Solution	Marks	Remarks
mx = 2(m+c)		
mx = 2m + 2c		,
mx - 2m = 2c	1M	for putting m on one side
m(x-2)=2c	1M	for factorization
• •		
$m = \frac{2c}{x-2}$	1A	·
* *	(3)	·
•		
3-5r		
For $\frac{3-5x}{4} \ge 2-x$, we have	·	
$3-5x \ge 4(2-x)$	ŀ	
$3 \sim 5x \geq 8 - 4x$		
$4x - 5x \ge 8 - 3$	1M	for putting x on one side
-x≥5	1141	101 paring 2 on one side
x ≤5	IA.	
For $x+8>0$, we have		
x > -8		·
So, the required solution is		
$x > -8$ and $x \le -5$. Thus, the required solution is	IA.	do not accept graphical solution
Thus, the required solution is $-8 < x \le -5$		
	(3)	
•		
(a) $x^2 - (y - z)^2$	1	
= (x + (y - z))(x - (y - z))		
•		
=(x+y-z)(x-y+z)	1A	·
(b) $ab-ad-bc+cd$	'	
= a(b-d) - c(b-d) $b(a-c) - d(a-c)$	1M	for taking out common factors
= (a-c)(b-d)	l	TOT CERTIFY OUT COULTHOU SECTORS
· — (w = 6)(D = w)	1A	-
	(3)	
$4^{x+1} = 8$		
$2^{2(x+1)} = 2^3$		
$2^{2x+2} = 2^3$	l 1M	for same base (2, 4 or 8 only)
2x + 2 = 3	13.6	for agustine 41
2x = 1	1M	for equating the powers
$x = \frac{1}{2}$	1A	
$4^{x+1} = 8$		
$\log(4^{x+1}) = \log 8$	1M	for taking log
$(x+1)\log 4 = \log 8$		
log 8	1M	for putting log on one side
$x + 1 = \frac{\log 8}{\log 4}$		
l .		
$x+1=\frac{3}{2}$	•	
	1.A	
$x = \frac{1}{2}$		
	(3)	
	Ι '΄	Provided by dse.li

2003-CE-MATH 1-4

	Solution	Mar	ks Remarks
i. (a)	The selling price of the handbag		
	= 400(1+20%)(0.75) 400(1+20%)(1-25%)	lA	
	= \$360	}	
(b)	Domonto de 1	1A	u-1 for missing unit
(0)	Percentage loss		
	$=\frac{400-360}{400}\times100\%$		ļ
	= 10%	1M	accept without 100%
		1A	f
			-(4)
ĭ a+	w headh and the second		
Then	x be the number of first-class tickets sold.		
× 1104	t, the number of economy-class tickets sold is $3x$. Therefore, we have $x+3x=600$		
	4x = 600	1A	1
	x = 150		J
_		1A	can be absorbed
Th	e sum of money for the tickets sold		Ì
	50)(850) + (3)(150)(500)	l 1M	
= \$35	52 500	ſ	
		lA	u-1 for missing unit r.t. \$353 000
The	e number of first-class tickets sold		:
= (60	$\left(0\right)\left(\frac{1}{1+2}\right)$	1A	
_160	(1+3)		
=150		1	}
The	number of economy-class tickets sold		
= 600 = 450	1-150) IA	for either one and can be absorbed
430		J	
The	sum of money for the tickets sold		
= (150	0)(850) + (450)(500)		
= \$352	•	1M	1.
		1A	u-1 for missing unit r.t. \$353 000
		(4	5 111 2333 000
(a)	Common difference		1
=	5-2		
	The 10th term	1A	
	2 + (10 - 1)(3)		
	29		
		1A	
(b) 7	The sum of the first 10 terms		
٠ _	10		
= -	$\frac{10}{2}(2+29) \qquad \qquad \frac{10}{2}((2)(2)+(10-1)(3))$	1M	for $\frac{10}{2}(2+(a))$
= 1	155	}	2 (2+(4))
<u> </u>		1A	
a) Note	that the arithmetic sequence is 2, 5, 8, 11, 14, 17, 20, 23, 26, 29,	1 4	
The	10th term = 29	1A 1A	
a) The	Sitte of the first 10	1	
=2+	sum of the first 10 terms 5+8+11+14+17+20+23+26+29		
= 155		1M	for 2++(a)
		IA.	
	·	(4)	
			D +1 11 1 1
-MATE	H 1-4		Provided by dse.

	Solution	Marks	Remarks
(a)	In $\triangle ABC$ and $\triangle CDA$, $\angle CAB = \angle ACD$ (alternate $\angle s$, $AB \parallel DC$) $\angle ACB = \angle CAD$ (alternate $\angle s$, $BC \parallel AD$) $AC = CA$ (common side) $\triangle ABC \cong \triangle CDA$ (ASA)		[(內)錯角、AB#DC] [(內)錯角、BC#AD] [公共 邊]
	Marking Scheme: Case 1 Any correct proof with correct reasons.	2	
(b)	Case 2 Any correct proof without reasons. $\Delta ABD \cong \Delta CDB$	1	
(0)	$\Delta ABE \cong \Delta CDE$ $\Delta AED \cong \Delta CEB$		
	Marking Scheme: Case 1 There are exactly three pairs of triangles and all of them are correct.	2	
	Case 2 Any one pair is correct.	1(4)	
(a)	The shortest distance = 100 sin 60°	1 M	
	= 50√3 ≈ 86 6025 2038 ≈ 87 km	1A	u–1 for missing unit
(b)	The distance travelled by S between 1:00 a.m. and when it is nearest to L		
	$= 100 \cos 60^{\circ}$ = 50 km	1M	accept $\frac{87}{\tan 60^\circ}$
	The distance travelled by S between 1:00 a.m. and when it is nearest to L		
	$= \sqrt{100^2 - (50\sqrt{3})^2}$ $= \sqrt{2500}$ $= 50 \text{ km}$	1M	for $\sqrt{100^2 - (a)^2}$
	The time taken $= \frac{50}{20}$	1M	
	= 2.5 hours		
	Therefore, S will be nearest to L at 3:30 arm.	1A (5)	do not accept 2.5 hours after 1:00 a.r

	Solution	Marks	Remarks
0. (a)	Let $V = aL^2 + bL$, where a and b are constants.	1A	
	When $L = 10$, $V = 30$, so we have $100a + 10b = 30$		
	$10a+b=3 \qquad \dots \dots (1)$	} IM	for substitution (either one)
	When $L = 15$, $V = 75$, so we have $225a + 15b = 75$]	
	15a+b=5 (2)		
	Solving (1) and (2), we have		
	$\begin{cases} a = \frac{2}{5} \\ b = -1 \end{cases}$) 1A	for both correct
	b = -1		161 Both correct
	$\therefore V = \frac{2}{5}L^2 - L$		
	5	(3	
	2 ,		
(b)	When $V \ge 30$, we have $\frac{2}{5}L^2 - L \ge 30$ (by(a)). Therefore, we have	ve IM	for putting the result of (a) into $V \ge 3$
	$2L^2 - 5L - 150 \ge 0$	1M	in the form $k_1L^2 + k_2L + k_3 \ge 0$
	$(2L+15)(L-10) \ge 0$ $L \ge 10$ or $L \le 75$	1 A	
	Since $5 \le L \le 25$, we have $10 \le L \le 25$.	1A	accept ' $L \ge 10$ ' and $L \le 25$ ' but
	No.	(4	do not accept graphical solution
		(4	1

		Solution		Marks	Remarks
11. (a)	(i)	The mode = 10		1A	
	(ii)	The median $= \frac{14 + 12}{2}$			
		= 11.5		1A	
	(iii)	= 10 + 10 + 12 + 13 + 16			
		= 12		1A	
	(iv)	The range			
		= 6		1A (4)	
(b)	(i)	The median will be the least when the four unknown data are at most equal to 10. The least possible value of the median			
		= 10	Ψ <u>.</u>	1 A	
		The median will be the greatest when the four unknown data are at least equal to 16. The greatest possible value of the median			·
		= 14.5		1A	
	(ii)	The required mean $= \frac{(12)(6) + (11)(4)}{6 + 4}$		1M	for $(11)(4) = \text{sum of the four unknown data}$
		= 11.6		1A (4)	

	Solution	Marks	Remarks
(a)	The slope of BC		
	$=\frac{3-0}{0-2}$	ļ	
	$=\frac{-3}{2}$	1A	accept -1.5 or $-1\frac{1}{2}$
	2	(1)	· -
(b)	The slope of AP		
	$=\frac{-1}{-1.5}$	1M	can be absorbed
		1141	can be absorbed
	$=\frac{2}{3}$		
	The equation of AP is:		
	$\frac{y-0}{x-(-1)} = \frac{2}{3}$	13.6	£ 1 6
	$x - (-1)^{-3}$	1M	for point-slope form
	2x - 3y + 2 = 0	1A	accept $y = \frac{2x}{3} + \frac{2}{3}$
	·	i	, ,
		(3)	-
(c)	(i) Let the coordinates of H be $(0, h)$.		
	Then, by (b), $2(0) - 3h + 2 = 0$	1M	for putting $x = 0$ into (b)
	$h = \frac{2}{3}$	1A	
	Thus, the coordinates of H are $(0, \frac{2}{3})$.		
	Thus, the coordinates of M are $(0, \frac{\pi}{3})$.		·
	3-0]	
	(ii) The slope of $AC = \frac{3-0}{0-(-1)} = 3$	1A	
	Suppose the altitude from B to AC cuts AC at Q .		
	The slope of $BQ = \frac{-1}{2}$	1M	
	3	11,71	
	The service of $p = 0$ $y = 0$ $y = 0$		or, the slope of $BH = \frac{0 - \frac{2}{3}}{2 - 0} = \frac{1}{3}$
	The equation of BQ is: $\frac{y-0}{x-2} = \frac{-1}{3}$		or, the slope of $BH = \frac{3}{2-0} = -\frac{3}{2}$
	x + 3y - 2 = 0		
	Note that $0 + (3)(\frac{2}{3}) - 2 = 2 - 2 = 0$		
	Hence, the three altitudes pass through the same point 覆.	1	-
	point 2g.		
	$0-\frac{2}{3}$		***
	Note that the slope of $BH = \frac{0 - \frac{2}{3}}{2 - 0} = \frac{-1}{3}$	1M	
	and the slope of $AC = \frac{3-0}{0-(-1)} = 3$,	
	, ,	1A	
	: (the slope of BH) (the slope of AC) = $(\frac{-1}{3})(3) = -1$.	
	∴ BH⊥AC		
	Hence, the three altitudes pass through the same point 程.	1	
		(5)	

Solution	Marks	Remarks
(a) (i) $\frac{x}{360} = \frac{30\pi}{(2\pi)(56+24)}$	1M	for $\frac{x}{360} = \frac{30\pi}{2\pi r}$
x = 67.5	1A	u-1 for having unit
$30\pi = (56 + 24)(\frac{x\pi}{180})$	1M	for $30\pi = r \left(\frac{x\pi}{180} \right)$
x = 67.5	IA_	u-1 for having unit
(ii) The required area = area of sector ODC - area of sector OAB = $\left(\frac{67.5}{360}\right)\left((56+24)^2\pi\right) - \left(\frac{67.5}{360}\right)\left(56^2\pi\right)$	1M	for either one
$= 1200\pi - 588\pi$ $= 612\pi \text{ cm}^2$	1A	u-1 for missing unit
The required area $= \text{area of sector } ODC - \text{ area of sector } OAB$ $= \frac{1}{2}(56 + 24)^2 \left(\frac{67.5\pi}{180}\right) - \frac{1}{2}(56^2) \left(\frac{67.5\pi}{180}\right)$	1M	for either one
$= 1200\pi - 588\pi$ $= 612\pi \text{ cm}^2$	1A	u-1 for missing unit
(b) (i) The required area $= (612\pi)(\frac{18}{24})^{2}$ $= \frac{1377}{4}\pi \text{ cm}^{2}$	1M	for $((a)(ii))(\frac{18}{24})^2$ accept 344.25π cm ² or $344\frac{1}{4}\pi$ cm ² u-1 for missing unit
I8 cm F		
$\frac{FO'}{BO} = \frac{FG}{BC}$ $\therefore \frac{FO'}{56} = \frac{18}{24}$ Hence, $FO' = 42$ cm		
The required area $= \frac{1}{2} (42 + 18)^2 \left(\frac{67.5\pi}{180} \right) - \frac{1}{2} (42^2) \left(\frac{67.5\pi}{180} \right)$	1M	ſ
$= \frac{1377}{4} \pi \text{ cm}^2$	1A	4
*		u-1 for missing unit

Solution	Marks	Remarks
(ii) $2\pi r = \left(\frac{18}{24}\right)(30\pi)$	1M+1M	1M for $\left(\frac{18}{24}\right)(30\pi) +$ 1M for equating $2\pi r$
$2\pi r = 22.5\pi$ $r = \frac{45}{4}$	1A	accept 11.25 r.t. 11.3
$ \begin{array}{c c} B & A \\ C & & D \end{array} $ $ \begin{array}{c c} 30\pi \text{ cm} \end{array} $		\$ cm
$\begin{array}{ccc} \therefore & 2\pi \ s = 30\pi \\ \therefore & s = 15 \end{array}$	MI	for equating $2\pi s$
$r = \left(\frac{FG}{BC}\right)s$ $r = \left(\frac{18}{24}\right)(15)$ 45	1M	accept 11.25
Thus, $r = \frac{45}{4}$		r.t. 11.3
	(5)	
	·	

			Solution		Marks	Remarks
14.	(a)	Вус	osine formula, we have			
			$\cos \angle OAC = \frac{3^2 + 6^2 - 4^2}{(2)(3)(6)}$		1A	
			(2)(3)(6) 2030 ≈ 36.33605751°			
			∠OAC ≈ 36.3°		1A	u-1 for missing unit
					(2)	
	(b)	(i)	$\tan 40^\circ = \frac{BC}{4}$	٦		
	` '	. ,	$ABC = 4 \tan 40^{\circ}$	}	1A	for either one correct
			DC = 3:356398525			
			BC ≈ 3.36 m			2.26
			$\tan 30^\circ = \frac{4\tan 40^\circ}{CD}$		1M	accept $\tan 30^{\circ} \approx \frac{3.36}{CD}$
			©D≈\$\$C352775			
			$CD \approx 5.81 \text{ m}$		1A -	u-1 for missing unit
				,	1	r.t. 5.81
		(ii)	By cosine formula, we have			
			$\cos \angle CAD = \frac{6^2 + 8^2 - CD^2}{(2)(8)(6)}$		1M	with CD substituted
			(2)(8)(0) cos£€2#0≈0\$\$9525#			
			ZCAD = 4639976083°			
			$\angle CAD \approx 46.4^{\circ}$		1A	u–1 for missing unit r.t. 46.4°
					}	1.6. 40.4
		(iii)	By sine formula, we have			
			$\frac{CE}{\sin \angle EAC} = \frac{6}{\sin \theta} \text{and} \frac{ED}{\sin \angle EAD} = \frac{8}{\sin(180^{\circ} - \theta)}$		IM	for either one with angle substitute
			So, $\frac{6\sin 36.33605751^{\circ}}{\sin \theta} + \frac{8\sin 10.06370296^{\circ}}{\sin(180^{\circ} - \theta)} \approx CD$		1M	with CD substituted
			3:555£215_1£3979444043 5id# \$10# 55813452775			
					13.6	C This are Order and in a
			sin θ ≈ 0.85200065 Θ ≈ 58.429942488		1M	for making $\sin \theta$ the subject
			$\theta \approx 58.4^{\circ}$ (: θ is acute)		1A	u-1 for missing unit
						r.t. 58.4°
			By cosine formula, we have	· · · · · ·	 	
			$\cos \angle ACD = \frac{6^2 + CD^2 - 8^2}{(2)(6)(CD)}$		23.5	with CD substituted
			(2)(6)(CD)		2M	with CD substituted
			cos ∠ACD ≈ 0.083086497			
			ZACD=85.234000000°			
			$\angle EAC + \angle ACD + \theta = 180^{\circ} (\angle sum \text{ of } \Delta)$			
			$36.33605751^{\circ} + 85.23400001^{\circ} + \theta = 180^{\circ}$		1M	
			$\theta \approx 58.42994248^{\circ}$ Thus, $\theta \approx 58.4^{\circ}$		1A	u-1 for missing unit
						r.t. 58.4°
					(9))
					- 1	i .

		Solution	Marks	Remarks
5. (a)	(i)	The required area = $\frac{1}{2}k(1-k)\sin 60^{\circ}$	lM	for $\frac{1}{2}ab\sin 60^{\circ}$
		$=\frac{\sqrt{3}}{4}k(1-k) \text{ m}^2$	1A	u-1 for missing unit
	(ii)	By cosine formula, we have $x^2 = k^2 + (1-k)^2 - 2k(1-k)\cos 60^{\circ}$ $x^2 = 3k^2 - 3k + 1$	1M	for $x^2 = a^2 + b^2 - 2ab \cos 60^c$
		$x = \sqrt{3k^2 - 3k + 1}$	1A	
	(iii)	By symmetry, $A_1B_1=B_1C_1=C_1A_1=x$ m. Thus, $A_1B_1C_1$ is an equilateral triangle.	1(5)	
(b)	(i)	$\therefore \frac{A_2B_1}{A_1B_0} = \frac{x(1-k)}{1-k} = x = \frac{xk}{k} = \frac{B_2B_1}{B_1B_0}$		·
		$\angle A_2 B_1 B_2 = 60^\circ = \angle A_1 B_0 B_1$		
		$\therefore \Delta A_1 B_0 B_1 \sim \Delta A_2 B_1 B_2 \qquad \text{(ratio of 2 sides, inc.} \angle \text{)}$		[兩邊成比例且夾角相等]
		Marking Scheme:		
		Case 1 Any correct proof with correct reasons. Case 2 Any correct proof without reasons.	1 2	
	(ii)	 ∴ Δ A₁B₀B₁ ~ Δ A₂B₁B₂ ~ Δ A₃B₂B₃ ~ ∴ their areas form a geometric sequence with a common ratio x². 		
		So, the total area	1A	can be absorbed
		$= \frac{\sqrt{3}}{4}k(1-k) + \frac{\sqrt{3}}{4}k(1-k)x^2 + \frac{\sqrt{3}}{4}k(1-k)x^4 + \cdots$:
		$=\frac{\frac{\sqrt{3}}{4}k(1-k)}{1-x^2}$	1M	for $\frac{(a)(i)}{1-r}$
		- -		1-7
		$= \frac{\frac{\sqrt{3}}{4}k(1-k)}{3k-3k^2}$ (by (a)(ii)) $= \frac{\sqrt{3}}{12} m^2$	1M	

Solution	Marks	Remarks
5. (a) The required probability = $\left(1 - \frac{1}{10}\right) \left(\frac{1}{2}\right)$	1M	for $\left(1 - \frac{1}{10}\right)P_1$, where $0 < P_1 < 1$
$=\frac{9}{20}$	1A	0.45
	(2)	
(b) (i) The required probability = $\left(1 - \frac{2}{25}\right)\left(\frac{1}{2}\right)$	1M	for $\left(1 - \frac{2}{25}\right)P_2$, where $0 < P_2 < 1$
$=\frac{23}{50}$	1A	0.46
(ii) (1) The required probability $= \left(\frac{2}{3}\right)\left(\frac{9}{20}\right) + \left(\frac{1}{3}\right)\left(\frac{23}{50}\right)$	IM+1M+1A	$1M \text{ for } \left(\frac{2}{3}\right)(a) +$ $1M \text{ for } \left(\frac{1}{3}\right)((b)(i))$ r.t. 0.453
2.4		1 M for $\left(\frac{1}{3}\right)$ $((b)(i))$
$=\frac{34}{75}$	1A	r.t. 0.453
(2) Transportation Transportation Cost Transportation Cost + \$15 Lunc	;h	
Bus and Train \$12 \$27	_	
Train and Train \$15 \$30	_	
The required probability $= 1 - \frac{34}{75}$	2M	for 1-(b)(ii)(1)
$=\frac{41}{75}$	1A	
75 The required probability	IA .	r.t. 0.547
= P(John will spend more than a total of \$30) = P(John will spend more than a total of \$22.5 for the morning trip and lunch) = $\left(\frac{2}{3}\right)\left(\frac{1}{10} + \left(1 - \frac{1}{10}\right)\left(\frac{1}{2}\right)\right) + \left(\frac{1}{3}\right)\left(\frac{2}{25} + \left(1 - \frac{2}{25}\right)\left(\frac{1}{2}\right)\right)$		
$= \frac{1}{15} + \frac{3}{10} + \frac{2}{75} + \frac{23}{150}$	lA+1A	1A for either one correct + 1A for all correct
$=\frac{41}{75}$	1A	r.t. 0.547
The required probability $= P(\text{John will spend more than a total of $30})$ $= P(\text{John will spend more than a total of $22.5 for the morning trip and lunch})$ $= \left(\frac{2}{3}\right)\left(\frac{1}{10}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{10}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{9}{10}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{25}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{25}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{23}{25}\right)\left(\frac{1}{2}\right)$		
$= \frac{1}{30} + \frac{1}{30} + \frac{3}{10} + \frac{1}{75} + \frac{1}{75} + \frac{23}{150}$ $= \frac{41}{75}$	1A+1A	1A for either one correct + 1A for all correct r.t. 0.547
	(9)	Provided by dse.li

	Solution	Marks	Remarks
7. (a) (i)	In $\triangle NPM$ and $\triangle NKP$, $\angle NPM = \angle NKP$ (\angle in alt. segment) $\angle MNP = \angle PNK$ (common angle) $\therefore \triangle NPM \sim \triangle NKP$ (\triangle sum of \triangle) So, $\frac{NP}{NK} = \frac{NM}{NP}$ Thus, we can conclude that $NP^2 = NK \cdot NM$.		[交錯弓形的圓周角]、[弦切角定理 [公共角] [公內角和 [等角] (AA) (equiangular)
	Marking Scheme: Case 1 Any correct proof with correct reasons. Case 2 Any correct proof without reasons.	3 2	
	Case 3 Any one line except the first line and the conclusion.	1	
(ii)	∴ $NP^2 = NK \cdot NM$ and $ON^2 = NK \cdot NM$ (by (a)(i)) ∴ $NP = ON$ ∴ $NP = ON$ ∴ $RS \# OP$ ∴ $\Delta KRM \sim \Delta KON$ (AAA) and $\Delta KMS \sim \Delta KNP$ (AAA)	1	
	$\frac{RM}{ON} = \frac{KM}{KN} \text{and} \frac{MS}{NP} = \frac{KM}{KN}$ $\frac{RM}{ON} = \frac{MS}{NP}$ $RM = MS$	1	
(b) (i)	FM = 2a $MG = 2(p-a)$	1A	for either one correct
	FG = 2a + 2(p - a) $= 2p$	IA	
	$ \begin{array}{l} \therefore x\text{-coordinate of } F \\ = -a \\ x\text{-coordinate of } G \\ = a + 2(p - a) \\ = 2p - a \end{array} $	1A	for either one correct
	FG = (2p - a) - (-a) $= 2p$	1A	
(ii	F = (-a, b) $\therefore FG = 2OP$ (by (b)(i)) and $FG \# OP$ (given) $\therefore O$ is the mid-point of F and Q . Thus, $Q = (a, -b)$	1A	
(ii	i) : x-coordinate of $Q = a = x$ -coordinate of M : $MQ \perp RS$: $RM = MS$ (by (a)(ii)) : $\Delta QMR \cong \Delta QMS$ (SAS) Thus, $QR = QS$ Hence, ΔQRS is an isosceles triangle.	1	