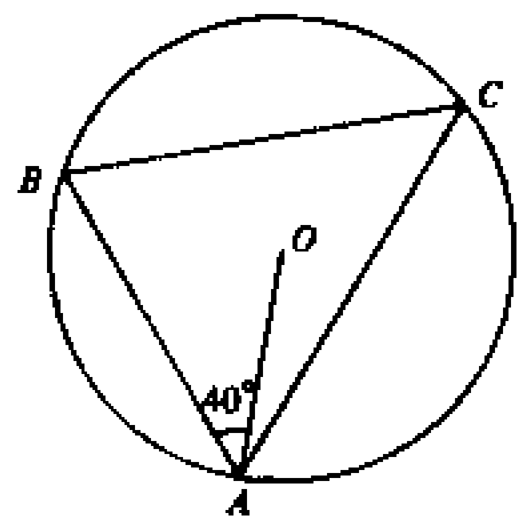

Figure 1

Candidate Number	Centre Number	Seat Number	Total Marks on this page

SECTION B Answer FIVE questions in this section.
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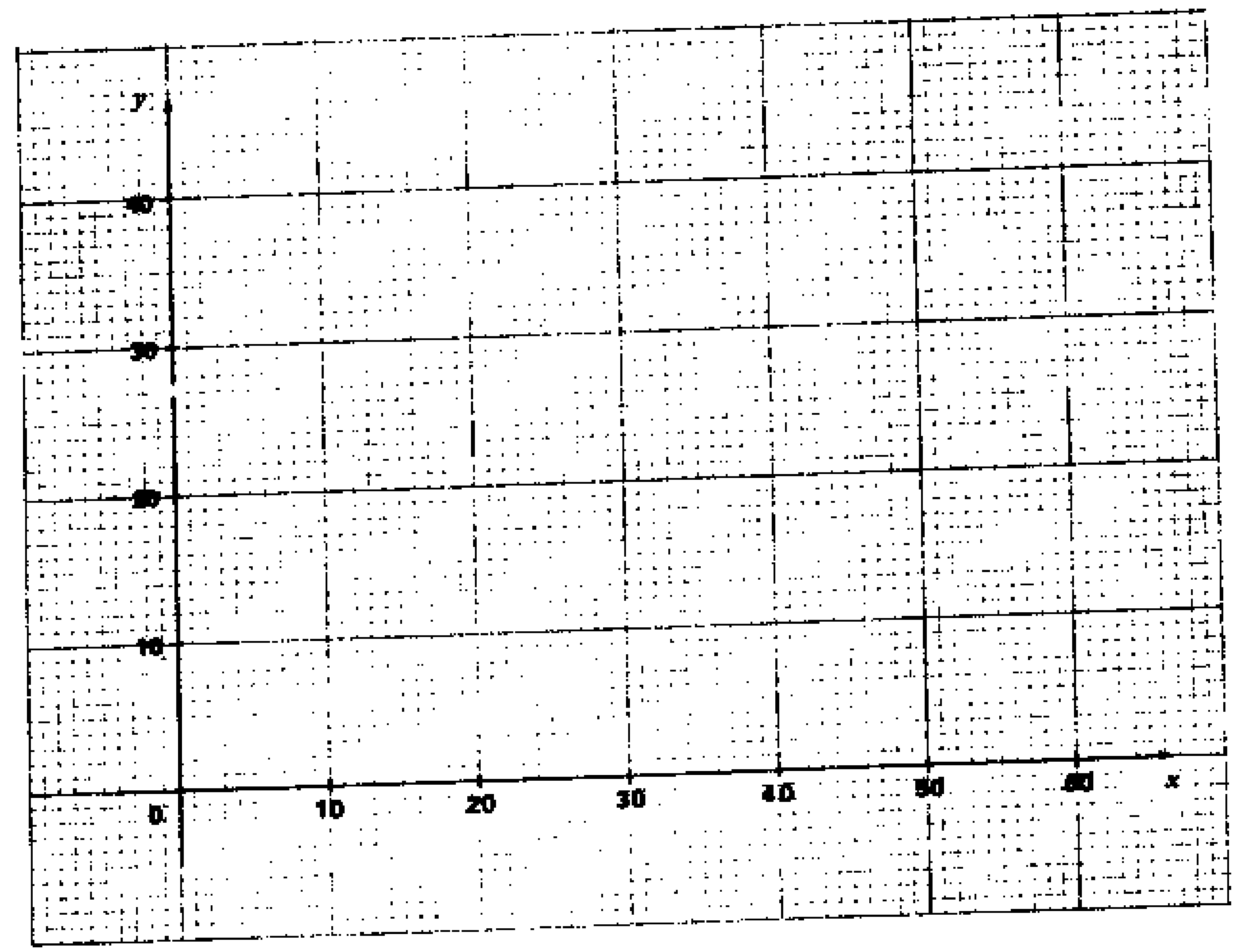
8. If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.

An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions :

(1) The hostel will accommodate at least 48 persons.
(2) Each single room will occupy an area of $10\ m^2$, each double room will occupy an area of $15\ m^2$ and the total available floor area for the rooms is $450\ m^2$.
(3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)



9. Normally, a factory produces 400 radios in x days. If the factory were to produce 20 more radios each day, then it would take 10 days less to produce 400 radios. Calculate x.

(12 marks)

10.

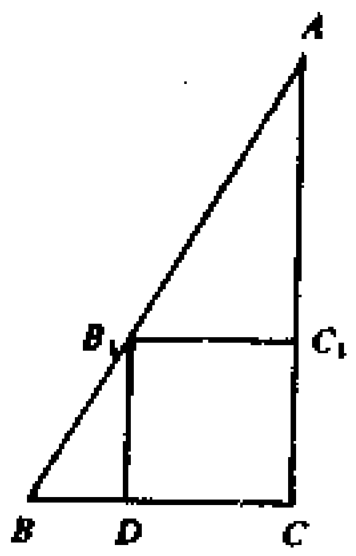


Figure 2(a)

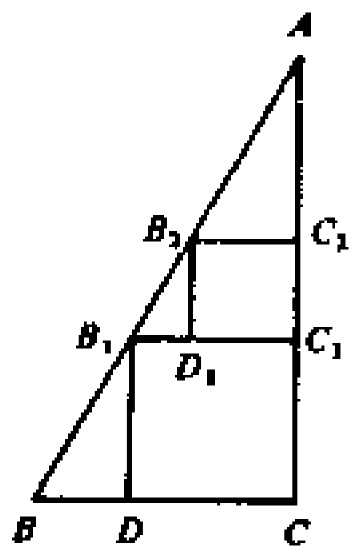


Figure 2(b)

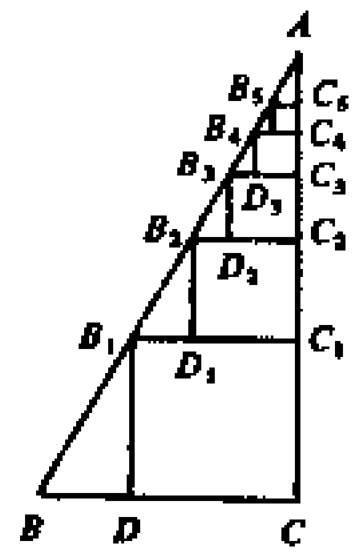


Figure 2(c)

In Figure 2(a), B_1C_1CD is a square inscribed in the right-angled triangle ABC. $\angle C = 90°$, $BC = a$, $AC = 2a$, $B_1C_1 = b$.

(a) Express b in terms of a.

(3 marks)

(b) $B_2C_2C_1D_1$ is a square inscribed in ΔAB_1C_1 (see Figure 2(b)).

(i) Express B_2C_2 in terms of b.

(ii) Hence express B_2C_2 in terms of a.

(2 marks)

(c) If squares $B_3C_3C_2D_2$, $B_4C_4C_3D_3$, $B_5C_5C_4D_4$, ... are drawn successively as indicated in Figure 2(c),

(i) write down the length of B_5C_5 in terms of a,

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(7 marks)

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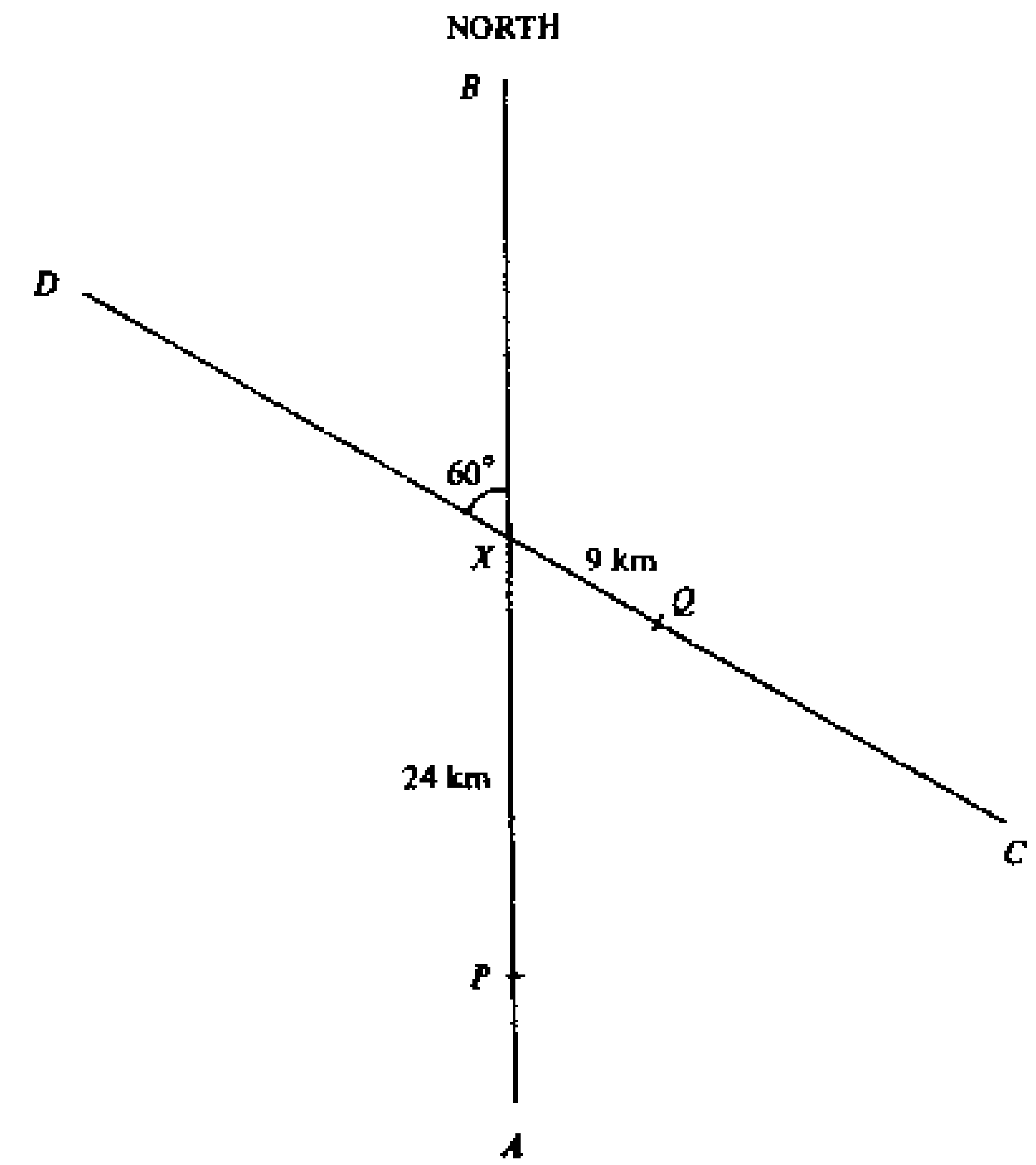

Figure 3

AB and CD are two straight roads intersecting at X. AB runs North and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in Figure 3. P walks at a speed of 4.5 km/h towards B and Q walks at a speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon. (4 marks)

(b) What are the distances of P and Q from X at 4 p.m.? (2 marks)

(c) Calculate the bearing of Q from P at 4 p.m. to the nearest degree. (6 marks)

12. Figure 4 shows a cylinder 10 metres high and 10 metres in radius used for storing coal-gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 2.4 radians at the point O, which is the centre of the top of the cylinder.

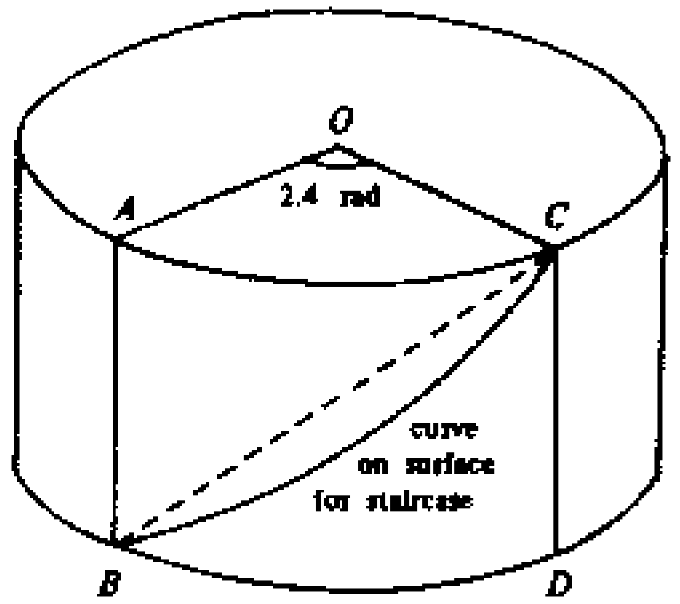

Figure 4

(a) Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures. (5 marks)

(b) Calculate the area of the curved surface $ABDC$ bounded by the minor arcs AC, BD and the lines AB, CD. (3 marks)

(c) A staircase from B to C is built along the shortest curve on the curved surface $ABDC$. Find the length of the curve. (4 marks)

3. In Figure 5, circles PMQ and QNR touch each other at Q. QT is a common tangent. PQR is a straight line. TP and TR cut the circles at M and N respectively.

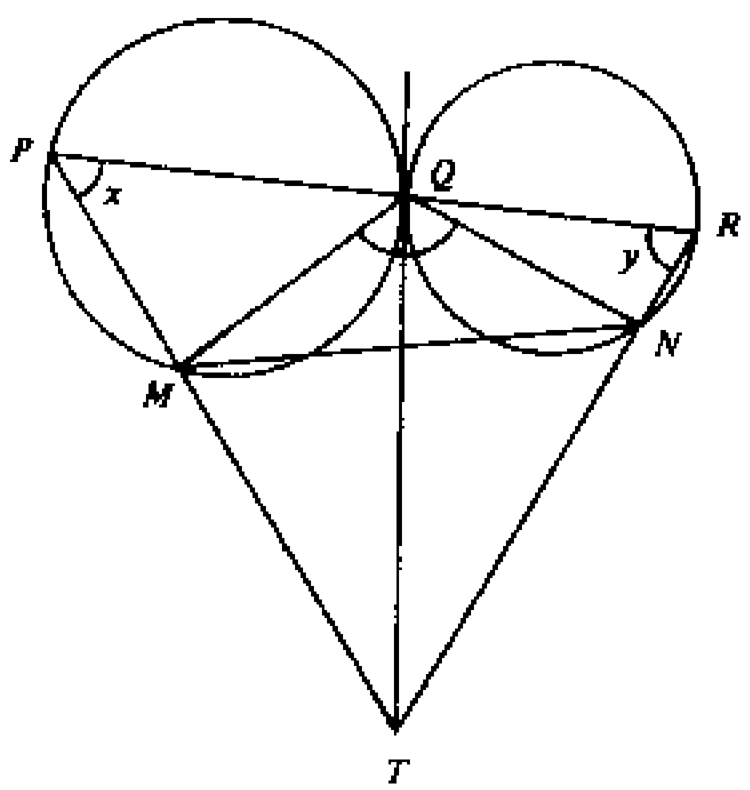

Figure 5

(a) If $\angle P = x$ and $\angle R = y$, express $\angle MQN$ in terms of x and y. (2 marks)

(b) Prove that Q, M, T and N are concyclic. (3 marks)

(c) Prove that P, M, N and R are concyclic. (4 marks)

(d) There are several pairs of similar triangles in the figure. Name any two pairs (no proof is required). (3 marks)

14.

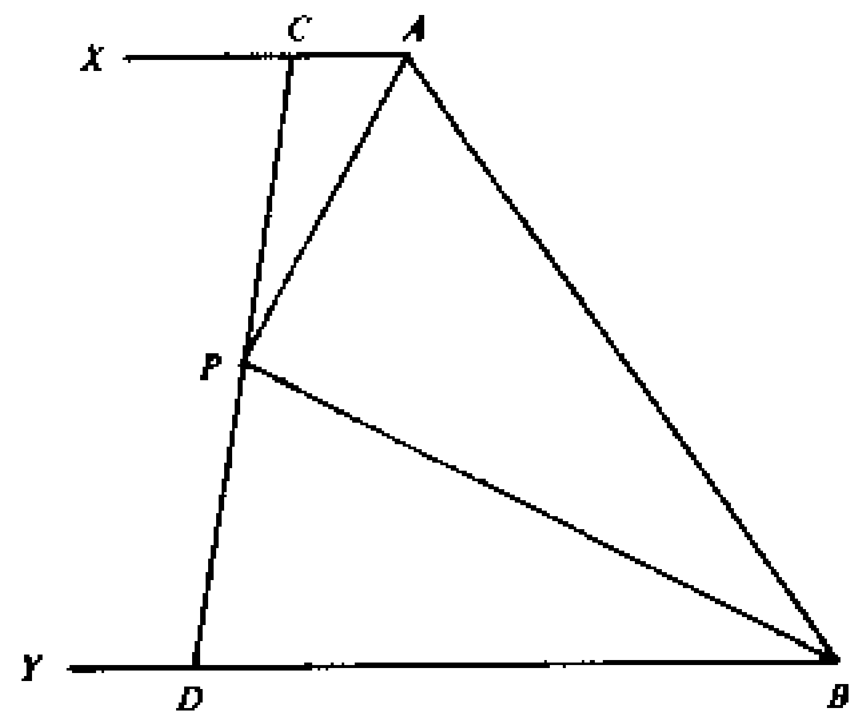

Figure 6

In Figure 6, $AX \parallel BY$. AP and BP bisect $\angle XAB$ and $\angle YBA$ respectively, and they meet at P. A straight line passing through P meets AX and BY at C and D respectively.

Prove that (a) $\angle APB = 90^\circ$, (4 marks)

(b) $CP = DP$, (5 marks)

(c) $AC + BD = AB$. (3 marks)

END OF PAPER

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數學(課程三) 試卷一

MATHEMATICS (SYLLABUS 3) PAPER I

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2. Let $f(x) = (x + 2)(x - 3) + 3$.
When $f(x)$ is divided by $(x - k)$, the remainder is k. Find k.
(5 marks)

3. There are 40 students in a class, including students A and B. If two students are to be chosen at random as class representatives, find the probability that both A and B are chosen.
(5 marks)

4. Solve $\cos(200° + \theta) = \sin 120°$ where $0° < \theta < 180°$.
(6 marks)

5. Factorize $(1 + x)^4 - (1 - x^2)^2$. (6 marks)

6. The heights of 1000 students form a symmetrical distribution with a mean of 1.70 m and a standard deviation of 0.02 m. If 67% of the students lie within one standard deviation of the mean and 97% lie within two standard deviations of the mean, find

(a) the number of students who are shorter than 1.74 m,

(b) the number of students whose heights lie between 1.68 m and 1.74 m. (6 marks)

7. The parabola $y^2 = 4ax$ passes through the points $A(1, 4)$ and $B(16, -16)$. A point P divides AB internally such that $AP : PB = 1 : 4$.

(a) Find the coordinates of P.

(b) Show that P is the focus of the given parabola. (6 marks)

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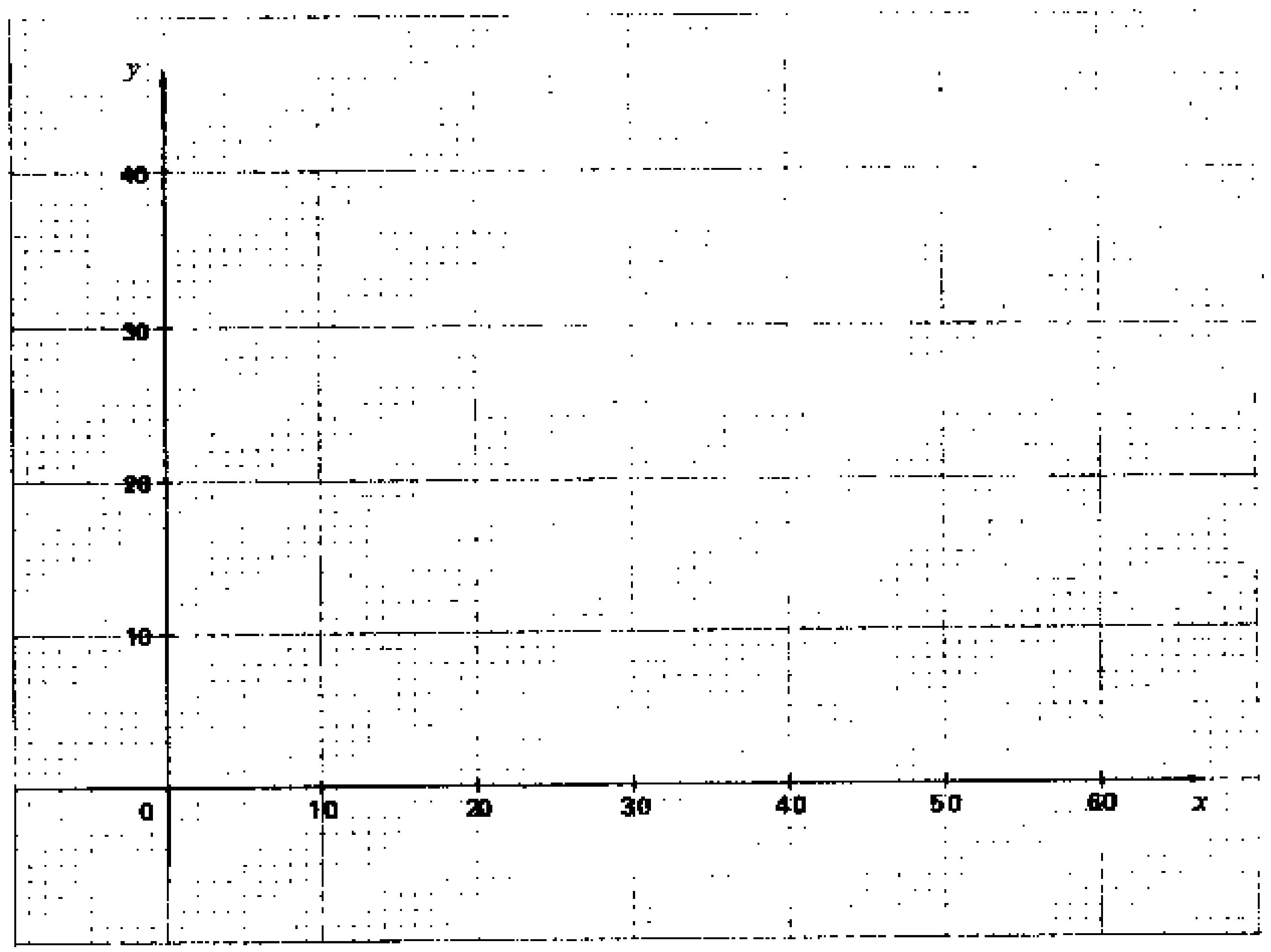

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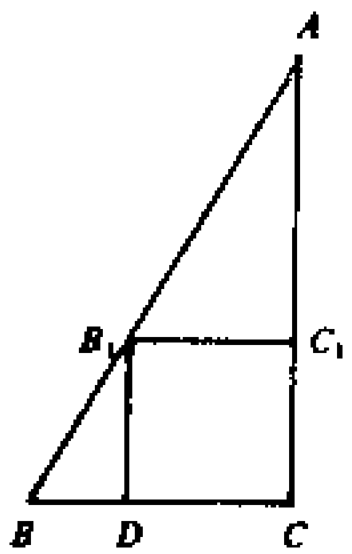
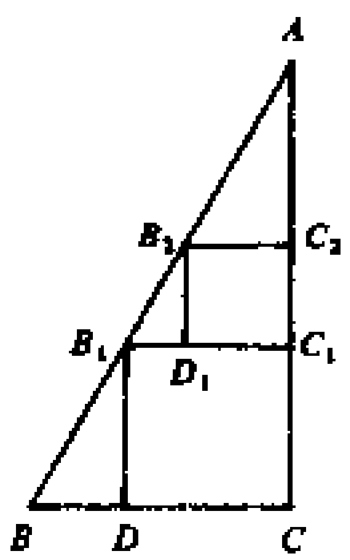


Figure 1(a)



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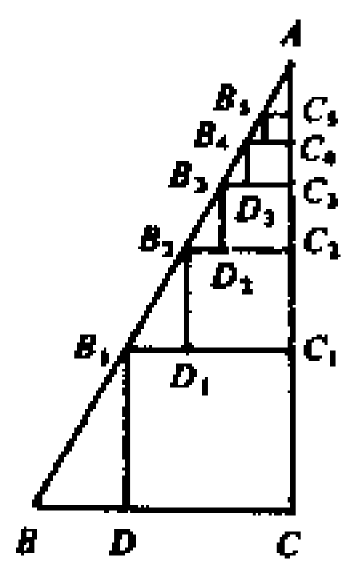

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(a) Express b in terms of a.

(3 marks)

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(ii) Hence express B_2C_2 in terms of a.

(2 marks)

(c) If squares $B_3C_3C_2D_2$, $B_4C_4C_3D_3$, $B_5C_5C_4D_4$, ... are drawn successively as indicated in Figure 1(c),

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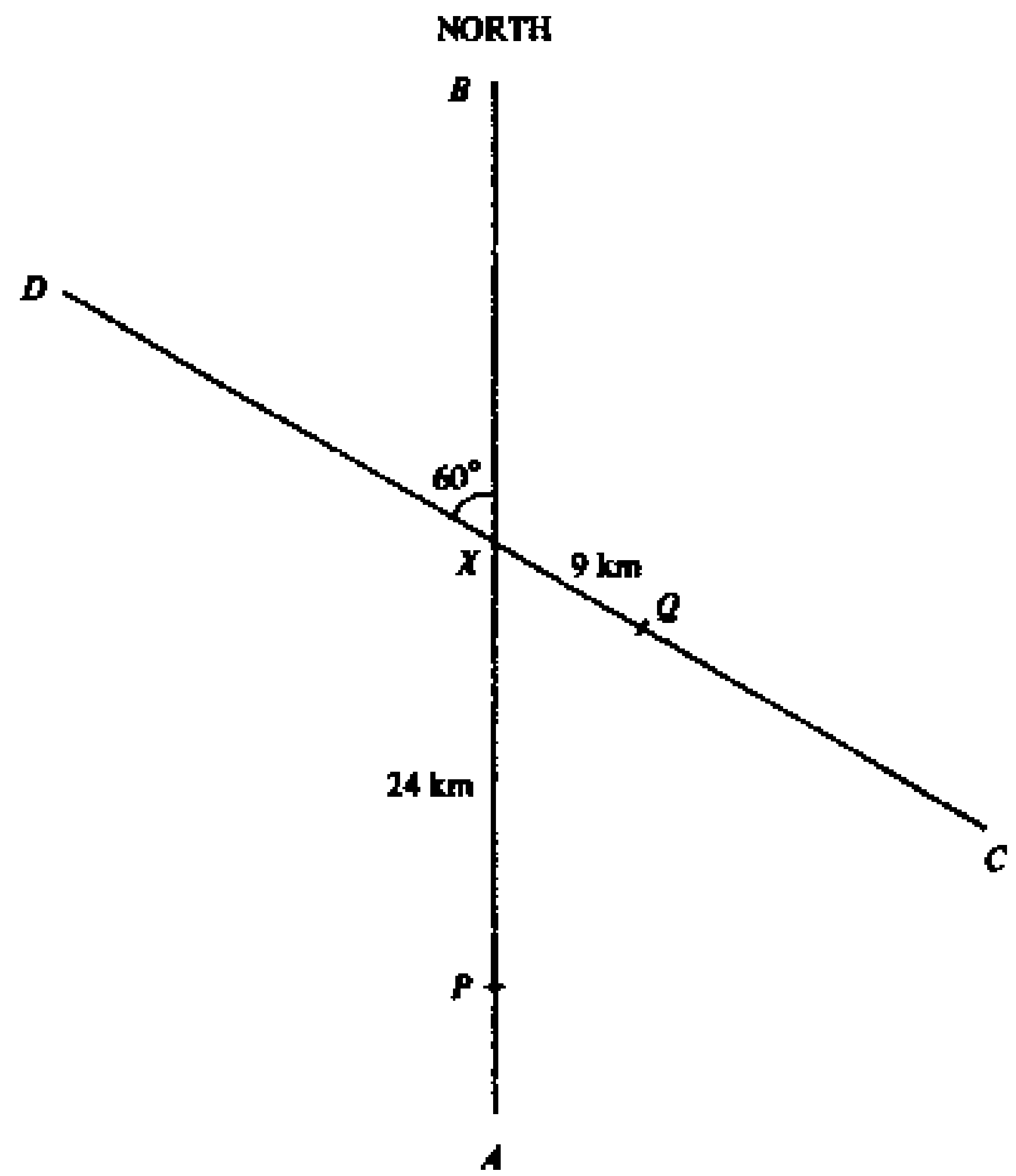

Figure 2

AB and CD are two straight roads intersecting at X. AB runs North and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in Figure 2. P walks at a speed of 4.5 km/h towards B and Q walks at a speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon. (4 marks)

(b) What are the distances of P and Q from X at 4 p.m.? (2 marks)

(c) Calculate the bearing of Q from P at 4 p.m. to the nearest degree. (6 marks)

12. Figure 3 shows a cylinder 10 metres high and 10 metres in radius used for storing coal-gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 2.4 radians at the point O, which is the centre of the top of the cylinder.

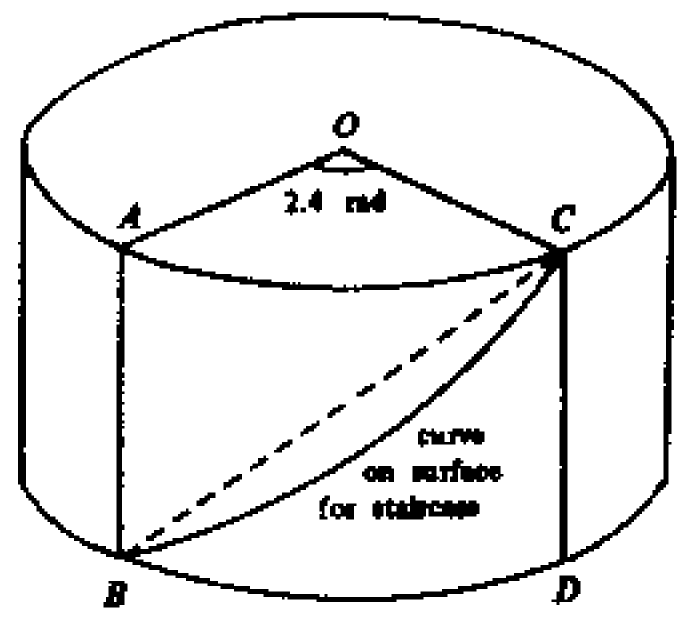

Figure 3

(a) Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures. (5 marks)

(b) Calculate the area of the curved surface $ABDC$ bounded by the minor arcs AC, BD and the lines AB, CD. (3 marks)

(c) A staircase from B to C is built along the shortest curve on the curved surface $ABDC$. Find the length of the curve. (4 marks)

13.

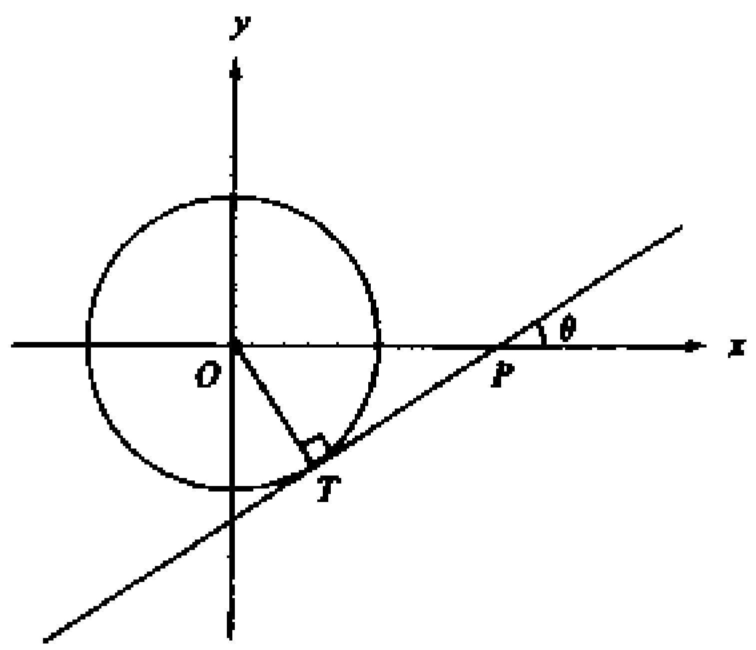

Figure 4(a)

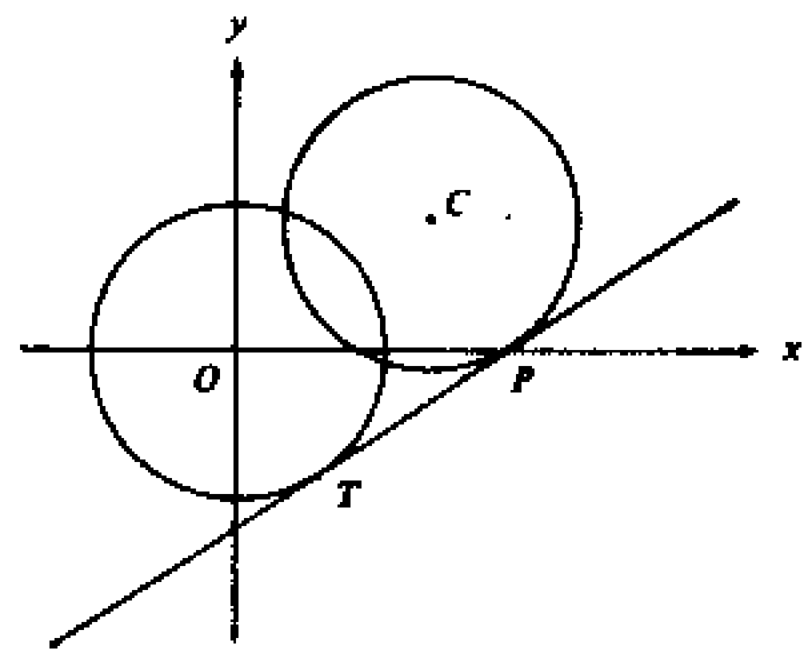

Figure 4(b)

Figure 4(a) shows a circle of radius 15 with centre at the origin O. The line TP, of slope $\frac{3}{4}$ ($= \tan\theta$), touches the circle at T and cuts the x-axis at P.

(a) Find the equation of the circle. (1 mark)

(b) Calculate the length of OP. (3 marks)

(c) Find the equation of the line TP. (2 marks)

Another circle, with centre C and radius 15, is drawn to touch TP at P (see Figure 4(b)).

(d) Find the equation of the line OC. (1 mark)

(e) Find the equation of the circle with centre C. (5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

14. **If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.**

The relationship between the height y of a flying object and time x is given by

$$y = x^3 + ax^2 + bx,$$

where y is in kilometres above sea-level and x is the number of hours after 12 : 00 noon. Figure 5 shows the graph of $y = x^3 + ax^2 + bx$.

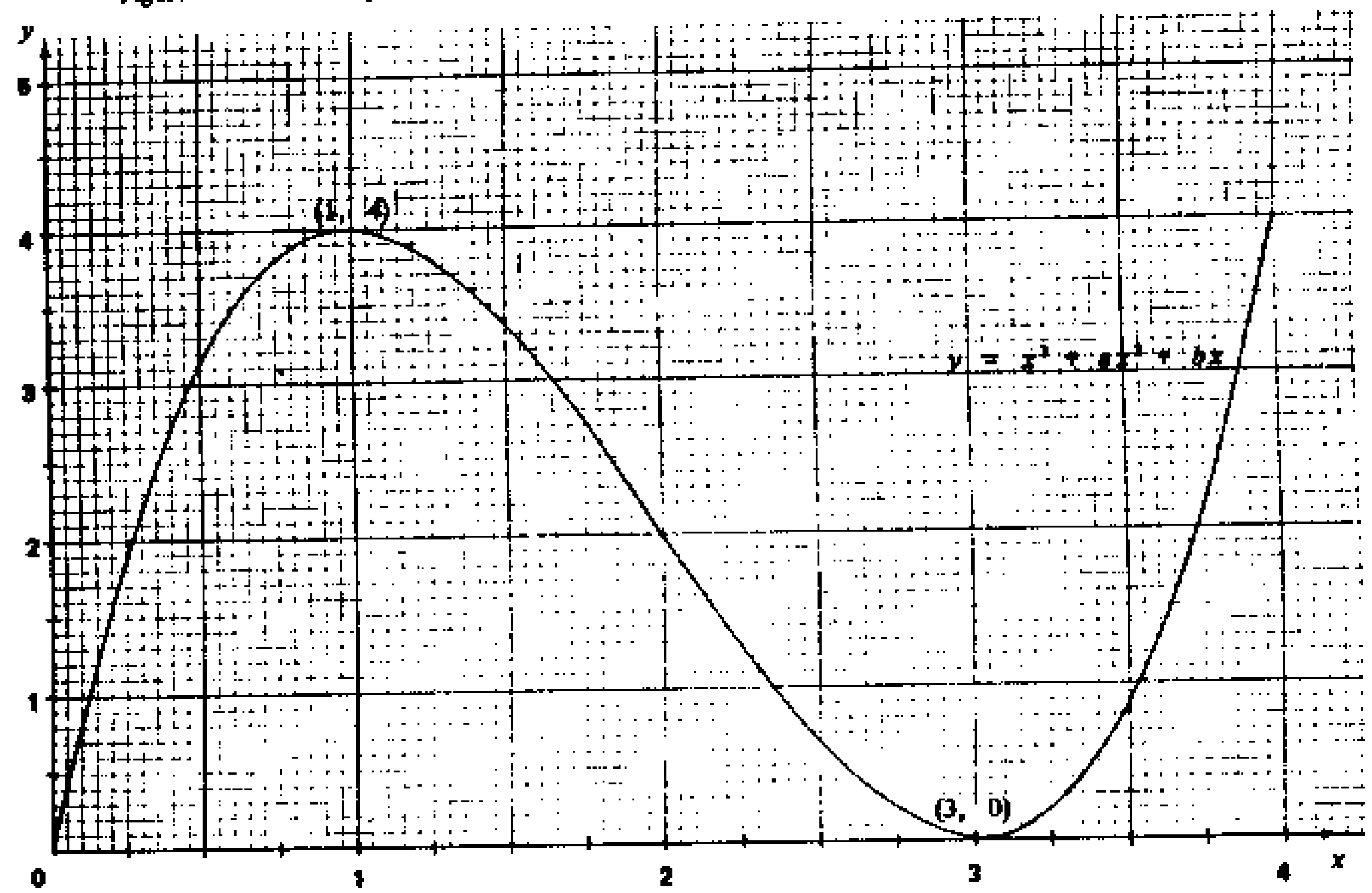

Figure 5

(a) Using Figure 5,

(i) find the values of a and b,

(ii) write down the time interval in which the flying object is descending. (4 marks)

(b) At 1:00 p.m., a balloon rises vertically from sea-level with a constant speed of 4 km/h.

(i) Add a straight line to Figure 5 to show the relationship between the height of the balloon and time x.

(ii) Hence, write down the value of x to 2 significant figures, for which the balloon and the flying object are at the same height. (3 marks)

(c) Use the method of magnification to find the value of x in (b)(ii) to 3 significant figures. (5 marks)

END OF PAPER