10 Inequalities and Linear Programming

10A Linear inequalities in one unknown

10A.1 HKCEE MA 1989 I – 2

Consider $x+1>\frac{1}{5}(3x+2)$.

(a) Solve the inequality.

(b) In addition, if $-4\le x\le 4$, find the range of x.

10A.2 HKCEE MA 1995 I – 1(a)

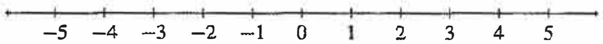
Solve the inequality $3x+1\ge 7$.

10A.3 HKCEE MA 1999 – I 3

Find the range of values of x which satisfy both $3x-4>2\ (x\ \ 1)$ and $x<6$.

10A.4 HKCEE MA 2000 – I – 5

Solve $\frac{11-2x}{5}<1$ and represent the solution in the figure.



10A.5 HKCEE MA 2002 – I 7

(a) Solve the inequality $3x+6\ge 4+x$.

(b) Find all integers which satisfy both the inequalities $3x+6\ge 4+x$ and $2x-5<0$.

10A.6 HKCEE MA 2003 I – 2

Find the range of values of x which satisfy both $\frac{3-5x}{4}\ge 2\ \ \ x$ and $x+8>0$.

10A.7 HKCEE MA 2005 I – 4

Solve the inequality $\frac{-3x+1}{4}>x-5$.

Also write down all integers which satisfy both the inequalities $\frac{-3\ \ x+1}{4}>x\ \ \ 5$ and $2\ \ x+1\ge 0$.

10A.8 HKCEE MA 2006 – I – 2

(a) Solve the inequality $x+1<\frac{x+25}{6}$.

(b) Write down the greatest integer satisfying the inequality $x+1<\frac{x+25}{6}$.

10A.9 HKCEE MA 2008 I – 2

(a) Solve the inequality $\frac{14x}{5}\ge 2\ \ x+7$.

(b) Write down the least integer satisfying the inequality $\frac{14x}{5}\ge 2x+7$.

10A.10 HKCEE MA 2010 – I – 2

(a) Solve the inequality $\frac{29x\ \ 22}{7}\le 3x$.

(b) Write down the greatest integer satisfying the inequality in (a).

10A.11 HKDSE MA 2012 I – 6

(a) Find the range of values of x which satisfy both $\frac{4x+6}{7}>2(x\ \ \ 3)$ and $2x-10\le 10$.

(b) How many positive integers satisfy both the inequalities in (a)?

10A.12 HKDSE MA 2013 – I – 5

(a) Solve the inequality $\frac{19-7x}{3}>23-5x$.

(b) Find all integers satisfying both the inequalities $\frac{19-7x}{3}>2\ \ 3-5x$ and $18\ \ \ 2x\ge 0$.

10A.13 HKDSE MA 2015 I – 5

(a) Find the range of values of x whichsatisfy both $\frac{7-3x}{5}\le 2(x+2)$ and $4x\ \ \ 13>0$.

(b) Write down the least integer which satisfies both inequalities in (a).

10A.14 HKDSE MA 2016 – I – 6

Consider the compound inequality $x+6<6(x+11)$ or $x\le\ \ 5$ ......(*).

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

10A.15 HKDSE MA 2017 I – 5

(a) Find the range of values of x which satisfy both $7(x-2)\le\frac{11x+8}{3}$ and $6\ \ \ x<5$.

(b) How many integers satisfy both inequalities in (a)?

10A.16 HKDSE MA 2018 – I 6

(a) Find the range of values of x which satisfy both $\frac{3-x}{2}>2x+7$ and $x+8\ge 0$.

(b) Write down the greatest integer satisfying both inequalities in (a).

10A.17 HKDSE MA 2019 – I – 6

(a) Solve the inequality $\frac{7x+26}{4}\le 2(3x-1)$.

(b) Find the number of integers satisfying both inequalities $\frac{7x+26}{4}\le 2(3x-1)$ and $45\ \ \ 5x\ge 0$.

10A.18 HKDSE MA 2020 I 6

Consider the compound inequality

$3\ \ x>\frac{7-x}{2}$ or $5+x>4$ .... (*).

(a) Solve (*).

(b) Write down the greatest negative integer satisfying (*).

(4 marks)

10B Quadratic inequalities in one unknown

10B.1 HKCEE MA 1982(1/2/3) I – 3

Solve $2x^2 - x < 36$.

10B.2 HKCEE MA 1988 – I – 3

Solve the inequality $2x^2 \geq 5x$.

10B.3 HKCEE MA 1990 – I – 4

(a) Solve the following inequalities:
  (i) $6x+1 \geq 2x-3$,
  (ii) $(2-x)(x+3) > 0$.
(b) Using (a), find the values of x which satisfy both $6x+1 \geq 2x-3$ and $(2-x)(x+3) > 0$.

10B.4 HKCEE MA 1993 – I – 4

Solve the inequality $x^2 - x - 2 < 0$.
Hence solve the inequality $(y-100)^2 - (y-100) - 2 < 0$.

10B.5 HKCEE MA 1996 – I – 5

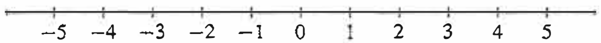
Solve (i) $\frac{x+5}{2} > 4$; (ii) $x^2 - 6x + 8 < 0$.
Hence write down the range of values of x which satisfy both the inequalities in (i) and (ii).

10B.6 HKCEE MA 1997 – I – 4

Solve (i) $2x - 17 > 0$, (ii) $x^2 - 16x + 63 > 0$.
Hence write down the range of values of x which satisfy both the inequalities in (i) and (ii).

10B.7 HKCEE MA 2001 – I – 4

Solve $x^2 + x - 6 > 0$ and represent the solution in the figure.



10B.8 HKCEE AM 1985 – I – 3

Solve the inequality $x^2 - ax - 4 \leq 0$, where a is real.

If, among the possible values of x satisfying the above inequality, the greatest is 4, find the least.

10B.9 HKCEE AM 1986 – I – 7

Solve $x > \frac{3}{x} + 2$ for each of the following cases:
(a) $x > 0$;
(b) $x < 0$.

10B.10 (HKCEE AM 1994 – I – 1)

Solve the inequality $\frac{2(x+1)}{x-2} \geq 1$ for each of the following cases:
(a) $x > 2$;
(b) $x < 2$.

10B.11 HKCEE AM 1995 – I – 4

Solve the inequality $x - \frac{5}{x} > 4$ for each of the following cases:
(a) $x > 0$;
(b) $x < 0$.

10B.12 (HKCEE AM 1996 – I – 3)

Solve the inequality $\frac{2x-3}{x+1} \leq 1$ for each of the following cases:
(a) $x > -1$;
(b) $x < -1$.

10B.13 HKCEE AM 1998 – I – 6(a)

Solve $x^2 - 6x - 16 > 0$.

10B.14 (HKCEE AM 1999 – I – 2)

Solve the inequality $\frac{x}{x-1} > 2$ for each of the following cases:
(a) $x > 1$;
(b) $x < 1$.

10B.15 (HKCEE AM 2000 – I – 1)

Solve the inequality $\frac{1}{x} \geq 1$ for each of the following cases:
(a) $x > 0$;
(b) $x < 0$.

10B.16 HKCEE AM 2011 – 3

Solve the following inequalities:
(a) $5x - 3 > 2x + 9$;
(b) $x(x-8) \leq 20$;
(c) $5x - 3 > 2x + 9$ or $x(x-8) \leq 20$.

10C Problems leading to quadratic inequalities in one unknown

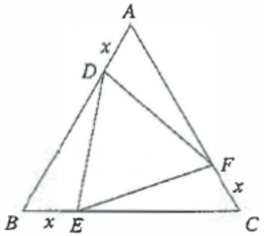
10C.1 HKCEE MA 1983(B) – I – 14 (Continued from **6C.3**.)

α and β are the roots of the quadratic equation $x^2 - 2mx + n = 0$, where m and n are real numbers.

(a) Find, in terms of m and n,
  (i) $(m-\alpha)+(m-\beta)$,
  (ii) $(m-\alpha)(m-\beta)$.
(b) Find, in terms of m and n, the quadratic equation having roots $m-\alpha$ and $m-\beta$.
(c) If $n=4$, find the range of values of m such that the equation $x^2-2mx+n=0$ has real roots.

10C.2 HKCEE MA 1985(A/B) – I – 13 (Continued from **7C.1**.)

In the figure, ABC is an equilateral triangle. $AB=2$. D, E, F are points on AB, BC, CA respectively such that $AD=BE=CF=x$.



(a) By using the cosine formula or otherwise, express DE^2 in terms of x.
(b) Show that the area of $\triangle DEF = \frac{\sqrt{3}}{4}(3x^2-6x+4)$.
  Hence, by using the method of completing the square, find the value of x such that the area of $\triangle DEF$ is smallest.
(c) If the area of $\triangle DEF \le \frac{\sqrt{3}}{3}$, find the range of the values of x.

10C.3 HKCEE MA 1987(B) – I – 14 (Continued from **8C.4**.)

Given $p=y+z$, where y varies directly as x, z varies inversely as x and x is positive. When $x=2$, $p=7$; when $x=3$, $p=8$.

(a) Find p when $x=4$.
(b) Find the range of values of x such that p is less than 13.

10C.4 HKCEE MA 1992 – I – 6

Find the range of values of k so that the quadratic equation $x^2+2kx+(k+6)=0$ has two distinct real roots.

10C.5 HKCEE MA 2003 – I – 10 (Continued from **8C.14**.)

The speed of a solar powered toy can is V cm/s and the length of its solar panel is L cm, where $5 \le L \le 25$. V is a function of L. It is known that V is the sum of two parts, one part varies as L and the other part varies as the square of L. When $L=10$, $V=30$ and when $L=15$, $V=75$.

(a) Express V in terms of L.
(b) Find the range of values of L when $V \ge 30$.

10C.6 HKCEE MA 2004 – I – 10 (Continued from **8C.15**.)

It is known that y is the sum of two parts, one part varies as x and the other part varies as the square of x. When $x=3$, $y=3$ and when $x=4$, $y=12$.

(a) Express y in terms of x.
(b) If x is an integer and $y<42$, find all possible value(s) of x.

10C.7 HKCEE AM 1983 – I – 1

Determine the range of values of λ for which the equation $x^2+4x+2+\lambda(2x+1)=0$ has no real roots.

10C.8 HKCEE AM 1988 – I – 5

Let $f(x)=x^2+4mx+4m+15$, where m is a constant. Find the discriminant of the equation $f(x)=0$. Hence, or otherwise, find the range of values of m so that $f(x)>0$ for all real values of x.

10C.9 HKCEE AM 1988 – I – 10 (Continued from **7B.10**.)

Let $f(x)=x^2+2x-1$ and $g(x)=-x^2+2kx-k^2+6$ (where k is a constant.)

(a) Suppose the graph of $y=f(x)$ cuts the x-axis at the points P and Q, and the graph of $y=g(x)$ cuts the x axis at the points R and S.
  (i) Find the lengths of PQ and RS.
  (ii) Find, in terms of k, the x-coordinate of the mid-point of RS.
  If the mid points of PQ and RS coincide with each other, find the value of k.
(b) If the graphs of $y=f(x)$ and $y=g(x)$ intersect at only one point, find the possible values of k; and for each value of k, find the point of intersection.
(c) Find the range of values of k such that $f(x)>g(x)$ for any real value of x.

10C.10 HKCEE AM 1991 – I – 7 (Continued from **6C.17**.)

p, q and k are real numbers satisfying the following conditions: $\begin{cases} p+q+k=2, \\ pq+qk+kp=1. \end{cases}$

(a) Express pq in terms of k.
(b) Find a quadratic equation, with coefficients in terms of k, whose roots are p and q.
  Hence find the range of possible values of k.

10C.11 HKCEE AM 1991 – I – 9 (Continued from **7B.11**.)

Let $f(x)=x^2+2x-2$ and $g(x)=-2x^2-12x-23$.

(a) Express $g(x)$ in the form $a(x+b)^2+c$, where a, b and c are real constants.
  Hence show that $g(x)<0$ for all real values of x.
(b) Let k_1 and k_2 $(k_1>k_2)$ be the two values of k such that the equation $f(x)+kg(x)=0$ has equal roots.
  (i) Find k_1 and k_2.
  (ii) Show that $f(x)+k_1g(x) \le 0$ and $f(x)+k_2g(x) \ge 0$ for all real values of x.
(c) Using (a) and (b), or otherwise, find the greatest and least values of $\frac{f(x)}{g(x)}$.

10C.12 HKCEE AM 1995 – I – 1

Let $f(x)=x^2+(1-m)x+2m-5$, where m is a constant. Find the discriminant of the equation $f(x)=0$. Hence find the range of values of m so that $f(x)>0$ for all real values of x.

10C.13 (HKCEE AM 1995 – I – 10) ***[Difficult]*** (Continued from **6C.20**.)

Let $f(x)=12x^2+2px-q$ and $g(x)=12x^2+2qx-p$, where p, q are distinct real numbers. α, β are the roots of the equation $f(x)=0$ and α, γ are the roots of the equation $g(x)=0$.

(a) Using the fact that $f(\alpha)=g(\alpha)$, find the value of α. Hence show that $p+q=3$.
(b) Express β and γ in terms of p.
(c) Suppose $-\frac{7}{24}<\beta^3+\gamma^3<\frac{7}{24}$.
  (i) Find the range of possible values of p.
  (ii) Furthermore, if $p>q$, write down the possible integral values of p and q.

10C.14 (HKCEE AM 1996 I 8)

The graph of $y = x^2 - (k-2)x + k + 1$ intersects the x-axis at two distinct points $(\alpha, 0)$ and $(\beta, 0)$, where k is real.

(a) Find the range of possible values of k.

(b) Furthermore, if $-5 < \alpha + \beta < 5$, find the range of possible values of k.

10C.15 (HKCEE AM 1997 – I 8)

Let α and β be the roots of the equation $x^2 + (k+2)x + 2(k-1) = 0$, where k is real.

(a) Show that α and β are real and distinct.

(b) If the difference between α and β is larger than 3, find the range of possible values of k.

10C.16 HKCEE AM 1999 – I – 4

Let $f(x) \quad 2x^2 + 2(k-4)x + k$, where k is real.

(a) Find the discriminant of the equation $f(x) = 0$.

(b) If the graph of $y = f(x)$ lies above the x axis for all values of x, find the range of possible values of k.

10C.17 HKCEE AM 2005 – 5

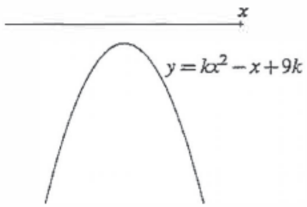
Find the range of values of k such that $x^2 - x - 1 > k(x-2)$ for all real values of x.

10C.18 HKCEE AM 2006 – 4

If $kx^2 + x + k > 0$ for all real values of x, where $k \neq 0$, find the range of possible values of k.

10C.19 HKCEE AM 2008 – 4

The graph of $y = kx^2 \quad x + 9k$ lies below the x axis, where $k \neq 0$ (see the figure). Find the range of possible values of k.



10C.20 HKCEE AM 2010 – 4

It is given that $(k \quad 1)x^2 + kx + k \geq 0$ for all real values of x. Find the range of possible values of k.

10D Linear programming (with given region)

10D.1 HKCEE MA 1984(A/B) – I 8

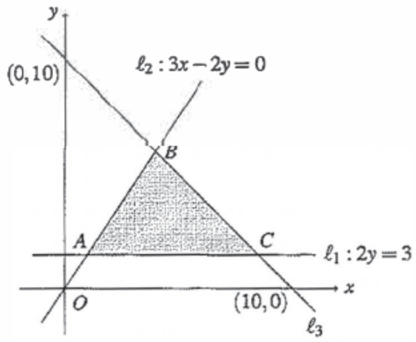
In the figure, $\ell_1 : 2y = 3$, $\ell_2 : 3x - 2y = 0$. The line ℓ_3 passes through $(0, 10)$ and $(10, 0)$.

(a) Find the equation of ℓ_3.

(b) Find the coordinates of the points A, B and C.

(c) In the figure, the shaded region, including the boundary, is determined by three inequalities. Write down these inequalities.

(d) (x, y) is any point in the shaded region, including the boundary, and $P = x + 2y - 5$. Find the maximum and minimum values of P.



10D.2 HKCEE MA 1988 – I – 12

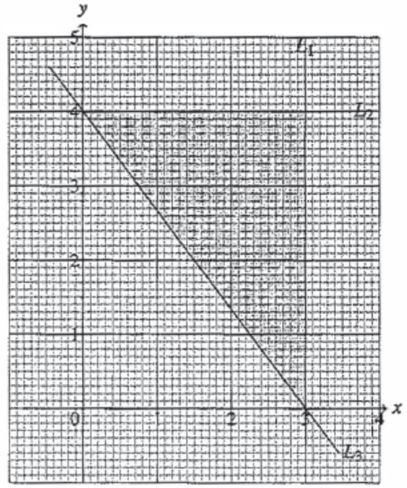
In the figure, L_1 is the line $x = 3$ and L_2 is the line $y = 4$. L_3 is the line passing through the points $(3, 0)$ and $(0, 4)$.

(a) Find the equation of L_3 in the form $ax + by = c$, where a, b and c are integers.

(b) Write down the three constraints which determine the shaded region, including the boundary.

(c) Let $P = x + 4y$. If (x, y) is any point satisfying all the constraints in (b), find the greatest and the least values of P.

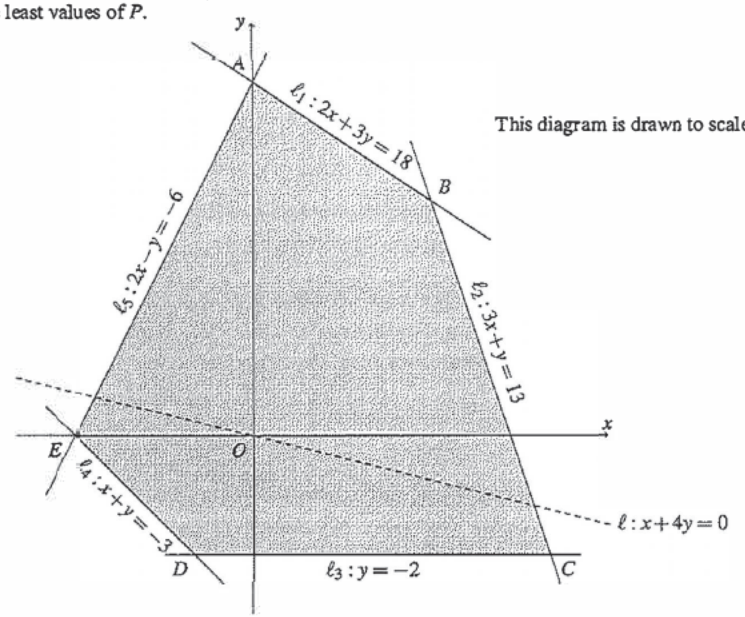
(d) If one more constraint $2x - 3y + 3 \leq 0$ is added, shade in the figure the new region satisfying all the four constraints.
For any point (x, y) lying in the new region, find the least value of P defined in (c).



10D.3 HKCEE MA 1990 I 5

In the figure, the shaded region $ABCDE$ is bounded by the five given lines ℓ_1, ℓ_2, ℓ_3, ℓ_4 and ℓ_5. The line $\ell: x+4y=0$ passes through the origin O.

Let $P=x+4y\ \ 2$, where (x,y) is any point in the shaded region including the boundary. Find the greatest and the least values of P.



This diagram is drawn to scale.

10D.4 HKCEE MA 1991 – I – 8

In the figure, L_1 is the line $x=4$, L_2 is the line passing through the point $(0,2)$ with slope 1, and L_3 is the line passing through the points $(5,0)$ and $(0,5)$.

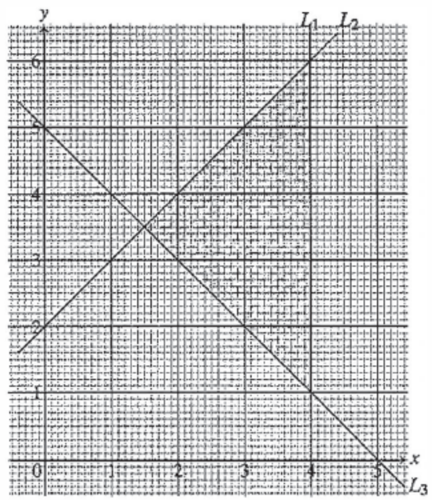
(a) Find the equations of L_2 and L_3.

(b) Write down the three inequalities which determine the shaded region, including the boundary.

(c) Suppose $P=x+2y-3$ and (x,y) is any point satisfying all the inequalities in (b).

(i) Find the point (x,y) at which P is a minimum. What is this minimum value of P?

(ii) If $P \geq 7$, by adding a suitable straight line to the figure, find the range of possible values of x.



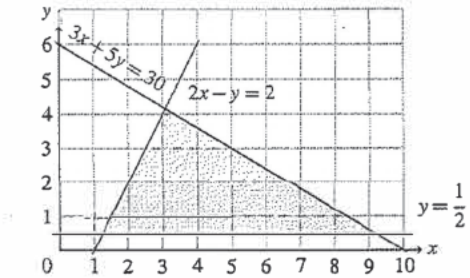
10D.5 HKCEE MA 1992 – I – 3

In this question, working steps are not required and you need to given the answers only.

In the figure, the shaded region, including the boundary, is determined by three inequalities.

(a) Write down the three inequalities.

(b) How many points (x,y), where x and y are both integers, satisfy the three inequalities in (a)?



10D.6 HKCEE MA 1993 I 1(d)

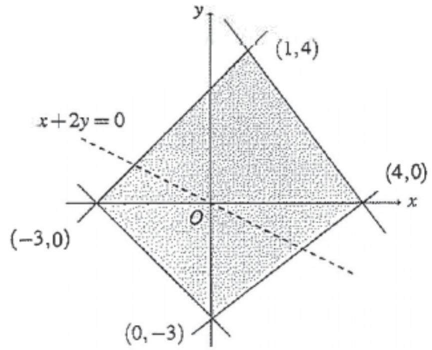
In this question, working steps are not required and you need to give the answers only.

In the figure, find a point (x,y) in the shaded region (including the boundary) at which the value of $x+2y$ is

(i) greatest,

(ii) least.

What are these greatest and least values?



10D.7 HKCEE MA 1995 – I – 12

A box of Brand X chocolates costs \$25 and contains 20 chocolates. A box of Brand Y chocolates costs \$37.50 and contains 40 chocolates.

Mrs. Chiu wants to spend not more than \$300 to buy at least 240 chocolates for her students. She wants to buy at least 3 boxes of each brand of chocolates but not more than 10 boxes altogether.

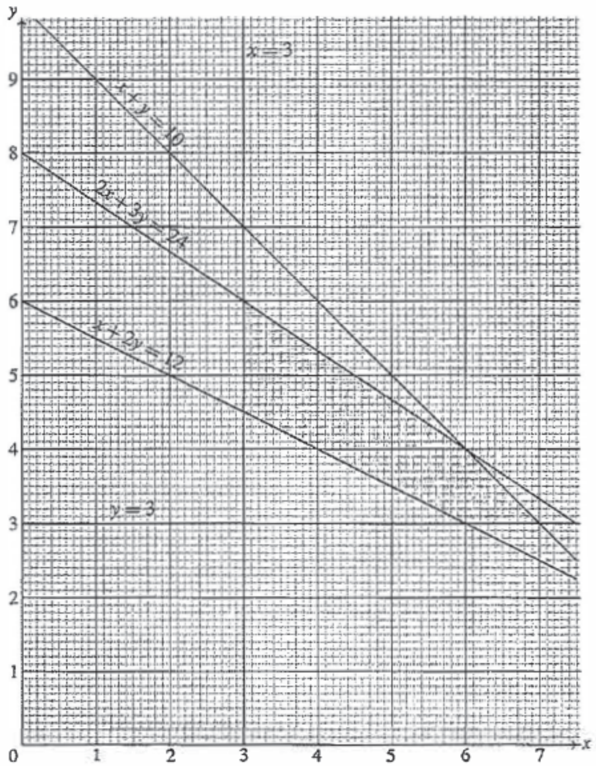
(a) If Mrs. Chiu buys x boxes of Brand X chocolates and y boxes of Brand Y chocolates, then x, y are integers such that $x \geq 3$ and $y \geq 3$. Write down the inequalities in terms of x and y which say
   (i) the total number of chocolates is at least 240;
   (ii) the total cost is not more than \$300;
   (iii) the total number of boxes is not more than 10.

(b) The points representing the ordered pairs (x, y) satisfying all the constraints in (a) are contained in the shaded region in the graph below. List all these ordered pairs (x, y).

(c) Find the least amount Mrs. Chiu has to pay in buying chocolates for her students.

(d) Mrs. Chiu goes to a shop to buy the chocolates. She finds that she can get a free gift for every purchase of \$300. In order to get the free gift, she decides to spend exactly \$300 on buying the chocolates. Find
   (i) all possible combinations (x, y) of the numbers of boxes of Brand X and Brand Y chocolates, and
   (ii) the greatest number of chocolates

   Mrs. Chiu can buy.



10D.8 HKCEE MA 1996 – I – 9

In the figure, $\mathscr{R}$ is the region (including the boundary) bounded by the three straight lines

$L_1 : 3x + 2y - 7 = 0,$

$L_2 : 3x - 5y + 7 = 0$

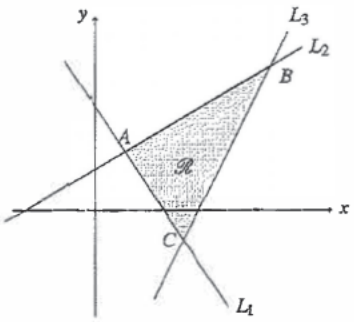
and $L_3 : 2x - y - 7 = 0.$

L_1 and L_2 intersect at $A(1, 2)$. L_2 and L_3 intersect at $B(6, 5)$.

(a) Find the coordinates of C at which L_1 and L_3 intersect.

(b) Write down the three inequalities which define the region $\mathscr{R}$.

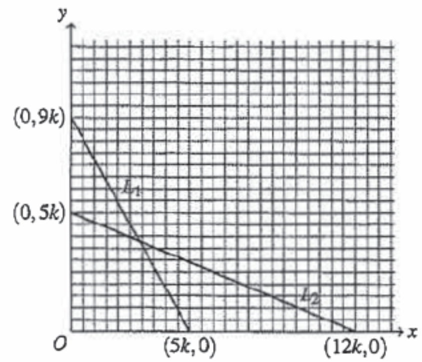
(c) Find the maximum value of $2x - 2y - 7$, where (x, y) is any point in the region $\mathscr{R}$.



10D.9 HKCEE MA 2002 – I – 17

(a) The figure shows two straight lines L_1 and L_2. L_1 cuts the coordinate axes at the points $(5k, 0)$ and $(0, 9k)$ while L_2 cuts the coordinate axes at the points $(12k, 0)$ and $(0, 5k)$, where k is a positive integer. Find the equations of L_1 and L_2.

(b) A factory has two production lines A and B. Line A requires 45 man-hours to produce an article and the production of each article discharges 50 units of pollutants. To produce the same article, line B required 25 man hours and discharges 120 units of pollutants. The profit yielded by each article produced by the production line A is \$3000 and the profit yielded by each article produced by the production line B is \$2000.
   (i) The factory has 225 man hours available and the total amount of pollutants discharged must not exceed 600 units. Let the number of articles produced by the production lines A and B be x and y respectively. Write down the appropriate inequalities and by putting $k = 1$ in the figure, find the greatest possible profit of the factory.
   (ii) Suppose now the factory has 450 man hours available and the total amount of pollutants discharged must not exceed 1200 units. Using the figure, find the greatest possible profit.



10D.10 HKCEE MA 2009 – I 16

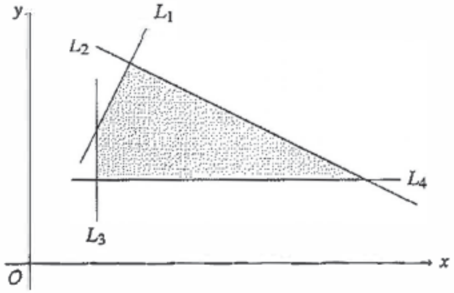
(a) In the figure, the straight lines L_1 and L_2 are perpendicular to each other. The equations of the straight lines L_3 and L_4 are $x = 8$ and $y = 10$ respectively. It is given that L_1 and L_2 intersect at the point $(12, 24)$ while L_1 and L_3 intersect at the point $(8, 16)$.

(i) Find the equations of L_1 and L_2.

(ii) In the figure, the shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities.

(b) There are two kinds of dining tables placed in a restaurant: square tables and round tables. The manager of the restaurant wants to place at least 8 square tables and 10 round tables. Moreover, the number of round tables placed is not more than 2 times that of the square tables placed. Each square table occupies a floor area of $4\,m^2$ and each round tables occupies a floor area of $8\,m^2$. The floor area occupied by the dining tables in the restaurant is at most $240\,m^2$. On a certain day, the profits on a square table and a round table at \$4000 and \$6000 respectively.

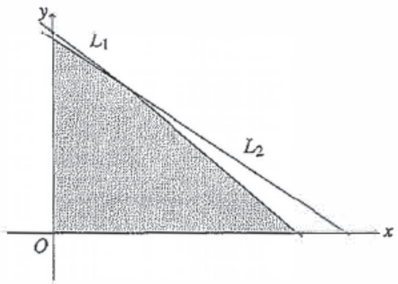
The manager claims that the total profit on the dining tables can exceed \$230000 that day. Do you agree? Explain your answer.



10D.11 HKDSE MA 2014 – I – 18

(a) In the figure, the equation of the straight line L_1 is $6x + 7y = 900$ and the x intercept of the straight line L_2 is 180. L_1 and L_2 intersect at the point $(45, 90)$. The shaded region (including the boundary) represents the solution of a system of inequalities. Find the system of inequalities.

(b) A factory produces two types of wardrobes, X and Y. Each wardrobe X requires 6 man-hours for assembly and 2 man-hours for packing while each wardrobe Y requires 7 man-hours for assembly and 3 man hours for packing. In a certain month, the factory has 900 man hours available for assembly and 360 man hours available for packing. The profits for producing a wardrobe X and a wardrobe Y are \$440 and \$665 respectively. A worker claims that the total profit can exceed \$80 000 that month. Do you agree? Explain your answer.



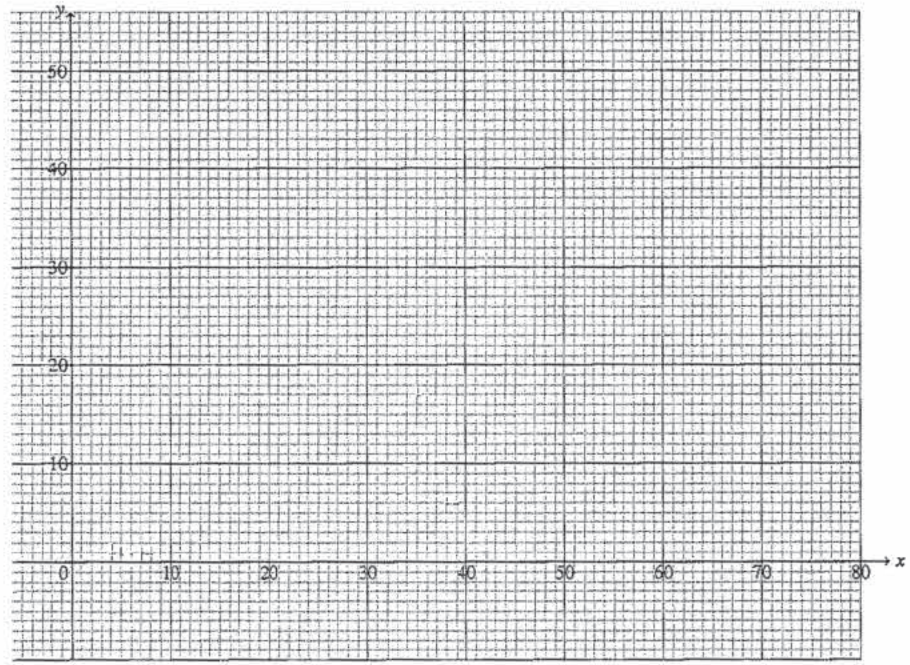
10E Linear programming (without given region)

10E.1 HKCEE MA 1980(1/1*/3) I 12

An airline company has a small passenger plane with a luggage capacity of 720 kg, and a floor area of $60\,m^2$ for installing passenger seats. An economy class seat takes up $1\,m^2$ of floor area while a first class seat takes up $1.5\,m^2$. The company requires that the number of first class seats should not exceed the number of economy class seats. An economy class passenger cannot carry more than 10 kg of luggage while a first-class passenger cannot carry more than 30 kg of luggage.

The profit from selling a first class ticket is double that from selling an economy-class ticket. If all tickets are sold out in every flight, find graphically how many economy-class seats and how many first class seats should be installed to give the company the maximum profit.

(Let x be the number of economy-class seats installed, y be the number of first-class seats installed.)



10E.2 HKCEE MA 1981(1/2/3) I – 8

An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions:

(1) The hostel will accommodate at least 48 persons.

(2) Each single room will occupy an area of $10\,\text{m}^2$, each double room will occupy an area of $15\,\text{m}^2$ and the total available floor area for the rooms is $450\,\text{m}^2$.

(3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

10E.3 HKCEE MA 1983(A/B) I 12

(a) On the graph paper provided below, draw the following straight lines:

$$y = 2x, \quad x + y = 30, \quad 2x + 3y = 120.$$

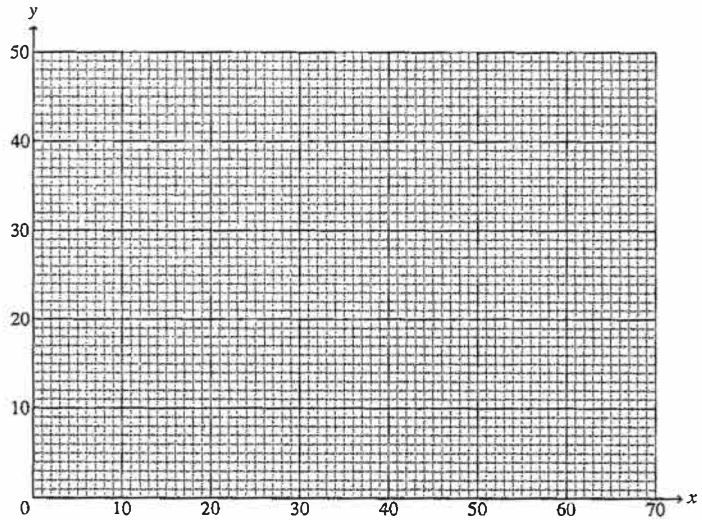
(b) On the same graph paper, shade the region that satisfies all the following inequalities:

$$\begin{cases} y \geq 0, \\ y \leq 2x, \\ x + y \geq 30, \\ 2x + 3y \leq 120. \end{cases}$$

(c) It is given that $P = 3x + 2y$. Under the constraints given by the inequalities in (b),

(i) find the maximum and minimum values of P, and

(ii) find the maximum and minimum values of P if there is the additional constraint $x \leq 45$.



10E.4 HKCEE MA 1986(A/B) – I 11

(a) (i) On the graph paper provided, draw the following straight lines:

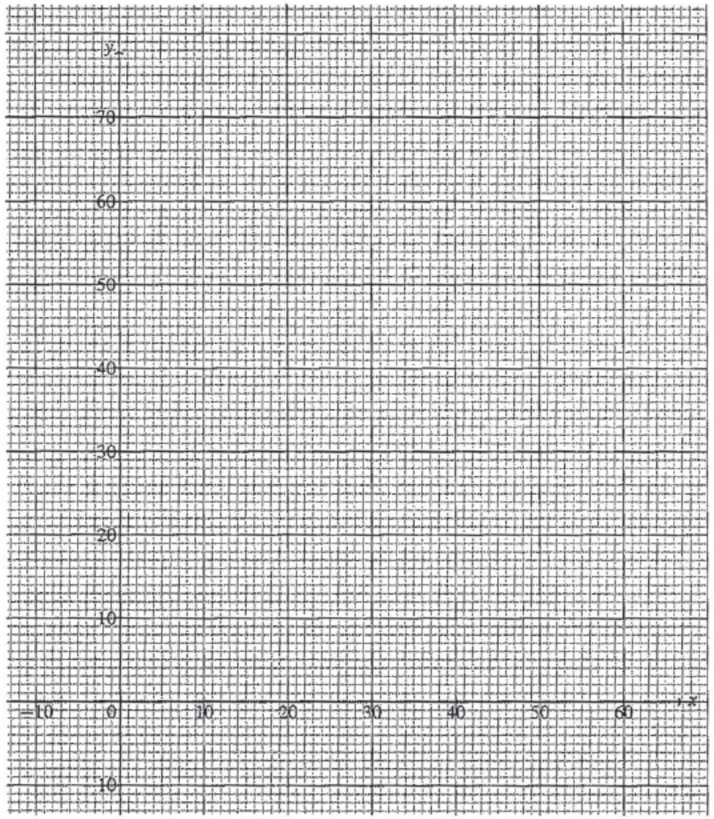
$$x+y=40,\quad x+3y=60,\quad 7x+2y=140.$$

(ii) On the same graph, paper, shade the region that satisfies all the following constraints:

$$x\geq 0,\quad y\geq 0,\quad x+y\geq 40,\quad x+3y\geq 60,\quad 7x+2y\geq 140.$$

(b) A company has two workshops A and B. Workshop A produces 1 cabinet, 1 table and 7 chairs each day. Workshop B produces 1 cabinet, 3 tables and 2 chairs each day. The company gets an order for 40 cabinets, 60 tables and 140 chairs. The expenditures to operate Workshop A and Workshop B are respectively \$1000 and \$2000 each day. Use the result of (a)(ii) to find the number of days each workshop should operate to meet the order if the total expenditure in operating the workshops is to be kept to a minimum.

(Denote the number of days that Workshops A and B should operate by x and y respectively.)



10E.5 HKCEE MA 1987(A/B) – I 12

A factory produces three products A, B and C from two materials M and N.

Each tonne of M produces 4000 pieces of A, 20 000 pieces of B and 6000 pieces of C.

Each tonne of N produces 6000 pieces of A, 5000 pieces of B and 3000 pieces of C.

The factory has received an order for 24 000 pieces of A, 60 000 pieces of B and 24 000 pieces of C. The costs of M and N are respectively \$4000 and \$3000 per tonne. By following the steps below, determine the least cost of the materials used so as the meet the order.

(a) Suppose x tonnes of M and y tonnes of N were used. By considering the requirement of A, B and C of the order, five constraints could be obtained. Three of them are:

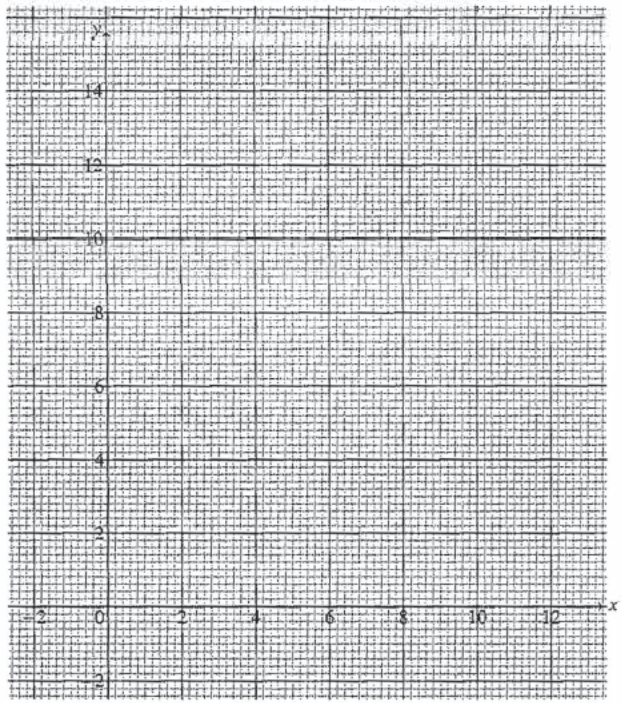
$$x\geq 0,\quad y\geq 0,\quad 4000x+6000y\geq 24000.$$

Write down the other two constraints on x and y.

(b) On the graph paper provided, draw and shade the region which satisfies the five constraints in (a).

(c) Express the cost of materials in terms of x and y.

Hence use the graph in (b) to find the least cost of materials used to meet the order.



10E.6 HKCEE MA 1989 – I – 14

(a) In the figure, draw and shade the region that satisfies the following inequalities:

$$\begin{cases} y \geq 20 \\ 2x \quad y \geq 40 \\ x + y \leq 100 \end{cases}$$

(b) The vitamin content and the cost of three types of food X, Y and Z are shown in the following table:

	Food X	Food Y	Food Z
Vitamin A (units/kg)	400	600	400
Vitamin B (units/kg)	800	200	400
Cost (dollars/kg)	6	5	4

A man wants to produce 100 kg of a mixture by mixing these three types of food. Let the amount of food X, food Y and food Z used by x, y and z kilograms respectively.

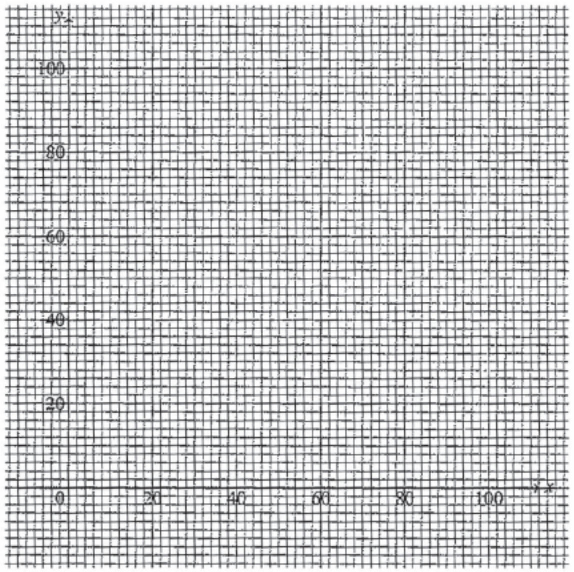
(i) Express z in terms of x and y.

(ii) Express the cost of the mixture in terms of x and y.

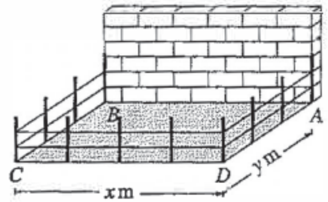
(iii) Suppose the mixture must contain at least 44 000 units of vitamin A and 48 000 units of vitamin B.

Show that $\begin{cases} y \geq 20 \\ 2x - y \geq 40 \\ x + y \leq 100 \end{cases}$

(iv) Using the result in (a), determine the values of x, y and z so that the cost is the least.



10E.7 HKCEE MA 1994 – I – 11



(a) Draw the following straight lines on the graph paper provided:

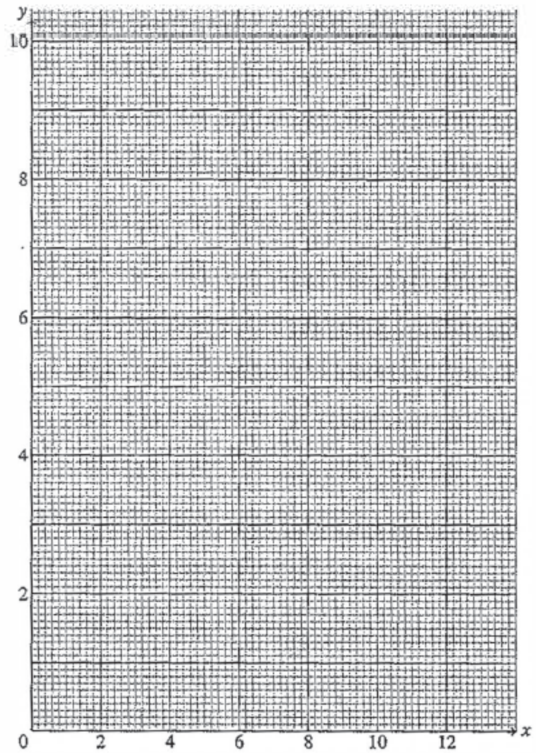
$x+y=10$, $x+2y=12$, $2x=3y$.

(b) Mr. Chan intends to employ a contractor to build a rectangular flower bed $ABCD$ with length AB equal to x metres and width BC equal to y metres. This project includes building a wall of length x metres along the side AB and fences along the other three sides as shown in the figure.

Mr. Chan wishes to have the total length of the four sides of the flower bed not less than 20 metres, and he also adds the condition that twice the length of the flower bed should not less than three times its width. However, no contractor will build the fences if their total length is less than 12 metres.

(i) Write down all the above constraints for x and y.

(ii) Mr. Chan has to pay the contractor $500 per metre for building the wall and $300 per metre for building the fences. Find the length and width of the flower bed so that the total payment for building the wall and fences is the minimum.
Find also the minimum total payment.



10E.8 HKCEE MA 1998 – I – 18

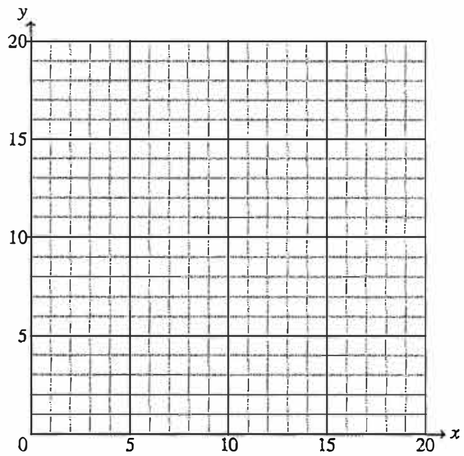
Miss Chan makes cookies and cakes for a school fair. The ingredients needed to make a tray of cookies and a tray of cakes are shown in the table.

	Flour	Sugar	Eggs
Cookies	0.32 kg	0.24 kg	2
Cakes	0.28 kg	0.36 kg	10

Miss Chan has 4.48 kg of flour, 4.32 kg of sugar and 100 eggs, from which she makes x trays of cookies and y trays of cakes.

(a) Write down the inequalities that represent the constraints on x and y. Let $\mathscr{R}$ be the region of points representing ordered pairs (x,y) which satisfy these inequalities. Draw and shade the region $\mathscr{R}$ in the figure below.

(b) The profit from selling a tray of cookies is \$90, and that from selling a tray of cakes is \$120. If x and y are integers, find the maximum possible profit.

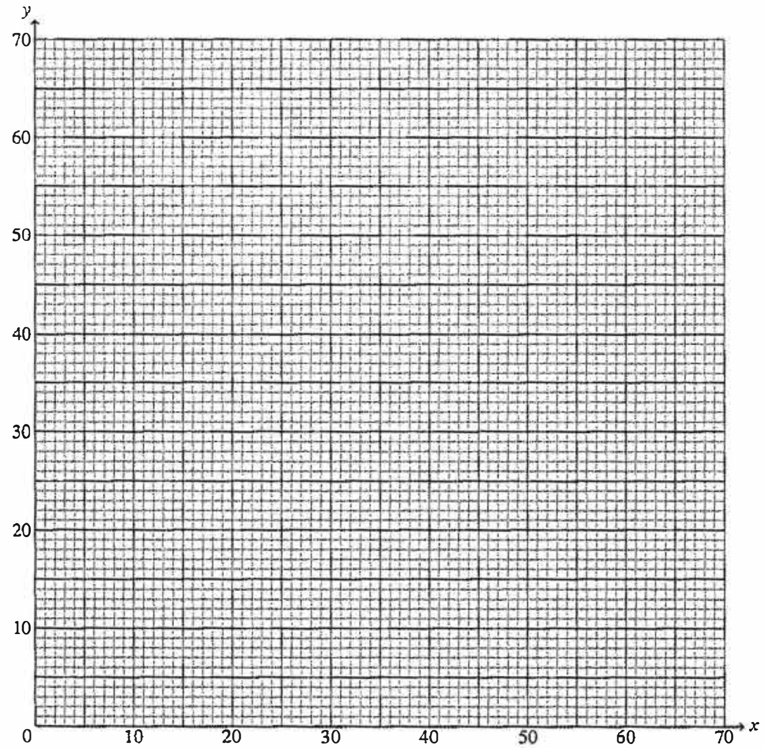


10E.9 HKCEE MA 2000 – I – 15

A company produces two brands, A and B, of mixed nuts by putting peanuts and almonds together. A packet of brand A mixed nuts contains 40 g of peanuts and 10 g of almonds. A packet of brand B mixed nuts contains 30 g of peanuts and 25 g of almonds. The company has 2400 kg of peanuts, 1200 kg of almonds and 70 carton boxes. Each carton box can pack 1000 brand A packets or 800 brand B packets.
The profits generated by a box of brand A mixed nuts and a box of brand B mixed nuts are \$800 and \$1000 respectively. Suppose x boxes of brand A mixed nuts and y boxes of brand B mixed nuts are produced.

(a) Using the graph paper provided, find x and y so that the profit is the greatest.

(b) If the number of boxes of brand B mixed nuts is to be smaller than the number of boxes of brand A mixed nuts, find the greatest profit.



10E.10 HKCEE MA 2001 – I – 15

(a) In Figure (1), shade the region that represents the solution to the following constraints: $\begin{cases} 1 \le x \le 9, \\ 0 \le y \le 9, \\ 5x - 2y > 15. \end{cases}$

(b) A restaurant has 90 tables. Figure (2) shows its floor plan where a circle represents a table. Each table is assigned a 2 digit number from 10 to 99. A rectangular coordinate system is introduced to the floor plan such that the table numbered $10x + y$ is located at (x, y) where x is the tens digit and y is the units digit of the table number. The table numbered 42 has been marked in the figure as an illustration.
The restaurant is partitioned into two areas, one smoking and one non smoking. Only those tables with the digits of the table numbers satisfying the constraints in (a) are in the smoking area.

(i) In Figure (2), shade all the circles which represent the tables in the smoking area.

(ii) *[Probability]*
Two tables are randomly selected, one after another and without replacement from the 90 tables. Find the probability that

(1) the first selected table is in the smoking area;

(2) of the two selected tables, one is in the smoking area, and the other is in the non smoking area and its number is a multiple of 3.

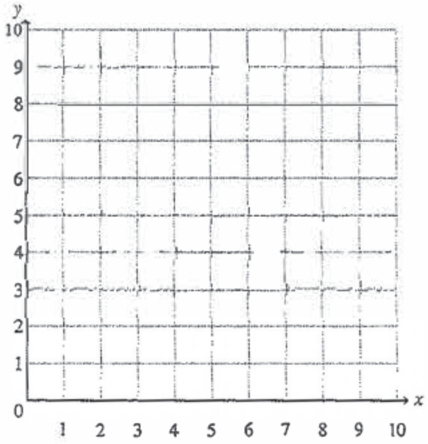


Figure (1)

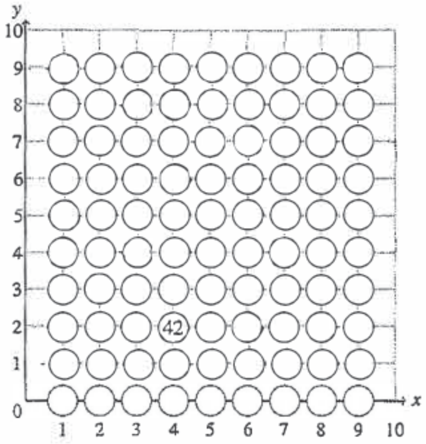


Figure (2)

10 Inequalities and Linear Programming

10A Linear inequalities in one unknown

10A.1 HKCEE MA 1989 – I – 2

(a) $5x+5>3x+2 \Rightarrow 2x>-3 \Rightarrow x>\frac{-3}{2}$

(b) $-\frac{3}{2}<x\le 4$

10A.2 HKCEE MA 1995 – I – 1(a)

$3x+1\ge 7 \Rightarrow 3x\ge 6 \Rightarrow x\ge 2$

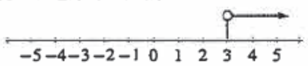
10A.3 HKCEE MA 1999 – I – 3

$3x-4>2(x-1) \Rightarrow 3x-4>2x-2 \Rightarrow x>2$

'And' with $x<6$: $2<x<6$

10A.4 HKCEE MA 2000 – I – 5

$11-2x<5 \Rightarrow 2x>6 \Rightarrow x>3$



10A.5 HKCEE MA 2002 – I – 7

(a) $3x+6\ge 4+x \Rightarrow 2x\ge -2 \Rightarrow x\ge -1$

(b) $2x-5<0 \Rightarrow x<\frac{5}{2}$

$\therefore$ 'And': $-1\le x<\frac{5}{2}$

10A.6 HKCEE MA 2003 – I – 2

$\frac{3-5x}{4}\ge 2-x \Rightarrow 3-5x\ge 8-4x \Rightarrow x\le -5$

$x+8>0 \Rightarrow x>-8$

$\therefore$ 'And': $-8<x\le -5$

10A.7 HKCEE MA 2005 – I – 4

$-3x+1>4x-20 \Rightarrow 7x<21 \Rightarrow x<3$

$2x+1\ge 0 \Rightarrow x\ge \frac{-1}{2}$

$\therefore$ 'And': $\frac{-1}{2}\le x<3$

10A.8 HKCEE MA 2006 – I – 2

(a) $6x+6<x+25 \Rightarrow 5x<19 \Rightarrow x<\frac{19}{5}$

(b) 3

10A.9 HKCEE MA 2008 – I – 2

(a) $14x\ge 10x+35 \Rightarrow 4x\ge 35 \Rightarrow x\ge \frac{35}{4}$

(b) 9

10A.10 HKCEE MA 2010 – I – 2

(a) $29x-22\le 21x \Rightarrow 8x\le 22 \Rightarrow x\le \frac{11}{4}$

(b) 2

10A.11 HKDSE MA 2012 – I – 6

(a) $\frac{4x+6}{7}>2(x-3) \Rightarrow 4x+6>14x-42 \Rightarrow x<\frac{24}{5}$

$2x-10\le 10 \Rightarrow x\le 10$

'And': $x<\frac{24}{5}$

(b) 4 (1, 2, 3 and 4)

10A.12 HKDSE MA 2013 – I – 5

(a) $\frac{19-7x}{3}>23-5x \Rightarrow 19-7x>69-15x \Rightarrow x>\frac{25}{4}$

(b) $18-2x\ge 0 \Rightarrow x\le 9$

$\therefore$ Integers satisfying both: 7, 8 and 9

10A.13 HKDSE MA 2015 – I – 5

(a) $\frac{7-3x}{5}\le 2(x+2) \Rightarrow 7-3x\le 10x+20 \Rightarrow x\ge -1$

$4x-13>0 \Rightarrow x>\frac{13}{4}$

$\therefore$ 'And'. $x>\frac{13}{4}$

(b) 4

10A.14 HKDSE MA 2016 – I – 6

(a) $x+6<6(x+11) \Rightarrow x>-12$

$\therefore$ 'Or': $x>-12$

(b) -1

10A.15 HKDSE MA 2017 – I – 5

(a) $7(x-2)\le \frac{11x+8}{3} \Rightarrow 21x-42\le 11x+8 \Rightarrow x\le 5$

$6-x<5 \Rightarrow x>1$

$\therefore$ 'And': $1<x\le 5$

(b) 4 (2, 3, 4 and 5)

10A.16 HKDSE MA 2018 – I – 6

(a) $\frac{3-x}{2}>2x+7 \Rightarrow 3-x>4x+14 \Rightarrow x<\frac{-11}{5}$

$x+8\ge 0 \Rightarrow x\ge -8$

$\therefore$ 'And': $-8\le x<\frac{-11}{5}$

(b) -3

10A.17 HKDSE MA 2019 – I – 6

(a) $\frac{7x+26}{4}\le 2(3x-1) \Rightarrow 7x+26\le 24x-8 \Rightarrow x\ge 2$

(b) $45-5x\ge 0 \Rightarrow x\le 9$

$\therefore$ 'And': $2\le x\le 9$

$\therefore$ 8 (2, 3, 4, 5, 6, 7, 8, 9)

10A.18 HKDSE MA 2020 – I – 6

6a

$3-x>\frac{7-x}{2}$ or $5+x>4$

$6-2x>7-x$ or $x>-1$

$x<-1$ or $x>-1$

Therefore, x can be any real numbers except -1.

b -2

10B Quadratic inequalities in one unknown

10B.1 HKCEE MA 1982(1/2/3) – I – 3

$2x^2-x-36<0$

$(2x-9)(x+4)<0 \Rightarrow -4<x<\frac{9}{2}$

10B.2 HKCEE MA 1988 – I – 3

$2x^2-5x\ge 0$

$x(2x-5)\ge 0 \Rightarrow x\le 0$ or $x\ge \frac{5}{2}$

10B.3 HKCEE MA 1990 – I – 4

(a) (i) $6x+1\ge 2x-3 \Rightarrow 4x\ge -4 \Rightarrow x\ge -1$

(ii) $(2-x)(x+3)>0 \Rightarrow -3<x<2$

(b) $-1\le x<2$

10B.4 HKCEE MA 1993 – I – 4

$x^2-x-2<0 \Rightarrow (x+1)(x-2)<0 \Rightarrow -1<x<2$

Hence, $-1<y-100<2 \Rightarrow 99<y<102$

10B.5 HKCEE MA 1996 – I – 5

(i) $x+5>8 \Rightarrow x>3$

(ii) $(x-2)(x-4)<0 \Rightarrow 2<x<4$

Hence, $3<x<4$

10B.6 HKCEE MA 1997 – I – 4

(i) $2x>17 \Rightarrow x>\frac{17}{2}$

(ii) $(x-9)(x-7)>0 \Rightarrow x<7$ or $x>9$

Hence, $x>9$

10B.7 HKCEE MA 2001 – I – 4

$x^2+x-6>0 \Rightarrow (x+3)(x-2)>0 \Rightarrow x<-3$ or $x>2$

10B.8 HKCEE AM 1985 – I – 3

$x^2-ax-4\le 0 \Rightarrow \frac{a-\sqrt{a^2+16}}{2}\le x\le \frac{a+\sqrt{a^2+16}}{2}$

$\therefore \frac{a+\sqrt{a^2+16}}{2}=4 \Rightarrow a^2+16=(8-a)^2 \Rightarrow a=3$

$\Rightarrow$ Least possible value of $x=\frac{(3)-\sqrt{(3)^2+16}}{2}=-1$

10B.9 HKCEE AM 1986 – I – 7

(a) $x>\frac{3}{x}+2 \Rightarrow x^2>3+2x$

$\Rightarrow x^2-2x-3>0 \Rightarrow x<-1$ or $x>3$

$\because x>0$

$\therefore x>3$

(b) $x>\frac{3}{x}+2 \Rightarrow x^2<3+2x$

$\Rightarrow x^2-2x-3<0 \Rightarrow -1<x<3$

$\because x<0$

$\therefore -1<x<0$

10B.10 (HKCEE AM 1994 – I – 1)

(a) $\frac{2(x+1)}{x-2}\ge 1 \Rightarrow 2x+2\ge x-2 \Rightarrow x\ge -4$

$\because x>2$

$\therefore x\ge -4$ 'and' $x>2 \Rightarrow x>2$

(b) $\frac{2(x+1)}{x-2}>1 \Rightarrow 2x+2\le x-2 \Rightarrow x\le -4$

$\because x<2$

$\therefore x\le -4$ 'and' $x<2 \Rightarrow x\le -4$

10B.11 HKCEE AM 1995 – I – 4

Solve the inequality $x-\frac{5}{x}>4$ for each of the following cases:

(a) $x-\frac{5}{x}>4 \Rightarrow x^2-5>4x$

$\Rightarrow x^2-4x-5>0 \Rightarrow x<-1$ or $x>5$

$\because x>0$

$\therefore x>5$

(b) $x-\frac{5}{x}>4 \Rightarrow x^2-5<4x$

$\Rightarrow x^2-4x-5<0 \Rightarrow -1<x<5$

$\because x<0$

$\therefore -1<x<0$

10B.12 (HKCEE AM 1996 – I – 3)

(a) $\frac{2x-3}{x+1}<1 \Rightarrow 2x-3\le x+1 \Rightarrow x\le 4$

$\because x>-1$

$\therefore -1<x\le 4$

(b) $\frac{2x-3}{x+1}<1 \Rightarrow 2x-3\ge x+1 \Rightarrow x\ge 4$

$\because x<-1$

$\therefore$ No solution

10B.13 HKCEE AM 1998 – I – 6(a)

$x^2-6x-16>0 \Rightarrow (x-8)(x+2)>0 \Rightarrow x<-2$ or $x>8$

10B.14 (HKCEE AM 1999 – I – 2)

(a) $\frac{x}{x-1}>2 \Rightarrow x>2(x-1) \Rightarrow x<2$

$\because x>1$

$\therefore 1<x<2$

(b) $\frac{x}{x-1}>2 \Rightarrow x<2(x-1) \Rightarrow x>2$

$\because x<1$

$\therefore$ No solution

10B.15 (HKCEE AM 2000 – I – 1)

Solve the inequality $\frac{1}{x}\ge 1$ for each of the following cases:

(a) $\frac{1}{x}>1 \Rightarrow 1\ge x \Rightarrow x\le 1$

$\because x>0$

$\therefore 0<x\le 1$

(b) $\frac{1}{x}\ge 1 \Rightarrow 1\le x \Rightarrow x\ge 1$

$\because x<0$

$\therefore$ No solution

10B.16 HKCEE AM 2011 – 3

Solve the following inequalities:

(a) $5x-3>2x+9 \Rightarrow 3x>12 \Rightarrow x>4$

(b) $x(x-8)\le 20 \Rightarrow x^2-8x-20\le 0 \Rightarrow -2\le x\le 10$

(c) 'Or': $x\ge -2$

10C Problems leading to quadratic inequalities in one unknown

10C.1 HKCEE MA 1983(B) – I – 14

(a) $\begin{cases}\alpha+\beta=2m\\ \alpha\beta=n\end{cases}$

(i) $(m-\alpha)+(m\ \ \beta)=2m\ \ (\alpha+\beta)=2m-(2m)=0$

(ii) $(m\ \ \alpha)(m\ \ \beta)=m^2-(\alpha+\beta)m+\alpha\beta$
$=m^2-(2m)m+(n)=n\ \ m^2$

(b) By (a), the equation is
$x^2\ \ (\text{sum})\ \ x+(\text{product})=0$
$x^2-(0)x+(n-m^2)=0 \Rightarrow x^2+n\ \ m^2=0$

(c) $x^2\ \ 2mx+4=0$
Real roots $\Rightarrow \Delta\geq 0$
$(2m)^2-4(4)\geq 0$
$m^2\geq 4 \Rightarrow m\leq -2$ or $m\geq 2$

10C.2 HKCEE MA 1985(A/B) – I – 13

(a) $DE^2=BD^2+BE^2\ \ 2\cdot BD\cdot BE\cos\angle B$
$=(2\ \ x)^2+x^2-2(2\ \ x)(x)\cos 60°$
$=3x^2-6x+4$

(b) Area of $\triangle DEF=\frac{1}{2}DE\cdot DE\sin 60°$
$=\frac{1}{2}(3x^2\ \ 6x+4)\ \frac{\sqrt{3}}{2}$
$=\frac{\sqrt{3}}{4}(3x^2\ \ 6x+4)$
$=\frac{3\sqrt{3}}{4}\left(x^2\ \ 2x+\frac{4}{3}\right)$
$=\frac{3\sqrt{3}}{4}\left(x^2\ \ 2x+1+\frac{1}{3}\right)$
$=\frac{3\sqrt{3}}{4}(x-1)^2+\frac{\sqrt{3}}{4}$
∴ Minimum area is attained when $x=1$.

(c) $\frac{3\sqrt{3}}{4}(x\ \ 1)^2+\frac{\sqrt{3}}{4}\leq\frac{\sqrt{3}}{3}$
$(x-1)^2\leq\frac{1}{9}$
$\frac{-1}{3}\leq x\ \ 1\leq\frac{1}{3} \Rightarrow \frac{2}{3}\leq x\leq\frac{4}{3}$

10C.3 HKCEE MA 1987(B) – I – 14

(a) Let $p=ax+\frac{b}{x}$.
$\begin{cases}7=2a+\frac{b}{2} \Rightarrow 4a+b=14\\ 8=3a+\frac{b}{3} \Rightarrow 9a+b=24\end{cases} \Rightarrow \begin{cases}a=2\\ b=6\end{cases}$
∴ $p=2x+\frac{6}{x}$.
When $x=4$, $p=2(4)+\frac{6}{(4)}=\frac{19}{2}$.

(b) $2x+\frac{6}{x}<13$
$2x^2+6<13x$ (∵ given $x>0$)
$2x^2\ \ 13x+6<0 \Rightarrow \frac{1}{2}<x<6$

10C.4 HKCEE MA 1992 – I – 6

$\Delta>0$
$(2k)^2+4(k+6)>0$
$(k+2)(k+3)>0 \Rightarrow k<-3$ or $k>-2$

10C.5 HKCEE MA 2003 – I – 10

(a) Let $V=hL+kL^2$.
$\begin{cases}30=10h+100k\\ 75=15h+225k\end{cases} \Rightarrow \begin{cases}h=-1\\ k=0.4\end{cases} \Rightarrow V=0.4L^2-L$

(b) $0.4L^2\ \ L\geq 30$
$2L^2\ \ 5L-150\geq 0 \Rightarrow L\leq\frac{-15}{2}$ or $L\geq 10$
Since $5\leq L\leq 25$, the solution is $10\leq L\leq 25$.

10C.6 HKCEE MA 2004 – I – 10

(a) Let $y=hx+kx^2$.
$\begin{cases}3=3h+9k\\ 12=4h+16k\end{cases} \Rightarrow \begin{cases}h=-5\\ k=2\end{cases} \Rightarrow y=2x^2-5x$

(b) $2x^2-5x<42 \Rightarrow 2x^2-5x\ \ 42<0 \Rightarrow -\frac{7}{2}<x<6$
∴ Possible values of x are 3, 2, 1, 0, 1, 2, 3, 4 and 5.

10C.7 HKCEE AM 1983 – I – 1

$x^2+4x+2+\lambda(2x+1)=0 \Rightarrow x^2+2(2+\lambda)x+(2+\lambda)=0$
No real roots $\Rightarrow \Delta<0$
$4(2+\lambda)^2\ \ 4(2+\lambda)<0$
$\lambda^2+3\lambda+2<0 \Rightarrow 2<\lambda<\ \ 1$

10C.8 HKCEE AM 1988 – I – 5

$\Delta\ \ (4m)^2\ \ 4(4m+15)=16m^2\ \ 16m+60$
If $f(x)>0$ for all real x, $\Delta<0$
$4(4m^2\ \ 4m+15)<0$
$(2m+3)(2m\ \ 5)<0 \Rightarrow \frac{3}{2}<m<\frac{5}{2}$

10C.9 HKCEE AM 1988 – I – 10

(a) (i) For $f(x)$, $\begin{cases}\text{Sum of rts}=\ \ 2\\ \text{Prod of rts}=-1\end{cases}$

For $g(x)$, $\begin{cases}\text{Sum of rts}=2k\\ \text{Prod of rts}=k^2\ \ 6\end{cases}$

PQ = Difference of rts of $f(x)$
$=\sqrt{(\ \ 2)^2-4(\ \ 1)}=\sqrt{8}$
RS = Difference of rts of $g(x)$
$=\sqrt{(2k)^2\ \ 4(k^2-6)}=\sqrt{24}$

(ii) Mid-pt of $RS=\left(\frac{\text{Sum of rts}}{2},0\right)=(k,0)$

If this is also the mid-point of PQ, $k=\frac{\ \ 2}{2}=-1$.

(b) $\begin{cases}y=f(x)\\ y=g(x)\end{cases} \Rightarrow x^2+2x-1=-x^2+2kx-k^2+6$
$2x^2+2(1-k)x+k^2\ \ 7=0\ \ldots\ (*)$
$\Delta=0$
$4(1\ \ k)^2\ \ 8(k^2\ \ 7)=0$
$k^2+2k-15=0 \Rightarrow k=-5$ or 3
For $k=-5$, $(*)$ becomes $2x^2+12x+18=0$
$2(x+3)^2=0$
$x=\ \ 3$
$\Rightarrow$ Intersection $=(\ \ 3,(\ \ 3)^2+2(\ \ 3)-1)=(\ \ 3,2)$
For $k=3$, $(*)$ becomes $2x^2-4x+2=0$
$2(x-1)^2=0$
$x=1$
$\Rightarrow$ Intersection $=(1,1^2+2(1)\ \ 1)=(1,2)$

(c) $f(x)>g(x)$
$2x^2+2(1-k)x+k^2\ \ 7>0$
If this is true for all real x, $\Delta<0$
$k^2+2k\ \ 15>0$
$k<-5$ or $k>3$

10C.10 HKCEE AM 1991 – I – 7

(a) From the first equation, $p+q=2\ \ k$
From the second equation, $pq+k(p+q)=1$
$pq=1\ \ k(2\ \ k)$
$=(k+1)^2$

(b) Sum of roots $=p+q=2-k$
Product of roots $=(k+1)^2$
∴ Required equation: $x^2-(2-k)x+(k+1)^2=0$
Hence, $\Delta\geq 0$
$(k\ \ 2)^2\ \ 4(k+1)^2\geq 0$
$3k^2+4k\leq 0 \Rightarrow \frac{-4}{3}\leq k\leq 0$

10C.11 HKCEE AM 1991 – I – 9

(a) $g(x)=-2x^2\ \ 12x\ \ 23=\ \ 2(x^2+6x+9\ \ 9)-25$
$=-2(x+3)^2-5$
$\leq -5<0$

(b) (i) $f(x)+kg(x)=0$
$(x^2+2x\ \ 2)+k(\ \ 2x^2\ \ 12x\ \ 23)=0$
$(1-2k)x^2+2(1\ \ 6k)x\ \ (2+23k)=0$
Equal rts $\Rightarrow \Delta=0$
$4(1-6k)^2+4(1\ \ 2k)(2+23k)=0$
$10k^2\ \ 7k-3=0$
$k=1$ or $\frac{-3}{10}$
∴ $k_1=1$, $k_2=\frac{-3}{10}$

(ii) $f(x)+k_1g(x)$
$=(x^2+2x-2)\ \ (2x^2+12x+23)$
$=\ \ x^2-10x\ \ 25=\ \ (x+5)^2\leq 0$
$f(x)+k_2g(x)$
$=(x^2+2x\ \ 2)+\frac{3}{10}(2x^2+12x+23)$
$=\frac{8}{5}\left(x^2+\frac{7}{2}x+\frac{49}{16}\right)=\frac{8}{5}\left(x+\frac{7}{4}\right)^2\geq 0$

(c) $f(x)+k_1g(x)\leq 0$
$f(x)\leq\ \ g(x)$
$\frac{f(x)}{g(x)}\geq -1$ (∵ $g(x)$ < 0 by (a))
∴ Least value $=\ \ 1$
(attained when $f(x)+k_1g(x)=0 \Leftrightarrow x=\ \ 5$)
$f(x)\ \ +k_2g(x)\geq 0$
$f(x)\geq\frac{3}{10}g(x)$
$\frac{f(x)}{g(x)}<\frac{3}{10}$
∴ Greatest value $=\frac{3}{10}$
(attained when $\left(x+\frac{7}{4}\right)^2=0 \Leftrightarrow x=\frac{\ \ 7}{4}$)

10C.12 HKCEE AM 1995 – I – 1

$\Delta=(1\ \ m)^2\ \ 4(2m-5)=m^2\ \ 10m+21$
If $f(x)>0$ for all real x, $\Delta<0$
$m^2\ \ 10m+21<0$
$(m-3)(m-7)<0 \Rightarrow 3<m<7$

10C.13 (HKCEE AM 1995 – I – 10)

(a) $f(\alpha)=g(\alpha)$
$12\alpha^2+2p\alpha\ \ q=12\alpha^2+2q\alpha-p$
$2\alpha(p-q)=\ \ (p-q)$ (∵ p,q are distinct)
$2\alpha=\ \ 1 \Rightarrow \alpha=\frac{-1}{2}$

(b) $\alpha+\beta=\ \ \frac{2p}{12} \Rightarrow \beta=\frac{-p}{6}+\frac{1}{2}$
$\alpha\gamma=\frac{-p}{12} \Rightarrow \gamma=\frac{-p}{12}\div\frac{\ \ 1}{2}=\frac{p}{6}$

(c) (i) $\beta^3+\gamma^3=(\beta+\gamma)(\beta^2-\beta\gamma+\gamma^2)$
$=\left(\frac{1}{2}\right)\left[\frac{p^2}{36}-\frac{p}{6}+\frac{1}{4}\ \ \frac{p}{6}\left(\frac{-p}{6}+\frac{1}{2}\right)+\frac{p^2}{36}\right]$
$=\frac{1}{2}\left(\frac{p^2}{12}\ \ \frac{p}{4}+\frac{1}{4}\right)$
Thus, the given inequality becomes
$\frac{7}{24}<\frac{p^2}{24}-\frac{p}{8}+\frac{1}{8}<\frac{7}{24}$
$\Rightarrow 7<p^2\ \ 3p+3<7$
$\Rightarrow \begin{cases}p^2\ \ 3p\ \ 4<0\\ p^2\ \ 3p+10>0\end{cases}$
$\Rightarrow \begin{cases}1<p<4\\ \text{All real nos}\end{cases} \Rightarrow -1<p<4$

(ii) $p=3$ and $q=0$
$p=2$ and $q=1$ (since $p+q=3$)

10C.14 (HKCEE AM 1996 – I – 8)

The graph of $y=x^2-(k-2)x+k+1$ intersects the x-axis at two distinct points $(\alpha,0)$ and $(\beta,0)$, where k is real.

(a) Two distinct roots $\Rightarrow \Delta>0$
$(k-2)^2\ \ 4(k+1)>0$
$k^2-8k>0 \Rightarrow k<0$ or $k>8$

(b) $-5<\alpha+\beta<5 \Rightarrow 5<k\ \ 2<5 \Rightarrow 3<k<7$
∴ 'And': $3<k<0$

10C.15 (HKCEE AM 1997 – I – 8)

(a) $\Delta=(k+2)^2\ \ 8(k-1)=k^2\ \ 4k+12=(k-2)^2+8$
$\geq 8>0$
∴ The roots are real and distinct.

(b) $\begin{cases}\alpha+\beta=\ \ (k+2)\\ \alpha\beta=2(k\ \ 1)\end{cases}$
$(\alpha\ \ \beta)^2>3^2$
$(\alpha+\beta)^2-4\alpha\beta>9$
$(k+2)^2-8(k-1)\ \ >9$
$(k\ \ 2)^2+8>9$
$(k-2)^2>1 \Rightarrow k\ \ 2<\ \ 1$ or $k-2>1$
$\Rightarrow k<1$ or $k>3$

10C.16 HKCEE AM 1999 – I – 4

Let $f(x)=2x^2+2(k\ \ 4)x+k$, where k is real.

(a) $\Delta=4(k\ \ 4)^2\ \ 8k=4k^2-40k+64$

(b) No intersection with x-axis $\Rightarrow \Delta<0$
$4(k^2\ \ 10k+16)<0$
$(k\ \ 2)(k-8)<0 \Rightarrow 2<k<8$

10C.17 HKCEE AM 2005 – 5

$x^2 \quad x \quad 1 > k(x-2) \Rightarrow x^2-(1+k)x+(2k-1)>0$

If this is true for all real x, $\Delta < 0$

$(1+k)^2-4(2k-1)<0$

$k^2-6k+5<0 \Rightarrow 1<k<5$

10C.18 HKCEE AM 2006 – 4

If $kx^2+x+k>0$ is true for all real x,

$\Delta<0$ and $k>0$

$1^2-4k^2<0$

$k^2>\frac{1}{4} \Rightarrow k<\frac{-1}{2}$ or $k>\frac{1}{2}$

$\therefore \quad k>\frac{1}{2}$

10C.19 HKCEE AM 2008 – 4

$\Delta<0$

$(-1)^2-4(k)(9k)<0$

$1-36k^2<0$

$k^2>\frac{1}{36} \Rightarrow k<\frac{-1}{6}$ or $k>\frac{1}{6}$ (rejected)

10C.20 HKCEE AM 2010 – 4

$k-1>0$ and $\Delta \le 0$

$k^2-4k(k \quad 1) \le 0$

$3k^2-4k\ge 0 \Rightarrow k\le 0$ or $k\ge\frac{4}{3}$

$\Rightarrow \quad k\ge\frac{4}{3}$

10D Linear programming (with given region)

10D.1 HKCEE MA 1984(A/B) – I – 8

(a) (Two-pt form) $\ell_3: \frac{y \quad 0}{x-10}=\frac{10-0}{0-10} \Rightarrow y= \quad x+10$

(Or intercept form) $\ell_3: \frac{x}{10}+\frac{y}{10}=1 \Rightarrow y=-x+10$

(b) $A: \begin{cases}\ell_1: 2y=3 \\ \ell_2: 3x-2y=0\end{cases} \Rightarrow \begin{cases}x=1 \\ y=\frac{3}{2}\end{cases} \Rightarrow A=\left(1,\frac{3}{2}\right)$

$B: \begin{cases}\ell_2: 3x \quad 2y=0 \\ \ell_3: y=-x+10\end{cases} \Rightarrow \begin{cases}x=4 \\ y=6\end{cases} \Rightarrow B=(4,6)$

$C: \begin{cases}\ell_1: 2y=3 \\ \ell_3: y=-x+10\end{cases} \Rightarrow \begin{cases}x=\frac{17}{2} \\ y=\frac{3}{2}\end{cases} \Rightarrow A=\left(\frac{17}{2},\frac{3}{2}\right)$

(c) $\begin{cases}2y\ge 3 \\ 3x-2y\ge 0 \\ y\le \quad x+10\end{cases}$

(d) At A, $P=(1)+2\left(\frac{3}{2}\right)-5=-1$

At B, $P=(4)+2(6)-5=11$

At C, $P=\left(\frac{17}{2}\right)+2\left(\frac{3}{2}\right)-5=\frac{13}{2}$

$\therefore$ Max of $P=11$, min of $P= \quad 1$

10D.2 HKCEE MA 1988 – I – 12

(a) (Two-pt form) $L_3: \frac{y-4}{x-0}=\frac{0-4}{3-0} \Rightarrow 4x+3y=12$

(Or intercept form) $L_3: \frac{x}{3}+\frac{y}{4}=1 \Rightarrow 4x+3y=12$

(b) $\begin{cases}x\le 3 \\ y\le 4 \\ 4x+3y\ge 12\end{cases}$

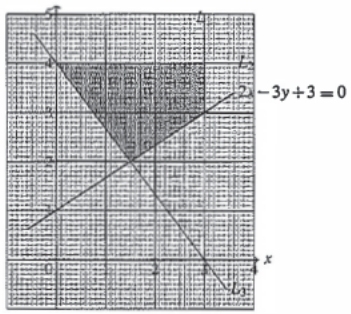
(c) At (0,4), $P=(0)+4(4)=16$

At (3,4), $P=(3)+4(4)=19$

At (3,0), $P=(3)+4(0)=3$

$\therefore$ Greatest $P=19$, least $P=3$

(d)



At (0,4), $P=(0)+4(4)=16$

At (3,4), $P=(3)+4(4)=19$

At (3,3), $P=(3)+4(3)=15$

At (1.5,2), $P=(1.5)+4(2)=9.5$

$\therefore$ Least $P=9.5$

10D.3 HKCEE MA 1990 – I – 5

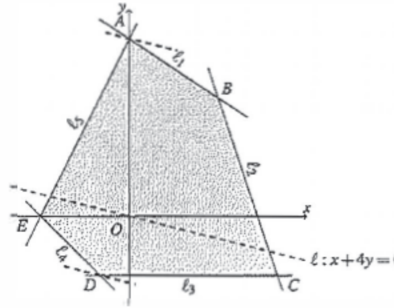
By sliding the dashed line, P attains its greatest value at A and least value at D.

$A: \begin{cases}\ell_1: 2x+3y=18 \\ \ell_5: 2x \quad y=-6\end{cases} \Rightarrow \begin{cases}x=0 \\ y=6\end{cases}$

$\therefore$ Greatest $P=(0)+4(6)-2=22$

$D: \begin{cases}\ell_3: y= \quad 2 \\ \ell_4: x+y=-3\end{cases} \Rightarrow \begin{cases}x= \quad 1 \\ y=-2\end{cases}$

$\therefore$ Least $P=(-1)+4(-2)-2=-11$



10D.4 HKCEE MA 1991 – I – 8

(a) (Slope-int form) $L_2: y=x+2$

(Two-pt form) $L_3: \frac{y \quad 0}{x-5}=\frac{5-0}{0-5} \Rightarrow y=-x+5$

(Or intercept form) $L_3: \frac{x}{5}+\frac{y}{5}=1 \Rightarrow y= \quad x+5$

(b) $\begin{cases}x\le 4 \\ y\le x+2 \\ y\ge -x+5\end{cases}$

(c) (i) At (4,6), $P=(4)+2(6)-3=13$

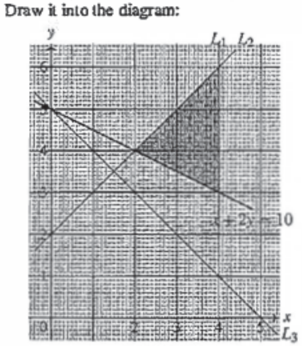
At (4,1), $P=(4)+2(1)-3=3$

At (1.5,3.5), $P=(1.5)+2(3.5)-3=5.5$

$\therefore$ Min of $P=3$, attained at (4,1)

(ii) $P=x+2y-3\ge 7 \Rightarrow x+2y\ge 10$

Draw it into the diagram:

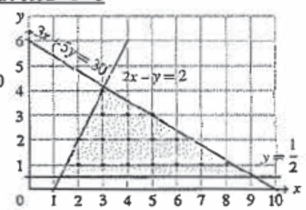


The range of x that covers the new feasible region is $2\le x\le 4$.

10D.5 HKCEE MA 1992 – I – 3

(a) $\begin{cases}y\ge\frac{1}{2} \\ 2x \quad y\ge 2 \\ 3x+5y\le 30\end{cases}$

(b) 16

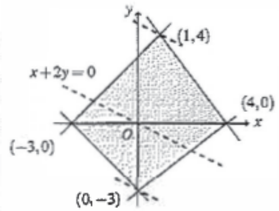


10D.6 HKCEE MA 1993 – I – 1(d)

By sliding the given line,

(i) Greatest value $=(1)+2(4)=9$, at (1,4)

(ii) Least value $=(0)+2(-3)=-6$, at $(0,-3)$



10D.7 HKCEE MA 1995 – I – 12

(a) (i) $20x+40y\ge 240 \Rightarrow x+2y\ge 12$

(ii) $25x+37.5y\le 300 \Rightarrow 2x+3y\le 24$

(iii) $x+y\le 10$

(b) (x and y must be integers!)

(3,5), (3,6), (4,4), (4,5), (5,4), (6,3), (6,4), (7,3)

(c) Cost $=25x+37.5y$

By sliding the line $25x+37.5y=0 \Rightarrow 2x+3y=0$, the least cost is attained at (4,4).

Least cost $=25(4)+37.5(4)=(\$)250$.

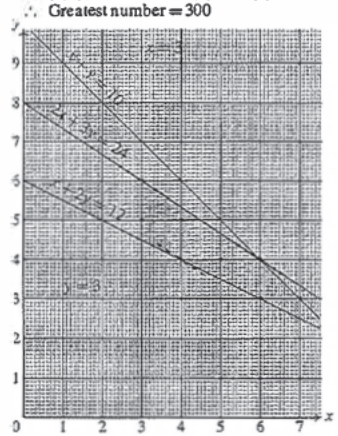
(d) (i) As Cost = 300, the only two points lying on the line $25x+37.5y=300$ are $(x,y)=(3,6)$ and (6,4).

(ii) Number of chocolates $=20x+40y$

At (3,6), Number $=20(3)+40(6)=300$

At (6,4), Number $=20(6)+40(4)=280$

$\therefore$ Greatest number $=300$



10D.8 HKCEE MA 1996 – I – 9

(a) $C:\begin{cases}L_1: 3x+2y-7=0\\ L_3: 2x-y-7=0\end{cases} \Rightarrow \begin{cases}x=3\\ y=-1\end{cases} \Rightarrow (3,-1)$

(b) $\begin{cases}3x+2y-7\ge 0\\ 3x-5y+7\ge 0\\ 2x-y-7\le 0\end{cases}$

(c) At A, $2(1)-2(2)-7=-9$
At B, $2(6)-2(5)-7=-5$
At C, $2(3)-2(-1)-7=1 \Rightarrow$ Max value $=1$

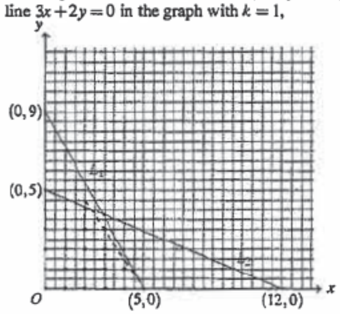
10D.9 HKCEE MA 2002 – I – 17

(a) $L_1: \dfrac{x}{5k}+\dfrac{y}{9k}=1 \Rightarrow 9x+5y=45k$

$L_2: \dfrac{x}{12k}+\dfrac{y}{5k}=1 \Rightarrow 5x+12y=60k$

(b) (i) $\begin{cases}45x+25y\le 225 \Rightarrow 9x+5y\le 45\\ 50x+120y\le 600 \Rightarrow 5x+12y\le 60\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

Let the profit be $P=3000x+2000y$. By sliding the line $3x+2y=0$ in the graph with $k=1$,

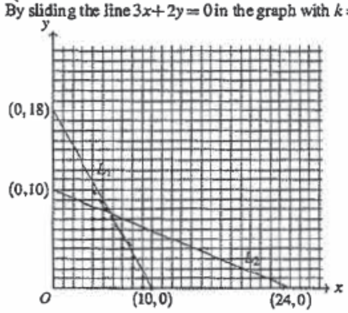


the greatest possible profit is attained at (3,3) and (5,0)

$\therefore$ Greatest profit $=3000(5)+0=(\$)15000$

(ii) $\begin{cases}45x+25y\le 450 \Rightarrow 9x+5y\le 90\\ 50x+120y\le 1200 \Rightarrow 5x+12y\le 120\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

By sliding the line $3x+2y=0$ in the graph with $k=2$,



the greatest possible profit is attained at (6,7)

$\therefore$ Greatest profit $=3000(6)+2000(7)=(\$)32000$

10D.10 HKCEE MA 2009 – I – 16

(a) (i) $L_1: \dfrac{y-24}{x-12}=\dfrac{24-16}{12-8}=2 \Rightarrow y=2x$

$L_2: y-24=\dfrac{-1}{2}(x-12) \Rightarrow x+2y-60=0$

(ii) $\begin{cases}y\le 2x\\ x+2y\le 60\\ x\ge 8\\ y\ge 10\end{cases}$

(b) The constraints are $\begin{cases}x\ge 8\\ y\ge 10\\ y\le 2x\\ 4x+8y\le 240 \Rightarrow x+2y\le 60\\ x \text{ and } y \text{ are integers.}\end{cases}$

Let the profit be $P=4000x+6000y$.
At (8,16), $P=4000(8)+6000(16)=128000$
At (12,24), $P=4000(12)+6000(24)=192000$
At (8,10), $P=4000(8)+6000(10)=92000$
At (40,10), $P=4000(40)+6000(10)=220000$
$\because$ Max profit $=\$220000<\$230000$
$\therefore$ NO.

10D.11 HKDSE MA 2014 – I – 18

(a) $L_2: \dfrac{y-90}{x-45}=\dfrac{90-0}{45-180}=\dfrac{-2}{3} \Rightarrow 2x+3y-360=0$

$\therefore$ The constraints are $\begin{cases}6x+7y\le 900\\ 2x+3y\le 360\\ x\ge 0\\ y\ge 0\end{cases}$.

(b) The constraints are $\begin{cases}6x+7y\le 900\\ 2x+3y\le 360\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

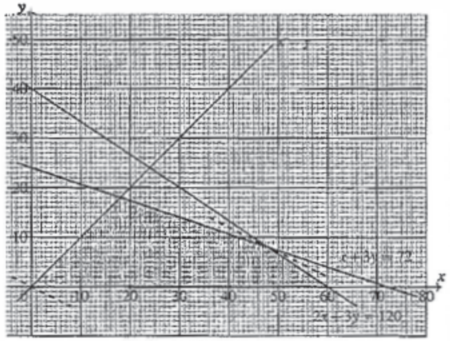
Let the profit be $P=440x+665y$.
At (0,0), $P=440(0)+665(0)=0$
At (0,120), $P=440(0)+665(120)=79800$
At (45,90), $P=440(45)+665(90)=79650$
At (150,0), $P=440(150)+665(0)=66000$
$\because$ Max profit $=\$79800$
$\therefore$ NO.

10E Linear Programming (without given region)

10E.1 HKCEE MA 1980(1/1*/3) – I – 12

$\begin{cases}10x+30y\le 720 \Rightarrow x+3y\le 72\\ x+1.5y\le 60 \Rightarrow 2x+3y\le 120\\ x\ge y\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

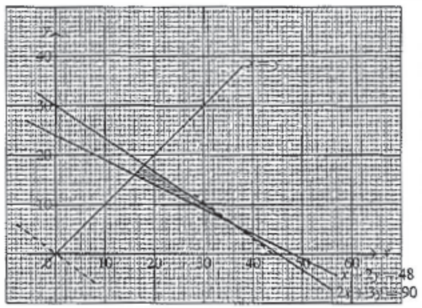
Let the profit be $P=kx+2ky$. By sliding the line $P=0$, the maximum is attained at (48,8).
$\therefore$ 48 economy- and 8 first-class seats respectively



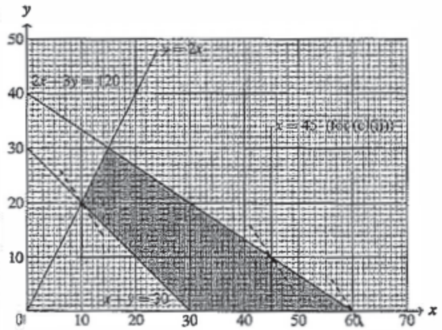
10E.2 HKCEE MA 1981(1/2/3) – I – 8

$\begin{cases}x+2y\ge 48\\ 10x+15y\le 450 \Rightarrow 2x+3y\le 90\\ x\ge y\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

Let the profit be $P=300x+400y$. By sliding the line $P=0$, the maximum is attained at $(x,y)=(36,6)$.

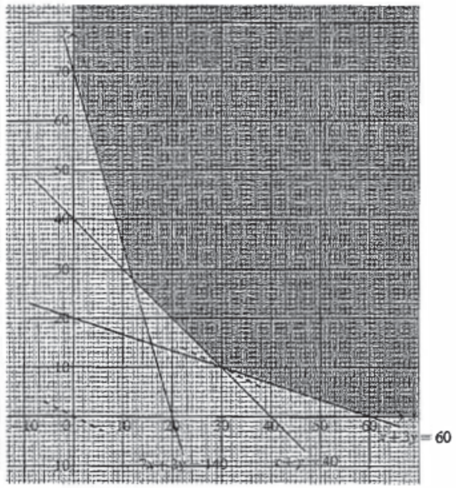


10E.3 HKCEE MA 1983(A/B) – I – 12



(c) (i) Max of $P=3(60)+2(0)=180$
Min of $P=3(10)+2(20)=70$
(ii) Max of $P=3(45)+2(10)=155$
Min of $P=3(10)+2(20)=70$ (unchanged)

10E.4 HKCEE MA 1986(A/B) – I – 11



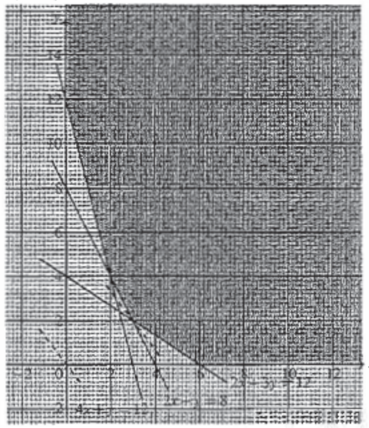
(b) Constraints: $\begin{cases}x+y\ge 40\\ x+3y\ge 60\\ 7x+2y\ge 140\\ x \text{ and } y \text{ are non-negative integers.}\end{cases}$

Let the cost be $C=1000x+2000y$. By sliding the line $C=0$, the minimum is attained at $(x,y)=(30,10)$.
$\therefore$ 30 days for A and 10 days for B

10E.5 HKCEE MA 1987(A/B) – I – 12

(a) $\begin{cases} 20000x+5000y \geq 60000 \\ 6000x+3000y \geq 24000 \end{cases}$

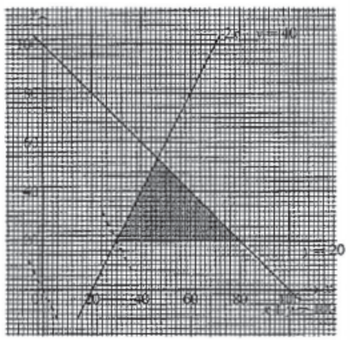
(b) $\begin{cases} x\geq 0, \quad y \geq 0 \\ 2x+3y\geq 12 \\ 4x+y \geq 12 \\ 2x+y\geq 8 \end{cases}$



(c) Let the cost be $C = 4000x+3000y$. By sliding the line $C=0$, the minimum is attained at $(3,2)$.
$\therefore$ Least cost $= 4000(3)+3000(2) = (\$)18000$

10E.6 HKCEE MA 1989 – I – 14

(a)



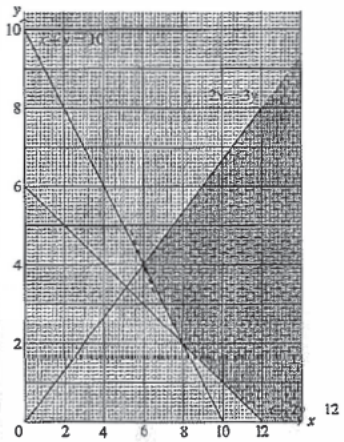
(b) (i) $z = 100-x-y$

(ii) Cost $= 6x+5y+4z \quad 6x+5y+4(100-x-y)$
$= (\$)2x+y+400$

(iii) $400x+600y+400z \geq 44000$
$\Rightarrow 2x+3y+2(100-x-y)\geq 220 \Rightarrow y\geq 20$
$800x+200y+400z\geq 48000$
$\Rightarrow 4x+y+2(100 \quad x \quad y) \geq 240 \Rightarrow 2x-y\geq 40$
$z \geq 0$
$\Rightarrow 100 \quad x \quad y\geq 0 \Rightarrow x+y\leq 100$

(iv) By sliding the line Cost $=0$, the minimum is attained at $(30,20)$.
i.e. $x=30, y=20, z=100 \quad 30 \quad 20=50$

10E.7 HKCEE MA 1994 – I – 11

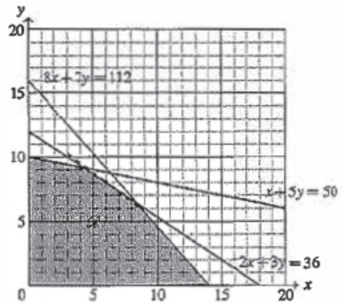


(b) (i) $\begin{cases} 2x+2y\geq 20 \Rightarrow x+y \geq 10 \\ 2x\geq 3y \\ x+2y\geq 12 \\ x\geq 0, \quad y \geq 0 \end{cases}$

(ii) The feasible region is shaded above.
Let the payment be $P = 500x+300(x+2y)$
$= 800x+600y$
By sliding the line $P=0$, the minimum is attained at $(x,y)=(6,4)$.
$\therefore$ Length $= 6$ m, Width $= 4$ m,
Payment $= 800(6)+600(4) = (\$)7200$

10E.8 HKCEE MA 1998 – I – 18

(a) $\begin{cases} 0.32x+0.28y\leq 4.48 \Rightarrow 8x+7y\leq 112 \\ 0.24x+0.36y \leq 4.32 \Rightarrow 2x+3y\leq 36 \\ 2x+10y\leq 100 \Rightarrow x+5y\leq 50 \\ x\geq 0, \quad y\geq 0 \end{cases}$

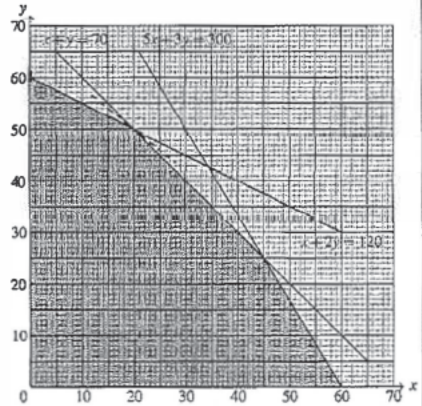


(b) Let the profit be $P = 90x+120y$. By sliding the line $P=0$ among the *lattice points* in $\mathscr{R}$, the maximum is attained at $(6,8)$.
$\therefore$ Max profit $= 90(6)+120(8) = (\$)1500$

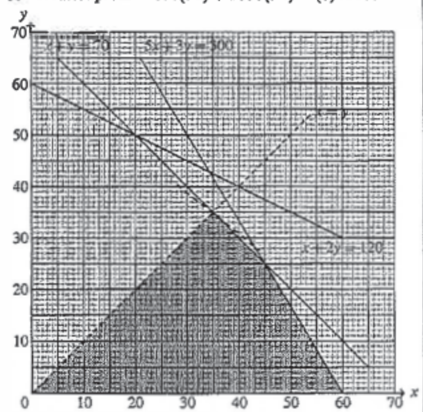
10E.9 HKCEE MA 2000 – I – 15

(a) The constraints are
$\begin{cases} 1000(0.04x)+800(0.03y)\leq 2400 \Rightarrow 5x+3y\leq 300 \\ 1000(0.01x)+800(0.25y)\leq 1200 \Rightarrow x+2y\leq 120 \\ x+y\leq 70 \\ x \text{ and } y \text{ are non-negative integers.} \end{cases}$

The feasible region consists of the *lattice points* in the shaded region below.
Let the profit be $P = 800x+1000y$. By sliding the line $P=0$, the maximum is attained at $(x,y)=(20,50)$.

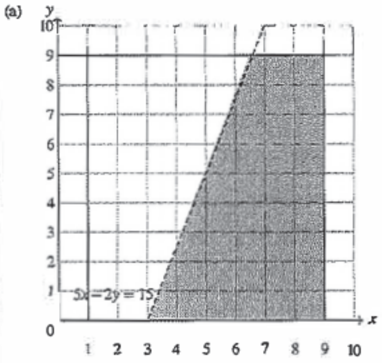


(b) Extra constraint: $x>y$.
The new feasible region consists of the lattice points in the (darker) shaded region below.
P now attains its maximum at $(36,34)$. (Note that $(35,35)$ is not in the feasible region.)
$\therefore$ Greatest profit $= 800(36)+1000(34) = (\$)62800$

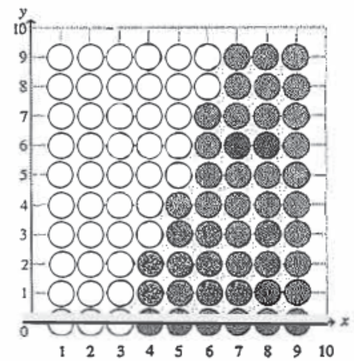


10E.10 HKCEE MA 2001 – I – 15

(a)



(b) (i)



(ii) (1) Number of tables in the smoking area $= 46$
$\therefore$ Prob $= \dfrac{46}{90} = \dfrac{23}{45}$

(2) Number of tables in the non-smoking area & multiple of 3   14
$\therefore$ Prob $\dfrac{46\times 14\times 2!}{90\times 89} = \dfrac{644}{4005}$