香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八〇年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1980

數學
評卷參考

MATHEMATICS
MARKING SCHEME

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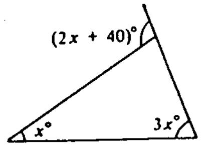
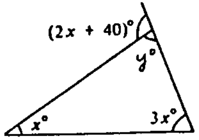
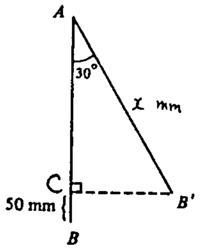
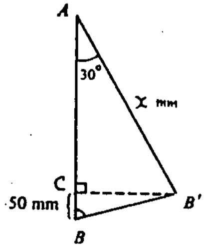
SOLUTION STEPS	MARKS	NOTES
 $2x + 40 = x + 3x$	1-1 1M + 1A	
$2x = 40$		
$x = 20$	2A	Do not penalize cand. for writing $x = 20^\circ$, $x^\circ = 20$ or $x^\circ = 20^\circ$.
ALTERNATIVELY,  $(2x + 40) + y = 180$	1M	
$y + x + 3x = 180$	1M	
$\vdots$ $x = 20$	2A	
ALTERNATIVELY, $x + 3x + 180 - (2x + 40) = 180$	2M	
$\vdots$ $x = 20$	2A	
2. (a) $a(3b - c) + c - 3b$		If a cand. writes $a(3b - c) + c - 3b = 0$ $a(3b - c) -(3b - c)= 0$ $(a - 1)(3b - c) = 0$, award 1 mark.
$= a(3b - c) - (3b -c)$ or $3b(a - 1) - c(a -1)$	1A	
$= (a - 1)(3b - c)$	1A	
(b) $x^4 - 1$		If a cand. writes $x^4 - 1 = 0$
$= (x^2 + 1)(x^2 - 1)$	2A	$(x^2+1)(x^2-1)=0$ 1A
$= (x^2 + 1)(x + 1)(x - 1)$	1A	$(x^2+1)(x+1)(x-1)=0$ 1A
ALTERNATIVELY, $f(x) = x^4 - 1$		If a cand. writes $f(1) = 0$, $(x-1)$ is a factor. 1A
$f(1) = 0$, $\therefore (x - 1)$ is a factor of $f(x)$.	1A	$f(-1)=0$ $(x+1)$ is a factor. 1A
By long division, $f(x) = (x - 1)(x^3 + x^2 + x + 1)$	1A	
$= (x - 1)(x + 1)(x^2 + 1)$	1A	

SOLUTION STEPS	MARKS	NOTES
Product of roots = $-\frac{5}{2}$	1A	
Let the other root be α		
$5\alpha = -\frac{5}{2}$ or $\alpha\beta = -\frac{5}{2}$	1A	同答案 $\alpha\beta = -\frac{5}{2}$ (2A).
$\alpha = -\frac{1}{2}$	1A	
$-\frac{k}{2} = -\frac{1}{2} + 5$ or $\alpha + \beta = -\frac{k}{2}$	1M	
$k = -9$	1A	
ALTERNATIVELY,		
Product of roots = $-\frac{5}{2}$	1A	
Since one of the roots is 5,		
$2(5)^2 + k(5) - 5 = 0$	1M	
$k = -9$	1A	
The equation is		
$2x^2 - 9x - 5 = 0$		
$(x - 5)(2x + 1) = 0$	1A	
$x = 5$ or $x = -\frac{1}{2}$		
The other root is $-\frac{1}{2}$	1A	
$\sin\theta = \cos 120^\circ$		
$= -\frac{1}{2}$	1A	
ALTERNATIVELY,		
$\sin\theta = \cos 120^\circ$		
$= -\cos 60^\circ$		
$= -\sin 30^\circ$	1A	
$\theta = 180^\circ + 30^\circ$ or $360^\circ - 30^\circ$		不寫 general solution 給2分
$= 210^\circ$ or 330°	2A + 2A	Accept $\theta = 210^\circ,\ 330^\circ$ Accept $\theta = 210^\circ$ and 330° 如題目所寫 0°—360° 以外答案扣壹分

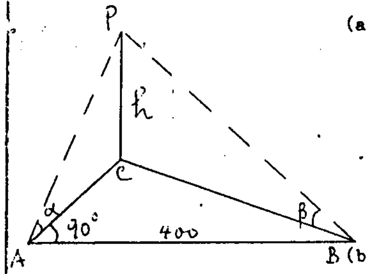


SOLUTION STEPS	MARKS	NOTES
Let the length of AB be x mm.		
$AC = x\cos 30^\circ$	2A	any equation in x. ~~for AC + CB = AB~~ Deleted.
$\therefore x\cos 30^\circ + 50 = x$	1M	
$x = \frac{50}{1 - \cos 30^\circ}$ or 373.2	1M	只寫 $\frac{x-50}{x} = \cos 30^\circ$, 3分
$= 373$ (corr. to 3 sig. fig.)	1A	
Length of the rod = 373 mm.		Wrong unit -1 pp. No unit O.K.
ALTERNATIVELY,		

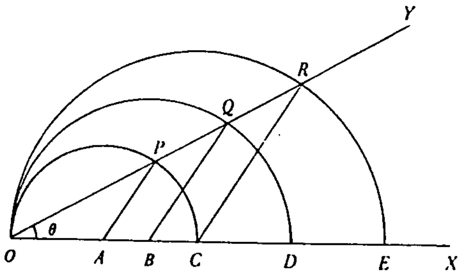


SOLUTION STEPS	MARKS	NOTES
Let AB = x mm		Alternatively, Let AC = h mm. $\frac{h}{h+50} = \cos 30^\circ$ --- 3M $h = \frac{50\cos 30^\circ}{1-\cos 30^\circ}$ or 323.2 1A AB = 373 ----- 1A.
$\angle ABB' = \angle AB'B$	1M	
$= 75^\circ$		
$\frac{CB'}{CB} = \tan 75^\circ$		
$CB' = 50\tan 75^\circ$	1A	
$= 186.6$		
$\frac{CB'}{x} = \sin 30^\circ$	1M + 1A	
$x = \frac{CB'}{\sin 30^\circ}$		
$= 373.2$		
$= 373$ (corr. to 3 sig. fig.)	1A	
Suppose x mothers lost only one of their children.		
Then (36 - x) mothers lost two of their children.	1A	If one answer given without explanation. 1A
$x + 2(36 - x) = 62$	2M	
$x = 10$	1A	If both answers given 3A
10 mothers lost only one of their children. 26 mothers lost both of their children.	1A	(i) With checking +1 (ii) With acceptable explanation +2
ALTERNATIVELY,		
Suppose x mothers lost only one of their children and y mothers lost two of their children.		
$x + y = 36$	1A	
$x + 2y = 62$	2M	
Solving, $x = 10$	1A	
$y = 26$	1A	

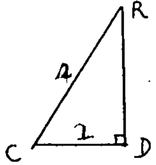
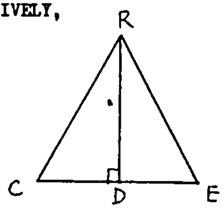
SOLUTION STEPS	MARKS	NOTES
$a(1+\frac{x}{100}) = b(1-\frac{x}{100})$		
$a+\frac{ax}{100} = b-\frac{bx}{100}$	1A	
$\frac{ax}{100}+\frac{bx}{100} = b-a$	1A	
$\frac{a+b}{100}x = b-a$	1A	
$x = \left(\frac{b-a}{a+b}\right)\cdot 100$	2A	
Daily wages for a skilled, semi-skilled and unskilled worker are $120, $90, $60 respectively.	1A	
Mean daily wage in dollars $= \frac{120 \times 10 + 90 \times 20 + 60 \times 30}{60}$	1M 1A	(for denominator)
$= \frac{4\ 800}{60}$	1A	(for 4 800)
$= 80$	1A	



SOLUTION STEPS	MARKS	NOTES
(a) (i) $\tan\alpha = \frac{h}{x}$	1A	
$x = \frac{h}{\tan\alpha}$	1A	Accept $x = h \cot\alpha$
(ii) $\tan\beta = \frac{h}{y}$	1A	
$y = \frac{h}{\tan\beta}$	1A	Accept $y = h \cot\beta$
(b) $BC^2 = AC^2 + AB^2$ $y^2 = x^2 + 400^2$	2M	
$\left(\frac{h}{\tan\beta}\right)^2 = \left(\frac{h}{\tan\alpha}\right)^2 + 400^2$	1M	For sub. x, y
$\left(\frac{h}{\tan 30^\circ}\right)^2 = \left(\frac{h}{\tan 60^\circ}\right)^2 + 400^2$	1A	
$(\sqrt{3}h)^2 = (\frac{1}{\sqrt{3}}h)^2 + 400^2$		
$2\frac{2}{3}h^2 = 400^2$		
$h^2 = \frac{3}{8} \times 400^2$		
$h = \sqrt{\frac{3}{8}} \times 400$ (or 244.9)	1A	or any figure which rounds off to 245
≈ 245 (corr. to 3 sig. fig.)	1A	
ALTERNATIVELY,		
$BC^2 = AC^2 + AB^2$	2M	
$y^2 = x^2 + 400^2$		
$x = \frac{h}{\tan 60^\circ}$ | $y = \frac{h}{\tan 30^\circ}$		
$\frac{x}{y} = \frac{\tan 30^\circ}{\tan 60^\circ}$	1A	
$= \frac{1}{3}$		
$3x = y$		
$9x^2 = x^2 + 400^2$	1M	
$8x^2 = 400^2$		
$x = \sqrt{\frac{400^2}{8}} = 141.42$		
$h = x \tan 60^\circ$	1A	
$= 245$	1A	

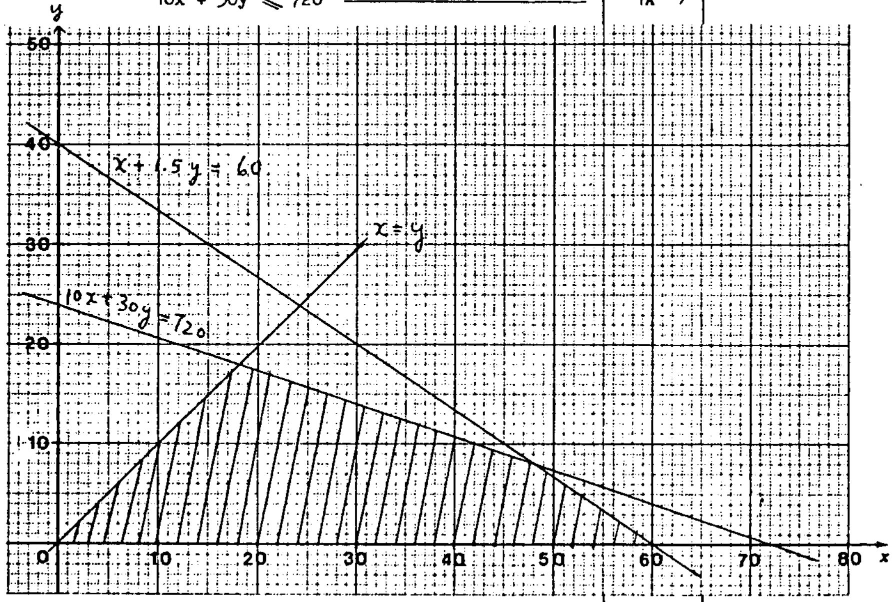


(a) $\angle PAX = 2\theta$ ($\because$ angle at centre is twice as great as angle at circumference)	2A	不以為理由
ALTERNATIVELY,		
$\angle APO = \theta$ or $\angle PAX = \theta + \angle APO$	1A	
$\angle PAX = 2\theta$	1A	
Similarly $\angle QBX = 2\theta$ $\angle RCX = 2\theta$	1A	Awarded only if both answers are correct
(b) $\angle PAO = \angle QBO = \angle RCO$	1M	
Sector PAO, sector QBO, sector RCO are similar.		
Area of sector PAO : area of sector QBO : area of sector RCO		Ratio of areas = ratio of radii^2
$= OA^2 : OB^2 : OC^2$	2M	
$= 2^2 : 3^2 : 4^2$ or $4 : 9 : 16$	1A	
ALTERNATIVELY,		
$\angle PAO = \angle QBO = \angle RCO = \phi$	1M	假設 $\angle PAO = \angle QBO = \angle RCO = 120°$ 只給M分
Area of sector PAO $= \frac{1}{2}OA^2\phi$ Area of sector QBO $= \frac{1}{2}OB^2\phi$ Area of sector RCO $= \frac{1}{2}OC^2\phi$	1A	(for any one of the three)
Area of sector PAO : area of sector QBO : area of sector RCO		
$= \frac{1}{2}OA^2\phi : \frac{1}{2}OB^2\phi : \frac{1}{2}OC^2\phi$	1	(provided all three expressions are correct)
$= OA^2 : OB^2 : OC^2$		
$= 2^2 : 3^2 : 4^2$ or $4 : 6 : 9$	1A	

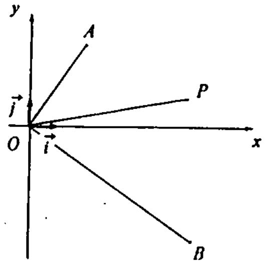
SOLUTION STEPS	MARKS	NOTES
(c) If $RD \perp OX$,		
$RD^2 = CR^2 - CD^2$	1M	
$= OC^2 - CD^2$		
$= 4^2 - 2^2$		
$= 12$		
$\tan\theta = \frac{RD}{OD}$	1M	
$= \frac{\sqrt{12}}{6} = \frac{\sqrt{3}}{3}$		
$\theta = 30°$	1A	
ALTERNATIVELY,  	1M	
$\angle RCD = 60°$	1A	
$\theta = 30°$	1A	
ALTERNATIVELY,  		
Mentioning RD is the perpendicular bisector of CE	1M	
$\therefore RC = RE$ $RC = CE$ (radii)		
Hence $\triangle RCE$ is an equilateral $\triangle$	1M	
$\therefore \angle RCE = 60°$		
$\therefore \theta = 30°$	1A 1A	

SOLUTION STEPS	MARKS	NOTES
1. (a) (i) Common ratio = 10 ————	1A	
(ii) Sum of n terms = $\frac{a(r^n - 1)}{r - 1}$ ————	1M	
$= \frac{k(10^n - 1)}{10 - 1}$		
$= \frac{k}{9}(10^n - 1)$ ————	1A	
(b) (i) One mark would be awarded if a cand. shows the correct **idea** of proving either 3rd term − 2nd term = 2nd term − 1st term or 1st term + 3rd term = 2 x 2nd term	1M	
$\log 10k - \log k = \log \frac{10k}{k}$		
= log 10 or 1 ————	1A	
$\log 100k - \log 10k = \log \frac{100k}{10k}$		
= log 10 or 1 ————	1A	
∴ It is an A.P.		
ALTERNATIVELY,		
$\frac{\log k + \log 100k}{2} = \frac{1}{2}\log 100k^2$		
= log 10k ————	2A	
It is an A.P.		
(ii) Quoting correct formula for the sum of A.P.		
$\frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}[T(1) + T(n)]$ ————	1M	This may be omitted
Sum of the first n terms		(ii) $S_n = \frac{n}{2}[T(1) + T(n)]$ [illegible] [illegible] 3分
$= \frac{n}{2}[2 \log k + (n - 1) \log 10]$ ————	1A	
$= \frac{n}{2}[2 \log k + (n - 1)]$		
Sum of the first ten terms		
$= \frac{10}{2}[2 \log k + 9 \log 10]$ ————	1A	
= 10 log k + 45 ————	1A	

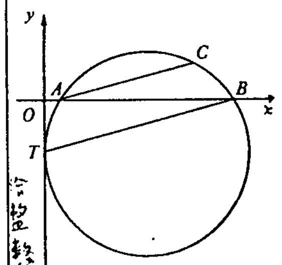
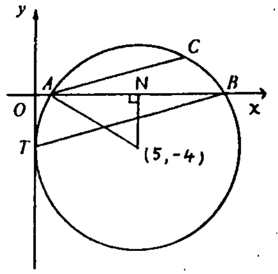
SOLUTION STEPS	MARKS	NOTES
2. Let x be the no. of economy class seats and y be the no. of first class seats.		
Constraints: $x, y \in N$. may be omitted		
$x + 1.5y \leqslant 60$ ————	1A	漏寫等號 扣一分
$x \geqslant y$ ————	1A	may be omitted
$10x + 30y \leqslant 720$ ————	1A	

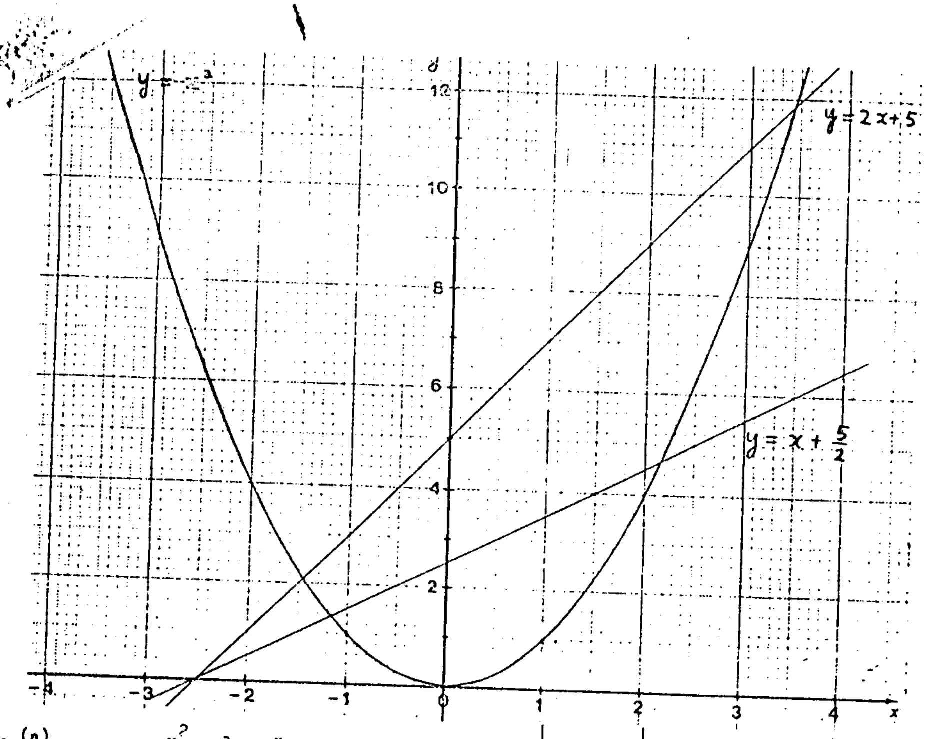


SOLUTION STEPS	MARKS	NOTES
Graphs of the lines: x = y ————	1A for graphs	Labelling of graphs not necessary.
x + 1.5y = 60 ————	1A	
10x + 30y = 720 ————	1A	
Correct region ————	1A	
Testing optimization ————	1M	
No. of first class seats = 8 ————	1A	Awarded only if region correct
No. of economy class seats = 48 ————	1A	
		畫錯一條，最多6分 二 . . 5分 三 . . 4分 漏 shaded region 扣一分

SOLUTION STEPS	MARKS	NOTES
(a) (i) $(3\vec{i} + 4\vec{j})\cdot(x\vec{i} + y\vec{j})$		If "." omitted, do not deduct mark.
$= 3x + 4y$	1A	
(ii) $\lvert\vec{OA}\rvert = \sqrt{3^2 + 4^2}$		$\lvert\vec{OA}\rvert = \lvert OA\rvert = OA$
$= 5$	1A	Accept
$\lvert\vec{OP}\rvert = \sqrt{x^2 + y^2}$	1A	$\lvert\vec{OA}\rvert = \sqrt{(3\vec{i})^2 + (4\vec{j})^2}$
(ii) $\cos\angle AOP = \dfrac{\vec{OA}\cdot\vec{OP}}{\lvert\vec{OA}\rvert\ \lvert\vec{OP}\rvert}$		
$= \dfrac{3x + 4y}{5\sqrt{x^2 + y^2}}$	1M	Accept $\begin{pmatrix}3\\4\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix} = 3x + 4y$, $(3, 4)\cdot(x, y) = 3x + 4y$
(b) $\vec{OB}\cdot\vec{OP} = 8x - 6y$	1A	
$\lvert\vec{OB}\rvert = 10$		
$\cos\angle BOP = \dfrac{\vec{OB}\cdot\vec{OP}}{\lvert\vec{OB}\rvert\ \lvert\vec{OP}\rvert}$		
$= \dfrac{8x - 6y}{10\sqrt{x^2 + y^2}}$	1M + 1A	
(c) Equation of internal bisector of $\angle AOB$:		
$\dfrac{3x + 4y}{5\sqrt{x^2 + y^2}} = \dfrac{8x - 6y}{10\sqrt{x^2 + y^2}}$	2M	
$3x + 4y = 4x - 3y$		
$x - 7y = 0$	1A	
		If "→" is omitted three times or more in the solution, deduct 1 mark for poor presentation.

SOLUTION STEPS	MARKS	NOTES
(a) Probability $= \frac{9}{10} \times \frac{2}{3}$	2A	Award 2 or 0
$= \frac{3}{5}$ or $\frac{6}{10}$ or $\frac{60}{100}$ or 60% or 0.6	1A ← $\frac{18}{30}$	不給
(b) Probability of obtaining the qualification with one re-examination of the theory paper		
$= \frac{1}{10} \times \frac{9}{10} \times \frac{2}{3}$	1A	
$= \frac{3}{50}$		
Probability of obtaining the qualification with one re-examination of the practical paper		
$= \frac{9}{10} \times \frac{1}{3} \times \frac{2}{3}$	1A	
$= \frac{1}{5}$		
Required probability		
$= \frac{3}{50} + \frac{1}{5}$	1M	Award this mark for the + sign. Even when $\frac{3}{50}$ and $\frac{1}{5}$ are both incorrect, still give this mark.
$= \frac{13}{50}$	1A	
(c) Probability that A (or B) does not obtain the qualification by sitting each paper once.		
$= 1 - \frac{3}{5}$	1A	
$= \frac{2}{5}$		
Probability that A <u>and</u> B do not obtain the qualification by sitting each paper once.		
$= \frac{2}{5} \times \frac{2}{5}$	cancelled ~~1M~~	
Required probability		$\frac{3}{5} + \frac{3}{5} - \frac{3}{5} \times \frac{3}{5}$
$= 1 - \frac{2}{5} \times \frac{2}{5}$	1M	This method mark should be given when the expression is of the form $1 - p^2$, where $0 < p < 1$.
$= \frac{21}{25}$	1A	
ALTERNATIVELY,		
Required probability		
$= \frac{3}{5} + \frac{2}{5} \times \frac{3}{5}$	1M + 1A	This method mark should be given when the expression is of the form $p_1 + p_2p_3$
$= \frac{21}{25}$	1A	
ALTERNATIVELY,		
Required probability		
$= \frac{3}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$	1M + 1A	This method mark should be given when the expression is of the form $p_1p_2 + p_3p_4 + p_5p_6$
$= \frac{21}{25}$	1A	

SOLUTION STEPS	MARKS	NOTES
$x^2 + y^2 - 10x + 8y + 16 = 0$ (*)		
(a) $\begin{cases} y = 0 \\ x^2 + y^2 - 10x + 8y + 16 = 0 \end{cases}$	1M	
$x^2 - 10x + 16 = 0$ $(x - 2)(x - 8) = 0$		
$x = 2$ or 8	$\frac{1}{2}$A + $\frac{1}{2}$A	給整數分
$A = (2, 0)$, $B = (8, 0)$	$\frac{1}{2}$A + $\frac{1}{2}$A	
$\begin{cases} x = 0 \\ x^2 + y^2 - 10x + 8y + 16 = 0 \end{cases}$	1M	eg. $\frac{1}{2}$分則給 1分.
$y^2 + 8y + 16 = 0$ $(y + 4)^2 = 0$ $y = -4$		
$T = (0, -4)$	1A	
(b) (i) Slope of BT $= \frac{0 - (-4)}{8 - (0)} = \frac{1}{2}$		
Equation of AC : $\frac{y - 0}{x - 2}$ = slope of BT	1M	
$\frac{y - 0}{x - 2} = \frac{1}{2}$		
$x = 2y + 2$ or $y = \frac{1}{2}x - 1$	1A	
(ii) Substitute $x = 2y + 2$ in (*),	1M	OR Sub. $y = \frac{x}{2} - 1$ in (*) 1M
$(2y + 2)^2 + y^2 - 10(2y + 2) + 8y + 16 = 0$ $5y^2 - 4y = 0$ $y(5y - 4) = 0$		$5x^2 - 28x + 36 = 0$ $(x - 2)(5x - 18) = 0$
$y = 0$ or $y = \frac{4}{5}$	1A	$x = 2$ or $\frac{18}{5}$ 1A
$y = 0$ is rejected, $y = \frac{4}{5}$ $x = \frac{18}{5}$		$x = 2$ is rejected, $x = \frac{18}{5}$ $y = \frac{4}{5}$
$C = (\frac{18}{5}, \frac{4}{5})$	1A	$C = (\frac{18}{5}, \frac{4}{5})$ 1A
(a) ALTERNATIVELY, $(x - 5)^2 + (y + 4)^2 = 25$		
Centre = $(5, -4)$	1A	
Radius = 5		
$T = (0, -4)$	1A	
$AN = BN = \sqrt{5^2 - 4^2} = 3$	1A	
$OA = 5 - 3 = 2$	½A	
$OB = 5 + 3 = 8$	½A	
$A = (2, 0)$, $B = (8, 0)$	½A + ½A	



(a) $x^2 - 2x - 5 = 0$ $x^2 = 2x + 5$		
Equation of straight line: $y = 2x + 5$	1A	May be written on the graph
Graph of the straight line: $y = 2x + 5$	1A	
$x_1 = -1.4$ (-1.5 to -1.4)	1A	Awarded only if the line is correctly drawn
$x_2 = 3.4$ (3.4 to 3.5) 2位以上小數位，不給分扣1分 ([illegible])	1A	
(b) $x^2 - 2x > 5$ $x < -1.4$ or $x > 3.4$	2M	(i) If the word "or" is omitted award 1 mark. (ii) If a candidate writes $-1.4 > x > 3.4$ or $x \nless -1.4$ and $x > 3.4$, award 1 mark.
(c) $2x^2 - 2x - 5 = 0$ $x^2 - x - \frac{5}{2} = 0$		
Equation of straight line: $y = x + \frac{5}{2}$	1A	May be written on the graph
Graph of the straight line: $y = x + \frac{5}{2}$	1A	
$x_1 = -1.2$ (-1.2 to -1.1)	1A	Awarded only if the line is correctly drawn
$x_2 = 2.2$ (2.1 to 2.2)	1A	
		Do not deduct mark if st. line not labelled or labelled as $x^2 - 2x - 5 = 0$. If answers in ordered pairs, deduct 1 mark as poor presentation.