

只限教師參閱

FOR TEACHERS' USE ONLY

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

2001年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2001

數學 試卷一

MATHEMATICS PAPER 1

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.

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2001-CE-MATH 1-1

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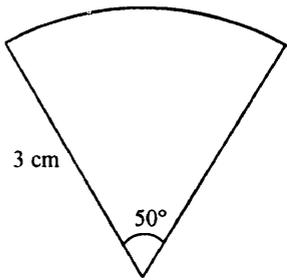
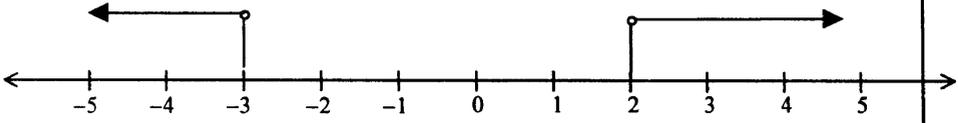
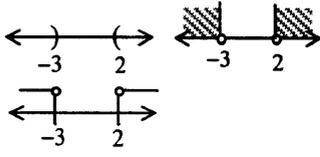
Hong Kong Certificate of Education Examination
Mathematics Paper 1

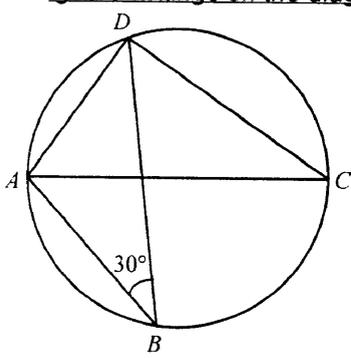
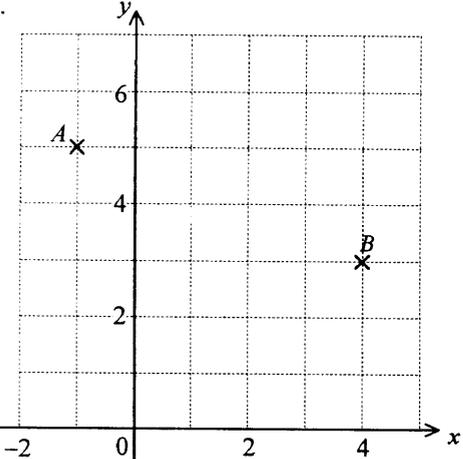
General Marking Instructions

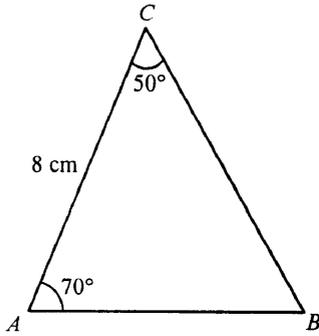
1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

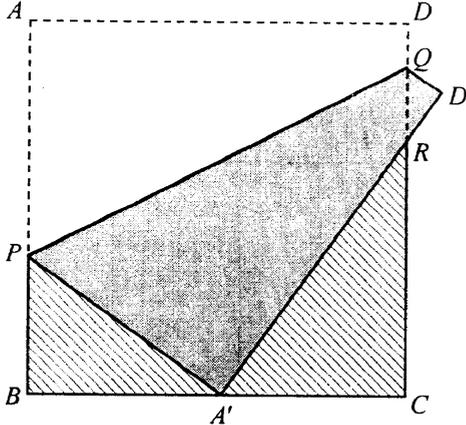
‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
 - a. The symbol $(u-1)$ should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* for the whole paper.
 - b. The symbol $(pp-1)$ should be used to denote 1 mark deducted for *pp*. At most deduct **2 marks** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
 - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
 - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. Marks entered in the Page Total Box should be the NET total scored on that page.
8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’, ‘f.t.’ stands for ‘follow through’ and ‘or equivalent’ means ‘accepting equivalent forms of the equation which has been simplified and without uncollected like terms’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\frac{m^3}{(mn)^2} = \frac{m^3}{m^2 n^2} = \frac{m^{3-2}}{n^2} = \frac{m}{n^2}$	1M 1M 1A ----- (3)	applying $(ab)^p = a^p b^p$ applying $\frac{a^p}{a^q} = a^{p-q}$
2. $\therefore f(2) = 2^3 - 2^2 + 2 - 1 = 5$ \therefore the remainder is 5.	2A 1A	
$\begin{array}{r} 1+1+3 \\ 1-2 \overline{) 1-1+1-1} \\ \underline{1-2} \\ 1+1-1 \\ \underline{1-2} \\ 3-1 \\ \underline{3-6} \\ 5 \end{array}$ <p>\therefore remainder = 5</p>	1A 1A 1A ----- (3)	
3. 	$\text{Perimeter} = [3 + 3 + \frac{50}{360}(2 \times 3 \times \pi)] \text{ (cm)}$	1M+1A 1M for summation (3+3+?) 1A for arc length
$\text{Perimeter} = [3 + 3 + 3 \times \frac{50}{180} \pi] \text{ (cm)}$	1M+1A	1M for summation 1A for arc length
$\approx 8.62 \text{ cm}$ $[6 + \frac{5}{6} \pi] \text{ cm}$	1A ----- (3)	r.t. 8.62, u-1 for missing unit
4. $x^2 + x - 6 > 0$ $(x+3)(x-2) > 0$ $x < -3$ or $x > 2$	1A 1M	For factorization only Distinct roots and ">0" No mark for $x < -3$, $x > 2$ etc.
	1M ----- (3)	or 

Solution	Marks	Remarks
<p>5. <u>Ignore writings on the diagram when marking this question.</u></p>  <p> $\therefore \angle ADC = 90^\circ$ $\angle DCA = 30^\circ$ $\therefore \angle DAC = 180^\circ - 90^\circ - 30^\circ = 60^\circ$ </p> <p>Joint BC, $\angle ABC = 90^\circ$ $\angle DBC = 90^\circ - 30^\circ$ $\angle DAC = \angle DBC = 60^\circ$ </p>	<p>1A 1A 1M 1A</p>	<p>Can be absorbed below Can be absorbed below $u-1$ for missing unit</p>
<p>6. $\therefore y = \frac{1}{2}(x+3)$</p> <p>$\therefore 2y = x+3$</p> <p>$x = 2y - 3$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $y = \frac{1}{2}x + \frac{3}{2}$ </div> <p>In the above linear relation, the coefficient of y is 2.</p>	<p>1A 1A 1M</p>	<p>Can be absorbed below 1M for $\angle DAC = \angle DBC$</p>
<p>Let $x_0 = 2y_0 - 3$. If $y = y_0 + 1$, then $x = 2(y_0 + 1) - 3 = x_0 + 2$.</p>	<p>1M</p>	<p>Putting $y = y_0 + 1$ or substituting two particular values of y which differ by 1</p>
<p>$\therefore x$ will be increased by 2 if y is increased by 1.</p>	<p>1M</p>	<p>(4)</p>
<p>7.</p>  <p>(a) The coordinates of A and B are $(-1, 5)$ and $(4, 3)$ resp.</p> <p>(b) Slope of $AB = \frac{5-3}{-1-4}$</p> <p>Equation of AB :</p> $\frac{y-3}{x-4} = -\frac{2}{5}$ $2x + 5y - 23 = 0$ <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $y = -\frac{2}{5}x + \frac{23}{5}$ </div>	<p>1A 1M 1M 1A</p>	<p>Irrespective of order Accept writing on the diagram</p> <p>or equivalent</p>
<p>8. (a) New price = $(\\$80) \times (1 + 20\%)$ = \$96</p> <p>(b) Peter pays : $(\\$96) \times (1 - 20\%)$ = \$76.8</p>	<p>1A 1A 1M 1A</p>	<p>$u-1$ for missing unit $u-1$ for missing unit</p>

	Solution	Marks	Remarks																		
9.	 <p> $\angle ABC = 60^\circ$ Using sine rule, $\frac{AB}{\sin 50^\circ} = \frac{8 \text{ (cm)}}{\sin 60^\circ}$ $AB \approx 7.0764 \text{ (cm)}$ $\approx 7.08 \text{ cm}$ </p> <p> Area of $\triangle ABC \approx \frac{1}{2}(8)(7.0764)\sin 70^\circ \text{ (cm}^2\text{)}$ $\approx 26.6 \text{ cm}^2$ </p>	1A 1M 1A 1M 1A -----(5)	Can be absorbed below r.t. 7.08 , u-1 for missing unit r.t. 26.6 , u-1 for missing unit																		
10. (a)	<table border="1" data-bbox="351 627 853 1008"> <thead> <tr> <th>Score (x)</th> <th>Frequency</th> <th>Relative Frequency</th> </tr> </thead> <tbody> <tr> <td>$44 \leq x < 52$</td> <td>48</td> <td>3</td> </tr> <tr> <td>$52 \leq x < 60$</td> <td>56</td> <td>9</td> </tr> <tr> <td>$60 \leq x < 68$</td> <td>64</td> <td>15</td> </tr> <tr> <td>$68 \leq x < 76$</td> <td>72</td> <td>11</td> </tr> <tr> <td>$76 \leq x < 84$</td> <td>80</td> <td>2</td> </tr> </tbody> </table>	Score (x)	Frequency	Relative Frequency	$44 \leq x < 52$	48	3	$52 \leq x < 60$	56	9	$60 \leq x < 68$	64	15	$68 \leq x < 76$	72	11	$76 \leq x < 84$	80	2	1A+1A+1A -----(3)	1A for each column
Score (x)	Frequency	Relative Frequency																			
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$60 \leq x < 68$	64	15																			
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$76 \leq x < 84$	80	2																			
(b)	Mean = 64 Standard deviation = 8	1A 1A -----(2)																			
(c)	Standard score = $\frac{76-64}{8}$ $= 1.5$	1M 1A -----(2)																			
(d)	Let her score in the second test be y , then $\frac{y-58}{10} = 1.5$ $y = 73$	1M 1A -----(2)																			

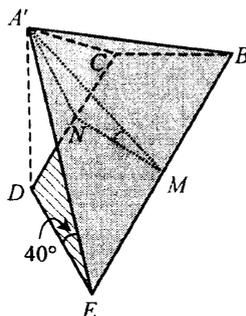
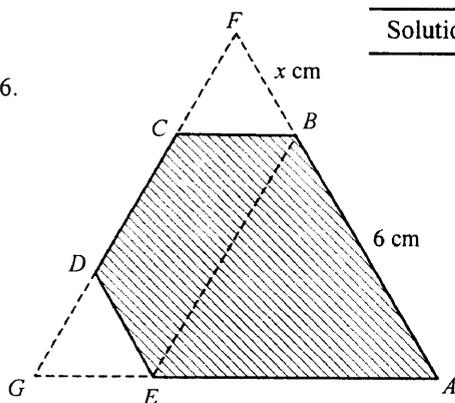
Solution	Marks	Remarks														
<p>11.</p> 																
<p>(a) Since $A'P = x$ (cm),</p> $\therefore (12 - x)^2 + 6^2 = x^2$ $144 - 24x + x^2 + 36 = x^2$ $x = 7.5$	<p>1A 1A 1A</p> <p>-----(3)</p>	<p>Can be absorbed below or written on the diagram</p> <p>$u-1$ for writing $x = 7.5$ cm</p>														
<p>(b) (L1)... In $\Delta s PBA'$ and $A'CR$,</p> <p>(L2)... (i) $\angle PBA' = \angle A'CR = 90^\circ$</p> <p>(L3)... Since $\angle A'PB + 90^\circ + \angle BA'P = 180^\circ$ (\angle sum of Δ)</p> <p>(L4)... and $\angle RA'C + 90^\circ + \angle BA'P = 180^\circ$ (adj. $\angle s$ on st. line)</p> <p>(L5)... \therefore (ii) $\angle A'PB = \angle RA'C$</p> <p>(L6)... Hence $\Delta PBA' \sim \Delta A'CR$ (AAA)</p>		<p>[Δ內角和] [直線上的鄰角] (equiangular), (AA) [等角]</p>														
<table border="1"> <thead> <tr> <th colspan="3">Marking Scheme :</th> </tr> </thead> <tbody> <tr> <td>Case 1</td> <td>Any correct proof with correct reasons.</td> <td>3</td> </tr> <tr> <td rowspan="2">Case 2</td> <td>Any correct proof without reasons.</td> <td>1</td> </tr> <tr> <td>In addition, any relevant correct argument with correct reason - L3, L4 or L6 (at most 1 mark)</td> <td>1</td> </tr> <tr> <td>Case 3</td> <td>Any relevant correct argument with correct reason - L2, L3 or L4 (at most 1 mark)</td> <td>1</td> </tr> </tbody> </table>			Marking Scheme :			Case 1	Any correct proof with correct reasons.	3	Case 2	Any correct proof without reasons.	1	In addition, any relevant correct argument with correct reason - L3, L4 or L6 (at most 1 mark)	1	Case 3	Any relevant correct argument with correct reason - L2, L3 or L4 (at most 1 mark)	1
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Case 3	Any relevant correct argument with correct reason - L2, L3 or L4 (at most 1 mark)	1														
<p>(c) Let $A'R = y$ cm and use the result of (b),</p> $\frac{A'R}{A'C} = \frac{PA'}{PB}$ $\frac{y}{6} = \frac{7.5}{12 - 7.5}$ $y = 10$ <p>i.e. $A'R = 10$ cm</p>	<p>1M 1A</p> <p>-----(2)</p>	<p>$u-1$ for missing unit</p>														

Solution	Marks	Remarks
12. (a) (i) Perimeter of $F_{40} = [10 + (40 - 1) \times 1]$ (cm) $= 49$ cm	1A 1A	$u-1$ for missing unit
(ii) The sum of the perimeters of the 40 figures $= [40 \times \frac{10 + 49}{2}]$ (cm) $[40 \times \frac{2 \times 10 + (40 - 1) \times 1}{2}]$ (cm) $= 1180$ cm	1M 1A -----(4)	$u-1$ for missing unit
(b) (i) Area of $F_2 = [4 \times (\frac{11}{10})^2]$ (cm ²) $= 4.84$ cm ² $\frac{121}{25}$ cm²	1M 1A	Using the square of linear ratio r.t. 4.84, $u-1$ for missing unit
(ii) Area of $F_3 = 4 \times (\frac{12}{10})^2$ (cm ²) = 5.76 (cm ²) \therefore Area of $F_2 -$ Area of $F_1 \neq$ Area of $F_3 -$ Area of F_2 (0.84 cm ² \neq 0.92 cm ²) \therefore the areas of figures F_1, F_2, \dots, F_{40} do not form an arithmetic sequence.	1M 1 -----(4)	

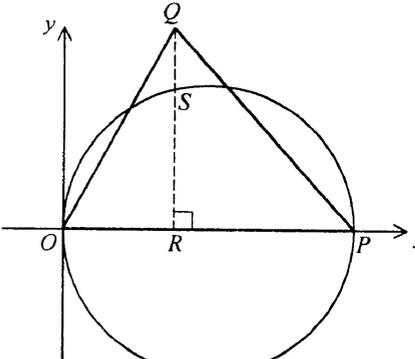
Solution	Marks	Remarks																																
14. (a) (i) <table border="1" style="margin-left: 20px; margin-top: 10px;"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> </tr> <tr> <td>1.05</td> <td>-0.0237</td> </tr> <tr> <td>1.1</td> <td>0.0105</td> </tr> <tr> <td>1.15</td> <td>0.111</td> </tr> </tbody> </table>	x	$f(x)$	1	0	1.05	-0.0237	1.1	0.0105	1.15	0.111	1A	r.t. -0.02, 0.01																						
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(ii) From (i), the root lies in the interval [1.05, 1.1]. Using the method of bisection, <table border="1" style="margin-left: 20px; margin-top: 10px;"> <thead> <tr> <th>a [$f(a) < 0$]</th> <th>b [$f(b) > 0$]</th> <th>$m = \frac{a+b}{2}$</th> <th>$f(m)$</th> </tr> </thead> <tbody> <tr> <td>1.0500</td> <td>1.1000</td> <td>1.0750</td> <td>-0.0144</td> </tr> <tr> <td>1.0750</td> <td>1.1000</td> <td>1.0875</td> <td>-0.0039</td> </tr> <tr> <td>1.0875</td> <td>1.1000</td> <td>1.0938</td> <td>0.0028</td> </tr> <tr> <td>1.0875</td> <td>1.0938</td> <td>1.0907</td> <td>-0.0006</td> </tr> <tr> <td>1.0907</td> <td>1.0938</td> <td>1.0923</td> <td>0.0011</td> </tr> <tr> <td>1.0907</td> <td>1.0923</td> <td>1.0915</td> <td>0.0002</td> </tr> <tr> <td>1.0907</td> <td>1.0915</td> <td></td> <td></td> </tr> </tbody> </table>	a [$f(a) < 0$]	b [$f(b) > 0$]	$m = \frac{a+b}{2}$	$f(m)$	1.0500	1.1000	1.0750	-0.0144	1.0750	1.1000	1.0875	-0.0039	1.0875	1.1000	1.0938	0.0028	1.0875	1.0938	1.0907	-0.0006	1.0907	1.0938	1.0923	0.0011	1.0907	1.0923	1.0915	0.0002	1.0907	1.0915			1M 1M 1M 1M	Can be absorbed in the table below Testing sign of mid-value Choosing the correct interval
a [$f(a) < 0$]	b [$f(b) > 0$]	$m = \frac{a+b}{2}$	$f(m)$																															
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1.0907	1.0915																																	
$\therefore 1.0907 < h < 1.0915$ $x \approx 1.091$ (correct to 3 decimal places)	1A	f.t.																																
------(5)																																		
(b) The given conditions lead to the equation $1000(1+r\%)^4 + 1000(1+r\%)^3 + 1000(1+r\%)^2 + 1000(1+r\%) = 5000$ Let $x = 1+r\%$, then $1000x^4 + 1000x^3 + 1000x^2 + 1000x = 5000$ $x^4 + x^3 + x^2 + x = 5$ $\frac{x(x^4 - 1)}{x - 1} = 5$ $x^5 - x = 5x - 5$ $x^5 - 6x + 5 = 0$ From (a), $x \approx 1.091$ i.e. $r \approx 9.1$	1M+1A 1M 1A	1M for compound interest 1M for sum of the sequence Using the result of (a)(ii) f.t.																																
------(6)																																		

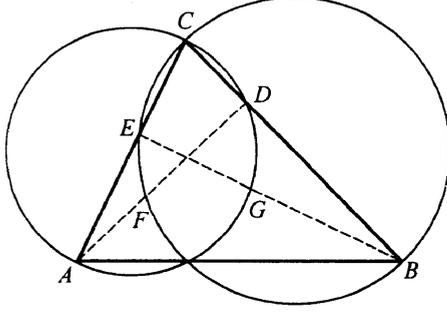
Solution		Marks	Remarks																
15. (a)		<p>1A+1A</p> <p>1A</p> <p>1A</p> <p>-----(4)</p>	<p>For drawing the lines $x=1$, $x=9$, $y=9$ and $5x-2y=15$. 1A for any one being correct. 1A for all.</p> <p>For the shaded region, excluding the boundaries</p> <p>For the dotted line</p>																
(b) (i)		<p>1A</p>	<p>Independent of (a)</p>																
(ii)	<table border="1"> <thead> <tr> <th>No. of tables</th> <th>Number of multiples of 14</th> <th>Number of multiples of 30</th> <th></th> </tr> </thead> <tbody> <tr> <td>Non-multiples of 14</td> <td>14</td> <td>30</td> <td>44</td> </tr> <tr> <td>Single multiples</td> <td>16</td> <td>30</td> <td>46</td> </tr> <tr> <td>Column sum</td> <td>30</td> <td>60</td> <td></td> </tr> </tbody> </table> <p>(I) Required probability = $\frac{46}{90}$ $= \frac{23}{45}$ 0.511 (p)</p> <p>(II) Required probability = $\frac{46}{90} \times \frac{14}{89} + \frac{14}{90} \times \frac{46}{89}$ or $\frac{46}{90} \times \frac{14}{89} \times 2$ $= \frac{644}{4005}$ 0.161</p>	No. of tables	Number of multiples of 14	Number of multiples of 30		Non-multiples of 14	14	30	44	Single multiples	16	30	46	Column sum	30	60		<p>1M</p> <p>1A</p> <p>1M+1M+1A</p> <p>1A</p> <p>-----(7)</p>	<p>r.t. 0.511</p> <p>1M for $p \times \frac{n}{89}$ 1M for multiplying by 2</p> <p>r.t. 0.161</p>
No. of tables	Number of multiples of 14	Number of multiples of 30																	
Non-multiples of 14	14	30	44																
Single multiples	16	30	46																
Column sum	30	60																	

16.



Solution	Marks	Remarks
<p>(a) In the trapezium $BCDE$,</p> <p>height = $x \sin 60^\circ$ (cm)</p> <div style="border: 1px solid black; display: inline-block; padding: 2px;"> $\frac{\sqrt{3}}{2}x$ (cm) </div> <p>$CD = (6-x)$ (cm)</p> <p>$\therefore \frac{6+(6-x)}{2} \times \frac{\sqrt{3}}{2}x = 5\sqrt{3}$</p> <p>$\frac{\sqrt{3}(12-x)x}{4} = 5\sqrt{3}$</p>	<p>1A</p> <p>1A</p>	<p>For either, can be absorbed below</p>
<div style="border: 1px solid black; padding: 5px;"> $2\left(\frac{1}{2}x^2 \sin 60^\circ\right) + 5\sqrt{3} + \frac{1}{2}(6^2) \sin 60^\circ = \frac{1}{2}(x+6)^2 \sin 60^\circ$ $2x^2 + 20 + 36 = (x+6)^2$ $x^2 - 12x + 20 = 0$ $(x-2)(x-10) = 0$ $x = 2 \text{ or } x = 10 \text{ (rejected)}$ </div>	<div style="border: 1px solid black; padding: 2px;"> $1A+1A$ </div>	<p>1A for the area of one triangle 1A for all</p>
<p>(b) (i) $A'D^2 = [6^2 + 2^2 - 2(6)(2) \cos 40^\circ]$ (cm²)</p> <p>≈ 21.6149 (cm²)</p> <p>$A'D \approx 4.6492$ (cm)</p> <p>≈ 4.65 cm</p>	<p>1</p> <p>1A</p> <p>------(4)</p> <p>1M</p> <p>1A</p>	<p>Accept "x = 2 only" u-1 for writing x = 2 cm</p> <p>r.t. 4.65, u-1 for missing unit</p>
<p>(ii) Let M, N be the mid-points of EB and DC respectively, then</p> <p>$A'M = 6 \sin 60^\circ$ (cm) = $3\sqrt{3}$ (cm),</p> <p>$MN = 2 \sin 60^\circ$ (cm) = $\sqrt{3}$ (cm), and</p> <p>$A'N = \sqrt{A'D^2 - DN^2}$</p> <p>$\approx \sqrt{21.6149 - 2^2}$ (cm)</p> <p>$\approx \sqrt{17.6149}$ (cm) 4.197 (cm)</p> <p>The angle between the planes $BCDE$ and $A'BE$ is $\angle A'MN$.</p> <p>$\cos \angle A'MN \approx \frac{(3\sqrt{3})^2 + (\sqrt{3})^2 - 17.6149}{2(3\sqrt{3})(\sqrt{3})}$</p> <p>$\approx 0.6881$</p> <p>$\angle A'MN \approx 46.5^\circ$</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>r.t. 46.5, u-1 for missing unit</p>
<p>(iii) Required volume = $\frac{1}{3}$(area of trapezium $CDEB$)($A'M \sin \angle A'MN$)</p> <p>$\approx \frac{1}{3}(5\sqrt{3})(3\sqrt{3} \sin 46.5^\circ)$ (cm³)</p> <p>≈ 10.9 cm³</p>	<p>1M</p> <p>1A</p> <p>------(7)</p>	<p>r.t. 10.9, u-1 for missing unit</p>

Solution	Marks	Remarks
<p>17. (a) (i)</p> 	1A+1A	Can be absorbed below
<p>Centre = $\left(\frac{p}{2}, 0\right)$, radius = $\frac{p}{2}$</p> <p>Equation of the circle OPS :</p> $\left(\frac{y}{x}\right)\left(\frac{y}{x-p}\right) = -1$ $x(x-p) + y^2 = 0$ $\left(x - \frac{p}{2}\right)^2 + y^2 = \left(\frac{p}{2}\right)^2$ $x^2 + y^2 - px = 0$ $x^2 + y^2 - px + pa - a^2 - b^2 = 0$	2A	
<p>Let the equation of the circle OPS be $x^2 + y^2 + Dx + Ey + F = 0$,</p> <p>then $\begin{cases} F = 0 \\ p^2 + pD + F = 0 \\ a^2 + b^2 + aD + bE + F = 0 \end{cases}$</p> <p>which gives $F = 0$, $D = -p$ and $E = -\frac{a^2 + b^2 - pa}{b}$.</p> <p>$\therefore$ the equation is $x^2 + y^2 - px - \frac{a^2 + b^2 - pa}{b}y = 0$.</p>	1A 1A 1A	for anyone for any two being correct
<p>(a) (ii) $\therefore S$ lies on the circle OPS,</p> <p>$\therefore a^2 + b^2 - pa = 0$</p>	1M	Sub. (a, b) into eqtn. in (a)
<p>Since $OS \perp SP$, $\therefore \left(\frac{b}{a-p}\right)\left(\frac{b}{a}\right) = -1$ or $a^2 + b^2 - pa = 0$.</p>	1M	
<p>$OR = OQ \cos \angle POQ$</p> <p>Using Pythagoras' Theorem,</p> $OS^2 = a^2 + b^2$ $= pa$ $= OP \cdot OR$ $= OP \cdot OQ \cos \angle POQ$	1A 1A 1	
<p>Since $\cos \angle SOR = \frac{OR}{OS} = \frac{OS}{OP}$</p> $OR = OQ \cos \angle POQ$ <p>$\therefore OS^2 = OP \cdot OR$</p> $= OP \cdot OQ \cos \angle POQ$	1M+1A 1A 1	1M for considering $\cos \angle SOR$ or the similar triangles
<p>Using cosine rule, $OP \cdot OQ \cos \angle POQ$</p> $= \frac{1}{2}(OP^2 + OQ^2 - PQ^2)$ $= \frac{1}{2}[(OS^2 + PS^2) + (OR^2 + QR^2) - (QR^2 + PR^2)]$ $= \frac{1}{2}(OS^2 + PS^2 + OR^2 - PR^2)$ $= \frac{1}{2}[OS^2 + (PR^2 + RS^2) + (OS^2 - RS^2) - PR^2]$ $= OS^2$	1A 1M 1A 1	Cosine rule Pythagoras' Theorem
<p>-----(7)</p>		

Solution	Marks	Remarks
<p>(b)</p>  <p>(i) $\because BC$ is a diameter of the circle $BCEF$, $\therefore \angle BEC = 90^\circ$ (\angle in semicircle) i.e. BE is an altitude of $\triangle ABC$.</p> <p>(ii) Since the points C, A, B, G and E are defined analogously as the points O, P, Q, S and R in (a), $\therefore CG^2 = CA \cdot CB \cos \angle ACB$. Similarly, AD is also an altitude of $\triangle ABC$ and $CF^2 = CB \cdot CA \cos \angle ACB$. Hence $CG = CF$.</p>	<p>1</p> <p>1A</p> <p>1A</p> <p>1</p> <p>----- (4)</p>	<p>[半圓上的圓周角] Stating that BC is a diameter or using \angle in semicircle</p>